

Aircraft Performance Calculations: Descent Analysis

CEE 5614
Analysis of Air Transportation Systems

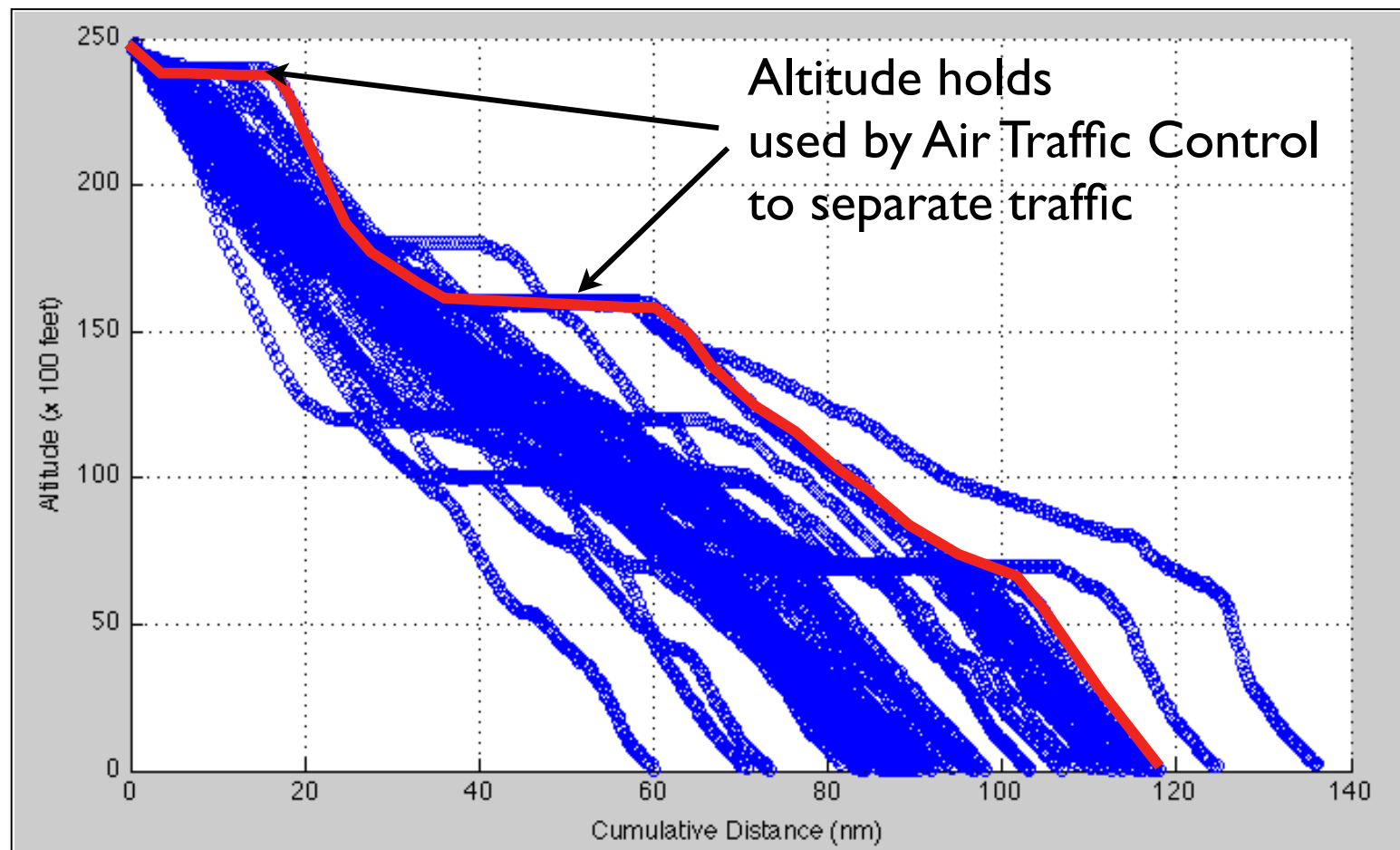
Dr. Antonio A. Trani
Professor

Aircraft Descent Performance

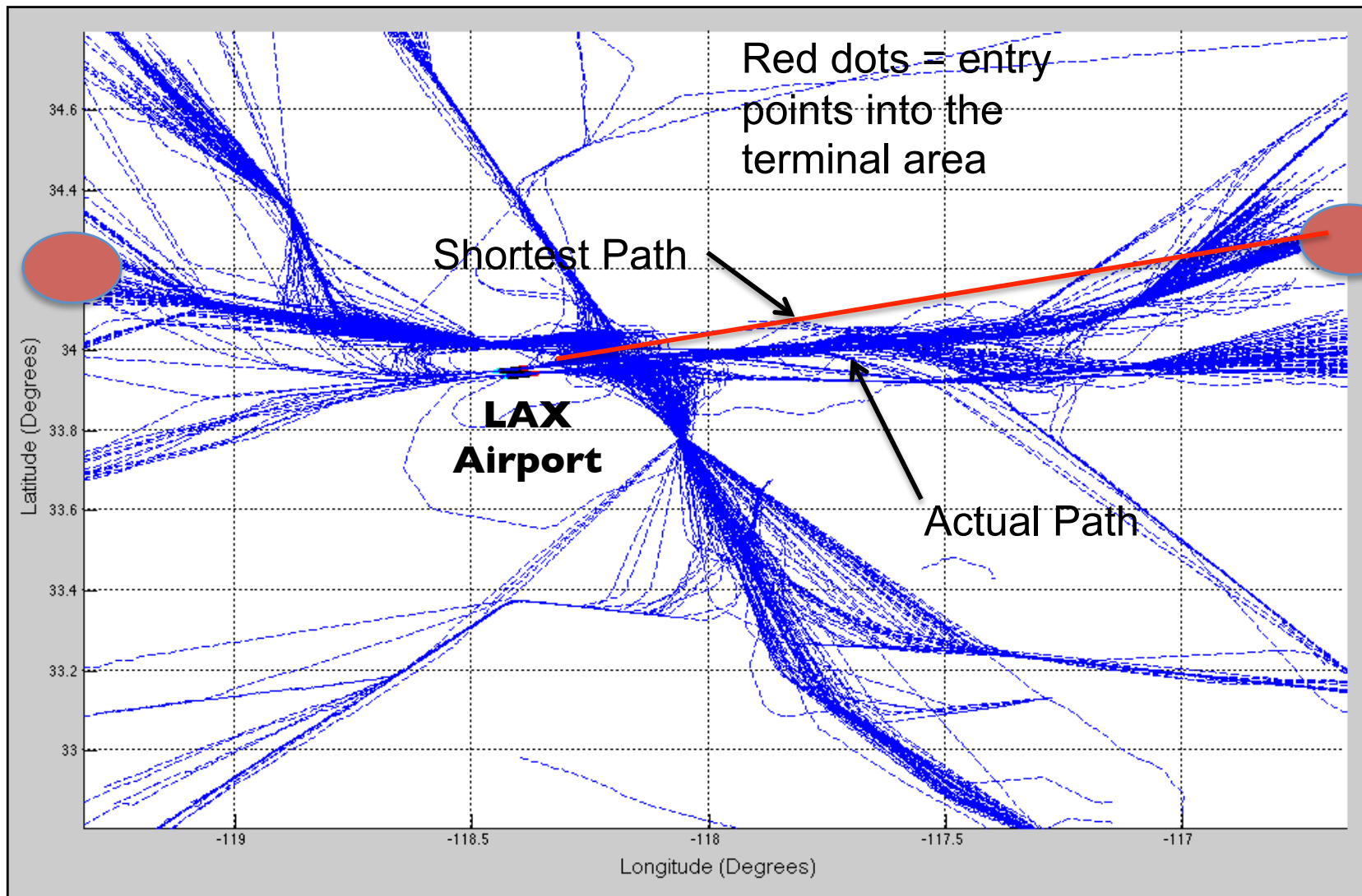
- The top of descent point typically starts 80-120 miles away from the destination airport (depending upon the cruise altitude assigned)
- A descent on commercial transport aircraft is initiated by setting the engine thrust to a very low power condition (i.e., idle thrust)
- The analysis done for climb is now reversed
- Once in the airport terminal area, thrust adjustments are necessary to compensate for altitude holds or flap configuration changes as needed

Sample Descent Profile (LAX Data)

- Shown are sample descent profiles for Boeing 767-300 flying into LAX International airport
- Clearly, not all aircraft fly the same descent profiles

