Example of Aircraft Climb and Maneuvering Performance

CEE 5614
Analysis of Air Transportation Systems

Dr. Antonio A. Trani
Professor
Example - Aircraft Climb Performance

- Aircraft maneuvering performance analysis (initial climb profile)
- Use the vehicle characteristics for the very large capacity transport aircraft in the Matlab files for CEE 5614 to solve this problem (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)
Departure Procedure (Sample Airport)

source: Google Earth (2009)
Problem Description

• The aircraft climbs at 180 knots (IAS) with 10 degrees of flaps down after departure as shown in the diagram.

• As the aircraft climbs past the runway threshold at an altitude of 100 meters (and with the landing gear already retracted), the aircraft loses the inner starboard engine.

• The pilot flies the departure profile shown in the figure to avoid terrain around the field.

• With the wing flaps deflected, the aircraft drag polar is modified with an extra term as follows:
Modified Drag Function

- Drag polar with an added term to account for drag due to flaps:

\[ C_d = C_{do} + C_{di} + C_{df} \]

\[ C_d = C_{do}(M) + \frac{C_l^2}{\pi A Re} + C_{df} \]

where:

- \( C_d \) = total drag coefficient (dim)
- \( C_{do} \) = zero lift drag coefficient (lift independent) (dim)
- \( C_{di} \) = induced drag coefficient (lift dependent) (dim)
- \( C_{df} \) = drag coefficient due to flaps (dim)
- \( AR \) = wing aspect ratio (dim)
- \( e \) = efficiency factor (dim)
- \( C_l \) = lift coefficient (dim)
Example - Aircraft Climb Performance Data File

- Very large capacity transport aircraft (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)

% Aircraft file to support other computational modules
% Aircraft = Similar to Airbus A380 (heavy transport)

global A e S neng tsfc macht Cdoct mass thrust_table mach_table lapse_rate_factor Vclimb altc g

S = 858; % wing area (square m)
A = 9.0; % Wings aspect ratio
e = 0.84; % Oswald's efficiency factor
g = 9.81;
neng = 4; % Number of engines
tsfc = 1.6e-4; % thrust specific fuel consumption (N/N/s)
mass = 540000; % mass at operating point (kg)

% Drag characteristics – CDO function (zero lift drag function)
Cdoct = [0.020 0.020 0.0204 0.022 0.037 0.038 0.040];
mach = [0.0 0.75 0.80 0.85 0.90 0.95 1.00];

% Thrust parameters for Eclipse aircraft (at sea level)
thrust_table = [338000 180000]; % Thrust limits (sea level in Newtons)
mach_table = [0.0 0.9]; % mach number limits to bound thrust
lapse_rate_factor = 0.96; % thrust lapse rate factor

% Computes the aircraft profile given altitude (Vcas given) – typical for
% four engine aircraft similar to the Airbus A380 aircraft
Vclimb = [180 200 220 250 270 280 290 300 300 300 300 300 300 300 300 300]; % knots IAS
Vdescent = [3 300 300 300 300 300 300 300 300 300 300 300 300 300 300 300]; % knots IAS
altc = [0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000]; % altitude (meters)
Effects of Wing Flaps on $C_{df}$

- The following table provides values of the drag coefficient due to flaps for various flap angles.
- Note that flap setting are discrete and unique to each aircraft.

<table>
<thead>
<tr>
<th>Flap Angle (degrees)</th>
<th>$C_{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.006</td>
</tr>
<tr>
<td>5</td>
<td>0.010</td>
</tr>
<tr>
<td>10</td>
<td>0.015</td>
</tr>
<tr>
<td>20</td>
<td>0.023</td>
</tr>
</tbody>
</table>
Example - Very Large Capacity Aircraft Data File

- An aircraft similar in size and performance as the Airbus A380
- Four turbofan engines each developing 34,400 kg (338,000 N) at sea level
- Takeoff mass is 450,000 kg. for this problem
More Problem Description

- Find if the aircraft clears the critical obstacle (Hill 3) with the required 305 meter (1000 ft.) margin of safety

- In your calculations, consider the degradation in the climb rate induced by the 45 degree turn maneuver

- During the climb maneuver the aircraft climbs at 180 knots (IAS) with 10 degrees of flaps
Example - Aircraft Climb Performance
Picture the Situation

- For this analysis we will ignore the second term in the Right Hand Side (RHS) of the differential equation (acceleration term)

- This simulates that the pilot is interested in climbing as fast as possible and thus using all the engine thrust to climb

\[
\frac{dh}{dt} = \frac{(T - D)V}{mg} - \frac{V}{g} \frac{dV}{dt}
\]
Calculation of Performance at Key points in the Climb Profile

- The analysis assumes the aircraft is studied as a point mass system. We evaluate the performance at discrete points.

- The analysis will estimate the climb performance at four key points in the initial climb maneuver. This will allow us to check if the aircraft clears the obstacle with a 1,000 foot clearance needed by Federal Aviation Regulations (FAR Part 25).