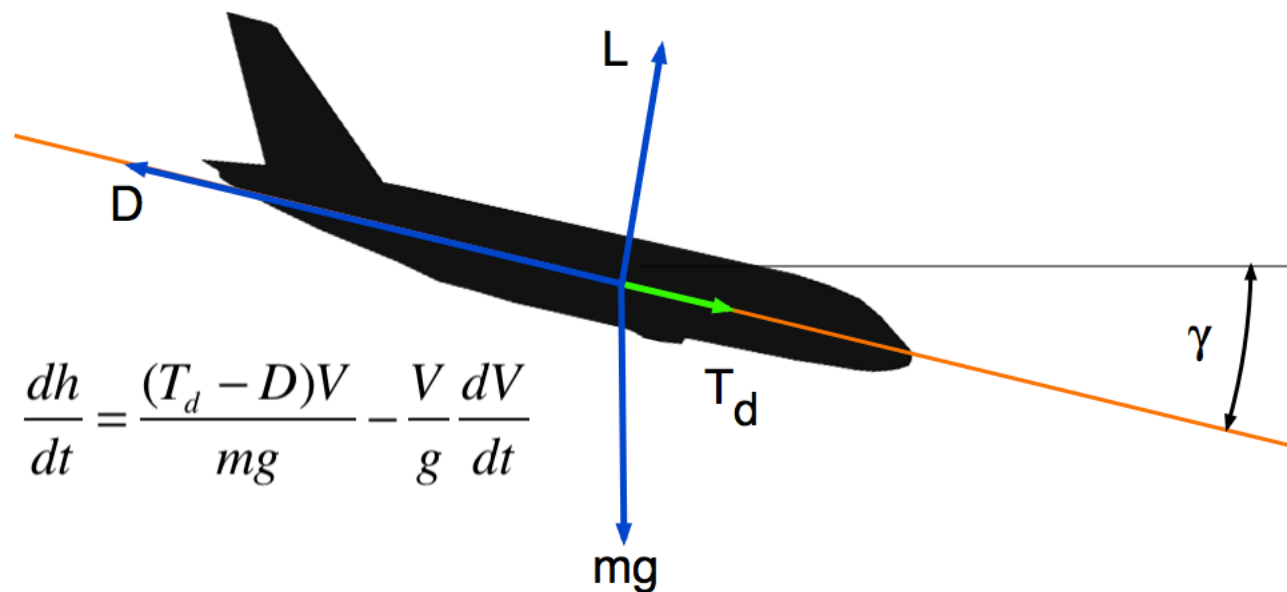


Arrival Flight Profiles into the LAX Airport Terminal Area



Aircraft Descent Performance Analysis

- The pilot reduces thrust to near idle conditions
- If we let the reduced thrust be T_d , then the analysis done for the climb procedure applies to the descent
- The most economical descent would be a continuous descent flown at idle conditions until a point where flaps and landing gear are deployed. At such point adjustments in thrust are required to maintain a safe rate of descent in the final approach



Example - Aircraft Descent Performance

Controlling the Speed Profile

- Very large capacity transport aircraft (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)
- The aircraft descent speed is controlled by the last **two lines of data** in the aircraft data file
- Vdescent is the vector of descent speed for altitudes (altc)
- altc is a vector of altitudes to complete the table function of speed vs altitude
- For example: The aircraft below descends at 300 knots indicated at the top of descent. However below 3000 meters, the aircraft slows down to 250 knots or below.

% Computes the aircraft profile given altitude (Vcas given) – typical for
 % four engine aircraft similar to the Airbus A380 aircraft

```
Vclimb = [210 210 220 230 250 260 290 290 290 290 290 300 300 300 300]; % knots IAS
Vdescent = [ 180 200 250 250 270 300 300 300 300 300 300 300 300 300 300]; % knots IAS
altc = [ 0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000]; % altitude (m)
```


Example - Very Large Capacity Aircraft Data File

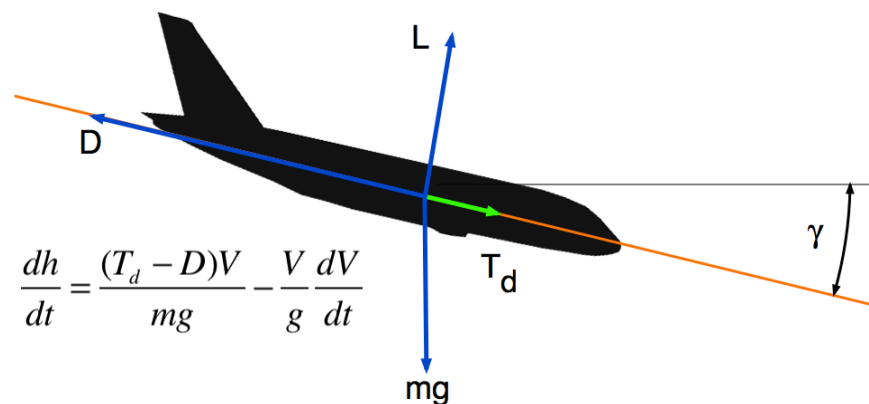
- An aircraft similar in size and performance as the Airbus A380
- Four turbofan engines each developing 34,400 kg (338,000 N) at sea level. **Assume idle thrust produces 1/10 of the full continuous thrust**
- **Top of descent mass is 400,000 kg.**



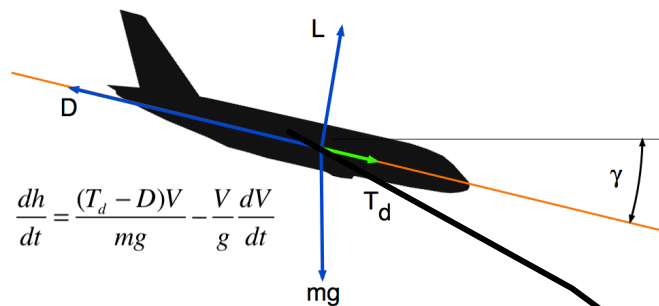
Airbus A380 taxis to the gate at LAX (A.A.Trani)

Descent Analysis Calculation Procedure

- Step 1: Estimate true airspeed using the atmospheric model
- Step 2: Estimate the lift coefficient needed to sustain flight using the basic lift equation
- Step 3: Estimate drag coefficient
- Step 4: Estimate total drag (D)
- Step 5: Estimate the reduced thrust produced by the engines at altitude (T_d)
- Step 6: Find the rate of descent (dh/dt)



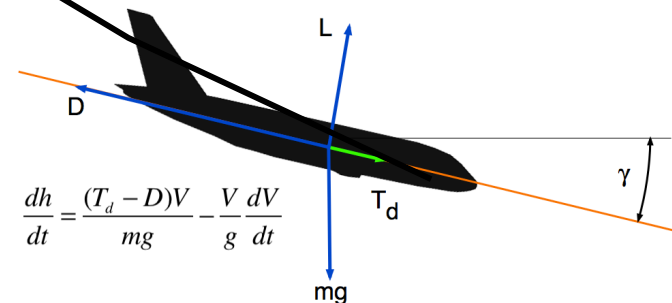
Sample Calculations for Two Aircraft States



$$\frac{dh}{dt} = \frac{(T_d - D)V}{mg} - \frac{V}{g} \frac{dV}{dt}$$

$h = 33,000$ feet
 $v = 300$ knots IAS

$h = 5,000$ feet
 $v = 240$ knots IAS



$$\frac{dh}{dt} = \frac{(T_d - D)V}{mg} - \frac{V}{g} \frac{dV}{dt}$$

Aircraft Descent Performance

33,000 feet (10,061 m) and 300 knots IAS

- Using the standard expression to estimate the true mach number of the aircraft at altitude,

$$M_{true} = \sqrt{5 \left[\left\{ \frac{\rho_0}{\rho} \left(\left[1 + 0.2 \left(\frac{V_{IAS}}{661.5} \right)^2 \right]^{3.5} - 1 \right) + 1 \right\}^{0.286} - 1 \right]}$$

- The true mach number is 0.751, the speed of sound at 10,061 meters is 299.2 m/s and the density of air is 0.41 kg/cu. m.
- The true airspeed (TAS) is 224.65 m/s or 437 knots
- Use the fundamental lift equation to estimate the lift coefficient under the known flight condition

$$L = mg = \frac{1}{2} \rho V^2 S C_l \quad \longrightarrow \quad C_l = \frac{2mg}{\rho V^2 S}$$

Aircraft Descent Performance

33,000 feet and 300 knots IAS

- The lift coefficient needed to maintain steady descent is,

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2 * (400,000)(9.81)}{(0.41)(224.65)^2 (858)} \frac{(kg)(m / s^2)}{(kg / m^3)(m / s)(m^2)} = 0.4421$$

- The lift coefficient is non-dimensional
- The drag coefficient can be calculated using the standard parabolic drag polar model

$$C_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi A Re} = 0.020 + \frac{0.4421^2}{\pi(9.0)(0.84)} = 0.0282$$

- Note that the value of C_{do} is found by interpolation in the table function relating C_{do} and Mach number (C_d is non-dimensional)

Aircraft Climb Performance

33,000 feet and 300 knots IAS

- The total drag is,

$$D = \frac{1}{2} \rho V^2 S C_d = \frac{1}{2} (0.410) (224.65)^2 (858) (0.0282) = 255,660 N$$

- The residual thrust developed is assumed to be 1/10 of the thrust produced at altitude for the given Mach number and altitude
- The calculation of thrust is done in the same way as before. However, the solution is multiplied by 1/10 (assumed idle residual thrust) as shown in the next page