Runway
Elevation = 1,300 m
Calculation Procedure

• Step 1: Estimate true airspeed using atmospheric model
• Step 2: Estimate the lift coefficient needed to sustain flight using the basic lift equation
• Step 3: Estimate drag coefficient
• Step 4: Estimate total drag (D)
• Step 5: Estimate the thrust produced by the engines at altitude (T)
• Step 6: Find the rate of climb (dh/dt)
Aircraft Climb Performance
1300 meters (above sea level) and 180 knots IAS
(Engine Failure Point - Point A)

- Using the standard expression to estimate the true mach number of the aircraft at altitude,
- The true mach number is 0.2896
- The true airspeed (TAS) is 97.1 m/s or 188.6 knots

\[
C_l = \frac{2mg}{\rho V^2 S} = \frac{2 \times (450,000)(9.81)}{(1.0797)(97.1)^2(858)} \frac{(kg)(m / s^2)}{(kg / m^3)(m / s)(m^2)} = 1.011
\]

\[
C_d = C_{do} + C_{di} + C_{df} = 0.020 + \frac{1.011^2}{\pi(9.0)(0.84)} + .015 = 0.078
\]
Aircraft Climb Performance
1300 meters (above sea level) and 180 knots IAS
(Engine Failure Point - Point A)

- The total drag is,

\[ D = \frac{1}{2} \rho V^2 S C_d = \frac{1}{2} (1.0797)(97.1)^2(858)(0.078) = 340,743 N \]

- The calculated drag has units of Newtons (verify by yourself)

- The thrust produced by 3 engines in the very large capacity transport is estimated using the simple linear model

\[ T_{0,M} = T_{0,M=0} - \lambda M_{true} \]

\[ T_{h,M} = T_{0,M} \left( \frac{\rho_h}{\rho_0} \right)^m \]
Aircraft Climb Performance
1300 meters (above sea level) and 180 knots IAS
(Engine Failure Point - Point A)

- The thrust developed by 3 remaining engines is calculated to be:
  - \( T_{\text{total}} = 762,801 \text{ N} \)
- The rate of climb is then,
  \[
  \frac{dh}{dt} = \frac{(T_{\text{total}} - D)V}{mg} = \frac{(762,801 - 340,743)(97.1)}{450,000(9.81)} \frac{(N - N)}{kg(m / s^2)}
  \]
  \[
  \frac{dh}{dt} = 9.28 \text{ m / s}
  \]
Aircraft Climb Performance
1300 meters (above sea level) and 180 knots IAS
(Engine Failure Point - Point A)

- The first segment is 2 nm long
- 2 nm is 3,704 meters
- The aircraft takes 38.2 seconds to cover the 2 nm.
- The aircraft gains 355 meters in 2nm
- The altitude reached at point B is 1,655 meters
Aircraft Climb Performance
1655 meters (above sea level) and 180 knots IAS (Turning - Point B)

- The 45-degree turn is performed in 15 seconds at 3 degrees per second
- Therefore, the aircraft gains another 114 meters while turning 45 degrees
- At the end of the 45-degree turn, the final altitude is estimated to be 1,769 meters

Rate of climb while turning is 16.4% lower than with wings leveled

Runway Elevation = 1,300 m

h_b = 1655 m

Elevation 1500 m

D

45 degree standard turn

1.0 miles

Hill1

h_c = 1759 m

Hill2

Hill3

2.0 miles

A
Aircraft Climb Performance
1655 meters (above sea level) and 180 knots IAS
(Turning Maneuver at Point B)

- The lift coefficient to maintain flight while turning and banking at 27.7 degrees is,

\[ C_l = \frac{2mg}{\rho V^2 S \cos(\phi)} = \frac{2 \times (450,000)(9.81)}{(1.024)(98.4)^2(858) \cos(27.7)} = 1.152 \text{ dim} \]

- The drag coefficient while turning and banking at 27.7 degrees is,

\[ C_d = C_{do} + C_{di} + C_{df} = 0.020 + \frac{1.152^2}{\pi(9.0)(0.84)} + 0.015 = 0.091 \]

- Note that this value is substantially higher than before (i.e., no turn).
Aircraft Climb Performance
1769 meters (above sea level) and 180 knots IAS
(End of Turning Maneuver at Point C)

- The aircraft is still configured with 10 degrees of flaps and traveling at 180 knots indicated airspeed
- This translates into 98.8 m/s (true airspeed)
- The aircraft returns to wings level flight with a climb rate of 8.62 m/s
- The final altitude at Point D is estimated to be 1,930 meters
- The hill has an elevation of 1,500 meters above sea level
- The aircraft clears the hill by 430 meters (1930-1500 m)
- Therefore the obstacle clearance accountability area is satisfied
Aircraft Climb Performance
Final Note

- The following diagram summarizes the calculations done for the engine failure condition.
Rate Of Climb Analysis

- Repeating the steps shown the previous pages we can estimate the speed in the initial climb that will maximize the rate of climb.

![Graph showing rate of climb analysis]

220 knots best speed for initial climb
Observations

• Rate of climb is sensitive to bank angle

• The analysis of obstacle accountability areas is critical in real airline operations

• The aircraft takeoff weight could be limited by the engine out condition in climbing above obstacles around the airport and not the runway length (WAT curves in aircraft performance)

• This will be illustrated in the homework

• A clear example of initial climb limitations is illustrated in the incident on United Flight 863 departing SFO. Read the article at:

  • http://www.post-gazette.com/headlines/19990320jumbojet4.asp