Aircraft Descent Performance

35,000 feet and 300 knots IAS

$$T_{0,M} = T_{0,M=0} - \lambda M_{true}$$

$$T_{0,M} = 338,000 - 175,560 M_{true}$$

$$T_{0,M} = 338,000 - 175,560(0.751)$$

$$T_{0,M} = 206,150 \text{ Newtons}$$

$$T_{h,M} = T_{0,M} \left(\frac{\rho_h}{\rho_0} \right)^m$$

$$T_{h,M} = 206,150 \left(\frac{0.410}{1.225} \right)^{0.96}$$

$$T_{h,M} = 72,087 \text{ Newtons}$$

But thrust is just 1/10 of that produced by the engine, therefore,

$$T_{produced} = \frac{1}{10} T_{h,M} = 7,209 \text{ Newtons}$$

For four engines,

$$T_{total} = n T_{produced} = (4)(7,209) = 28,835 \text{ Newtons}$$

Aircraft Descent Performance

33,000 feet and 300 knots IAS

- The rate of descent of the aircraft can be calculated,

$$\frac{dh}{dt} = \frac{(T_d - D)V}{mg} =$$

$$\frac{dh}{dt} = \frac{(28,835 - 255,660)(224.65)}{400,000(9.81)} \frac{(N - N)}{kg(m / s^2)}$$

$$\frac{dh}{dt} = -12.98 \text{ m / s}$$

- This is equivalent to 779 meters per minute or 2,556 feet per minute
- This descent rate is typical of transport aircraft at the TOD point
- The process is now repeated for state 2

Aircraft Descent Performance

5,000 feet and 240 knots IAS

- The true mach number is 0.390, the speed of sound is 334.3 m/s and the density of air is 1.056 kg/cu. m.
- The true airspeed (TAS) is 130.4 m/s or 253.4 knots
- The lift coefficient needed to maintain flight at 130.4 m/s is,

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2 * (400,000)(9.81)}{(1.056)(130.4)^2 (858)} \frac{(kg)(m / s^2)}{(kg / m^3)(m / s)(m^2)} = 0.5094$$

- The drag coefficient at 5,000 feet and 240 knots (IAS) can be calculated using the standard parabolic drag polar model

$$C_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi A Re} = 0.020 + \frac{0.5094^2}{\pi(9.0)(0.84)} = 0.0309$$

- Note that the value of C_{do} at Mach 0.390 is 0.020

Aircraft Descent Performance

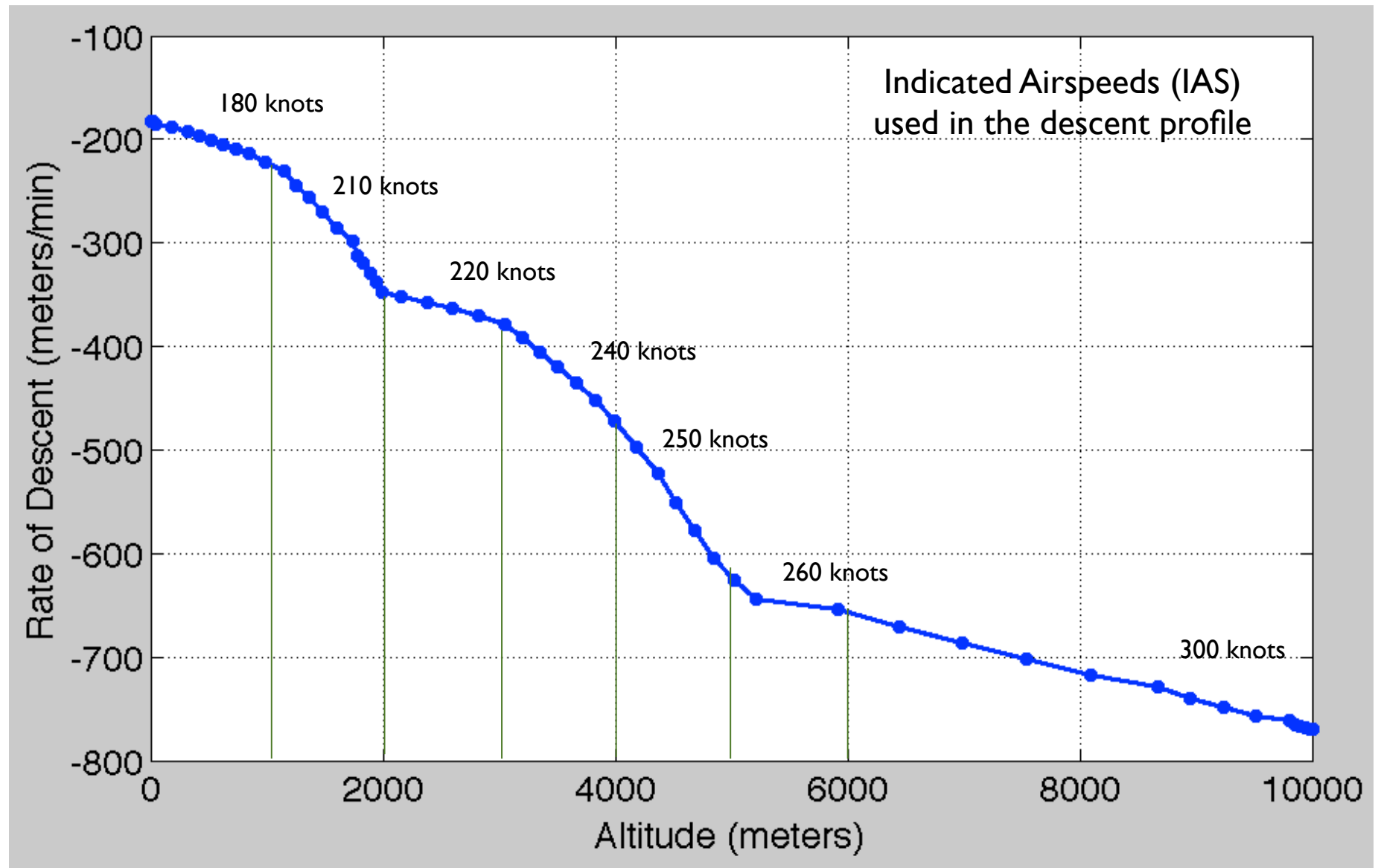
5,000 feet and 240 knots IAS

- The total drag is,

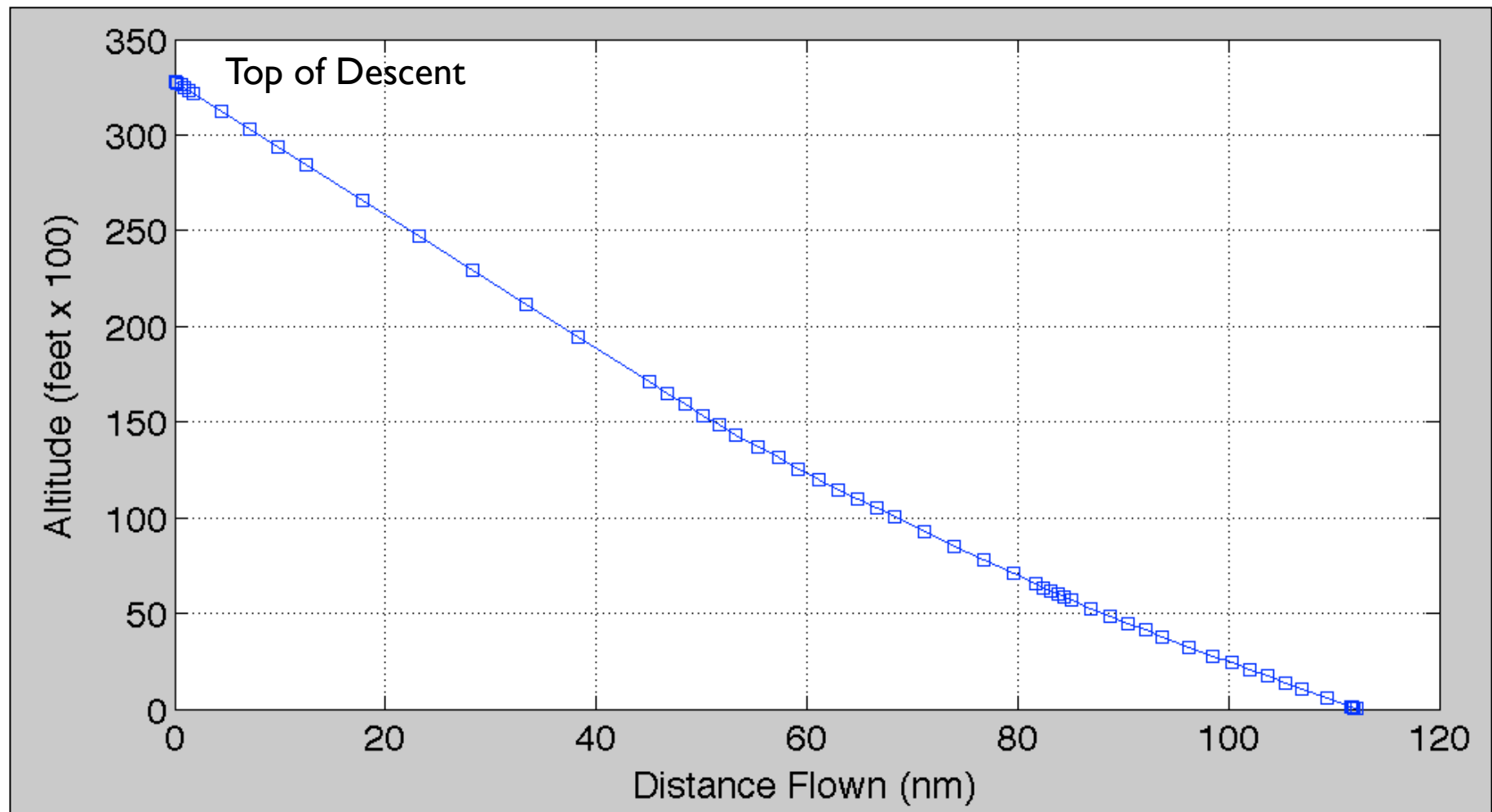
$$D = \frac{1}{2} \rho V^2 S C_d = \frac{1}{2} (1.056) (130.4)^2 (858) (0.0309) = 238,230 N$$

- The rest of the process can be easily computed
- Repeating the same steps outlined here we can derive a rate of descent equation for various altitudes
- The analysis presented in the following pages includes variations in aircraft weight as the the aircraft descends from teh Top of Descent (TOD) point to the airport elevation

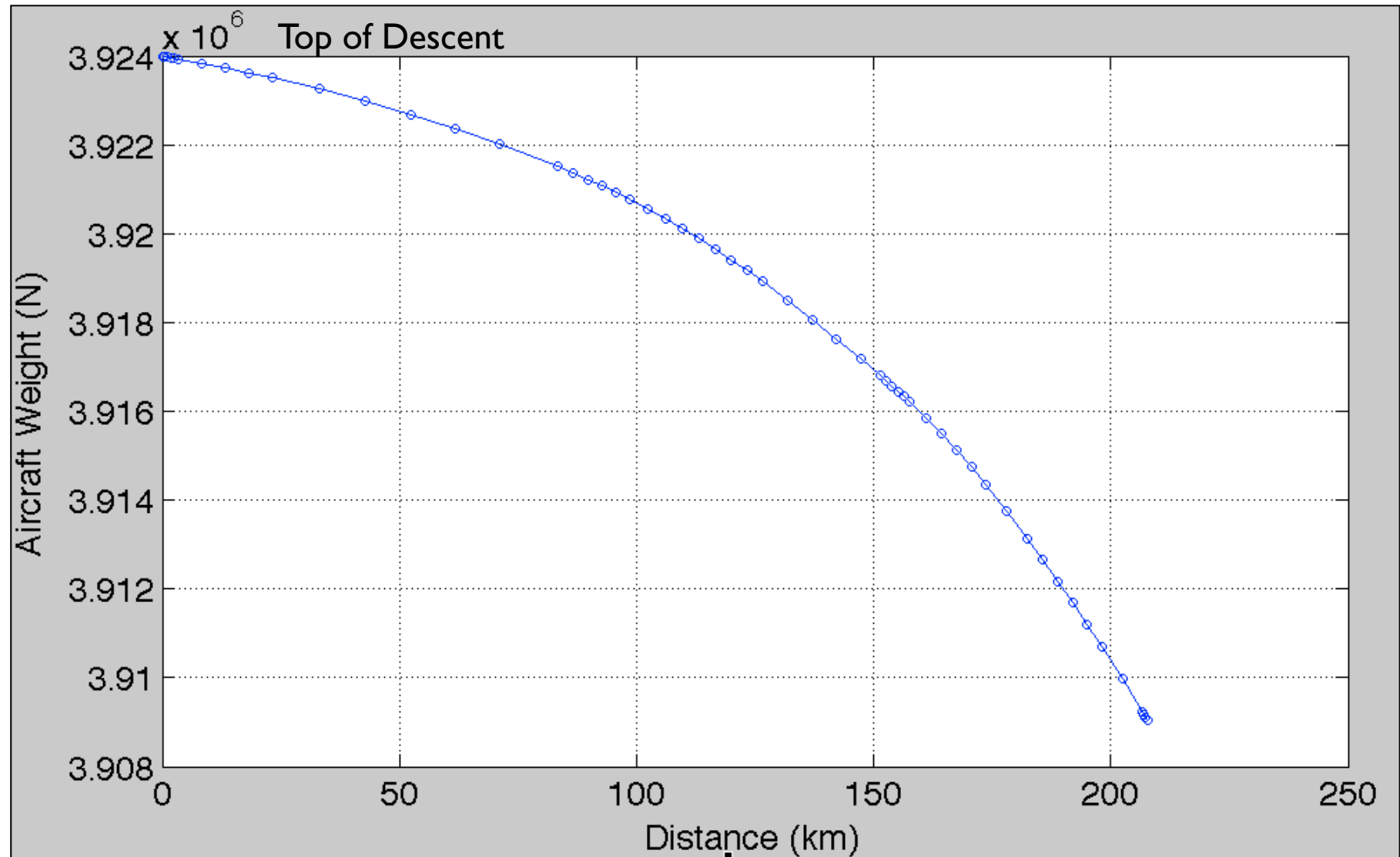
Rate Of Descent Analysis



Descent Profile for Very Large Capacity Aircraft



Descent Profile for Very Large Capacity Aircraft



Observations

- Rate of descent is controlled by the speed profile and the assumed residual thrust
- Typical rates of descent vary from 2600 (at TOD) to 700 feet per minute (at lower altitudes)
- The final approach phase is not well represented in this analysis because flaps and landing gear are usually deployed below 5,000 feet and change the character of the drag coefficient
- The aircraft mass changes by 1,526 kg in the descent. This a relatively small amount of fuel for a vehicle that could carry 182,000 kg of fuel at takeoff
- The aircraft performs a continuous descent from TOD to the airport elevation and travels 112 nautical miles

Implications for Real-world Aviation Operations

- The performance of the aircraft has profound effects in real-world flight planning applications
- Obstacle accountability analysis
 - Obstacle clearance procedures in the terminal area (before landing)
- Current terminal operations do not support continuous descent approaches but for a few, isolated flights
- Continuous descent profiles are expected to save fuel and time once NextGen technologies are implemented

Use of Matlab Code

- The previous analysis has been done using the **UnrestrictedDescentAnalysis.m** program
- This main file integrates numerically the equations of motion of the aircraft
- Four state variables:
 - Altitude ($y(1)$)
 - Aircraft weight ($y(2)$)
 - Distance traveled along path ($y(3)$)
 - Distance traveled along the plane of the earth ($y(4)$)
- The initial conditions of the states are specified in the file under line 54
- `yN = [h_TOD Mass_init 0 0];` % Vector of initial values of state variables

UnrestrictedDescentAnalysis.m

- Main program to execute the descent analysis
- Employs Matlab Ordinary Differential Equation solver (ODE15s)
- Function Calls:
 - **fdescent_06.m** - function that contains the equations of motion of the aircraft in the descent phase
 - **densityAltitudeoffISA.m** - function to estimate the atmospheric conditions for both ISA and non-ISA conditions
 - **drag03.m** - function to estimate the aircraft drag at any altitude (h) and Mach number (M)
 - **thrust_calculation.m** - function to estimate the thrust produced by the engine for any Mach number and altitude (h) condition

UnrestrictedDescentAnalysis.m

- Inputs to the Program
- Aircraft file to be used in analysis (line 36)

```

33 % Enter aircraft file desired – reads a file with aircraft characteristics
34 % eclipse500New_class
35 % regionalJetDescent
36 – AirbusA380_class % aircraft file used

```

- Speed profile. Descent speed profile specified as a table function in the aircraft file (lines 30 and 31 in aircraft file)

```

29 – Vclimb = [210 210 220 230 250 260 290 290 290 290 290 300 300 300 300]; % knots IAS
30 – Vdescent = [180 200 250 250 270 300 300 300 300 300 300 300 300 300 300]; % knots IAS
31 – altc = [ 0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000]; %

```

- Initial aircraft states (lines 38-41 in main program)
 - altitude, mass, distance traveled along path and distance traveled along a flat earth

```

38 – h_TOD = 10000; % initial altitude (m) – at Top of Descent Point
39 – mass_TOD = 400000; % mass at TOD point (kg)
40 – rhos = 1.225; % sea level density (kg/m-m-m)
41 – deltaTemp = 0; % ISA + deltaTemp conditions for analysis (de

```

UnrestrictedDescentAnalysis.m

- **Outputs of the Program**
 - Results of the four aircraft state variables in the climb profile (altitude, mass, distance traveled along path and distance along flat earth)
 - Plots of state variables vs. time
 - Plot of state variables vs. distance

