Analysis of Air Transportation Systems

Fundamentals of Aircraft Performance (2)

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Fall 2006
Blacksburg

Example of Aircraft Climb Performance

The following example gives an idea of the typical procedures in the estimation of the aircraft climbing performance. Assume that a heavy transport aircraft has drag polar of the form,

$$C_D = C_{Do} + \frac{C_L^2}{\pi A R e}$$

where: AR = 8.0, e =0.87 and C_{DO} (the zero lift drag coefficient) varies according to true airspeed (TAS) according to the following table:

Mach Number	**C_{DO} (nondimensional)**
0.0 to 0.75	0.0180
0.80	0.0192

Mach Number	C_{DO} (nondimensional)
0.85	0.023
0.90	0.037
0.95	0.038
1.00	0.040

The engine manufacturer supplies you with the following data for the engines of this aircraft:

True Airspeed (m/s)	Sea Level Thrust (Newtons)
0	250,000
300	150,000

For simplicity assume that thrust variations follow a linear behavior between 0 and 300 m/s. The thrust also decreases with altitude according to the following simple thrust lapse rate equation,

$$T_{altitude} = T_{Sea\ Level}\ (\rho/\rho_o)^{.90} \qquad (1)$$

where ρ is the density at altitude h and ρ_o is the sea level standard density value (1.225 kg./ m^3).

The aircraft in question has four engines and has a wing area of 525 m^2.

A) Calculate the thrust and drag for this vehicle while climbing from sea level to 10,000 m. under standard atmospheric conditions at a constant indicated airspeed of 280 knots. Simulate the climb performance equation of motion assuming that the takeoff weight is 360,000 kg.

B) Estimate the rate of climb of the vehicle if the fuel consumption is approximately proportional to the thrust as follows,

F_c = TSFC (T)

where TSFC = 2.1×10^{-5} (Kg/second)/Newton

C) Find the time to climb and the fuel consumed to 10,000 m.

D) What is the approximate distance traveled to reach 10,000 m. altitude?

Solution

- The process to estimate the complete climb profile for the aircraft is best done in a computer. There are numerous computations that need to be repeated for each altitude.
- A suitable algorithm to solve the equations of motion of the aircraft over time is presented in the following pages.

Computational Algorithm Flowchart

Initial Aircraft States
Mass (W_o), Altitude (h_o)
and Distance Traveled (S_o)

Given: Speed profile
Typically V as a
function of Altitude (h)

1) Compute Atmospheric values
for a given altitude
(density, speed of sound, etc.)

From Table

2) Compute lift coefficient (C_L)
C_L = f (mass, density, wing area, etc.)

Equation (31)

3) Compute the drag coefficient (C_D)
C_D = f (C_L, Mach, AR, e, etc.)

Equation (30)

Computational Algorithm (contd.)

Iterate

4) Compute total drag (D)

$D = f(C_D, V, S, \text{density})$

Equation (29)

5) Compute the thrust produced (T)

$T = f(\text{Mach and density})$

Equation (33)

6) Compute the rate of climb (dh/dt)

$dh/dt = f(T, D, V)$

Equation (32)

7) Compute fuel burn (dW/dt)

$dW/dt = -\,TSFC\,(T)$

ΔT is a suitable step size (say 5 seconds)

Computational Algorithm (contd.)

8) Compute the distance traveled (*dS/dt*)

$$dS/dt = V * \Delta T$$

9) Update new aircraft altitude (*h*)

$$h_t = h_{t-1} + \Delta T\,(dh/dt)$$

10) Update new aircraft mass (*W*)

$$W_t = W_{t-1} + \Delta T\,(dW/dt)$$

11) Update new aircraft distance (*S*)

$$S_t = S_{t-1} + \Delta T\,(dS/dt)$$

***ΔT* is a suitable step size (say 5 seconds)**

Solution Using Numeric Software Packages

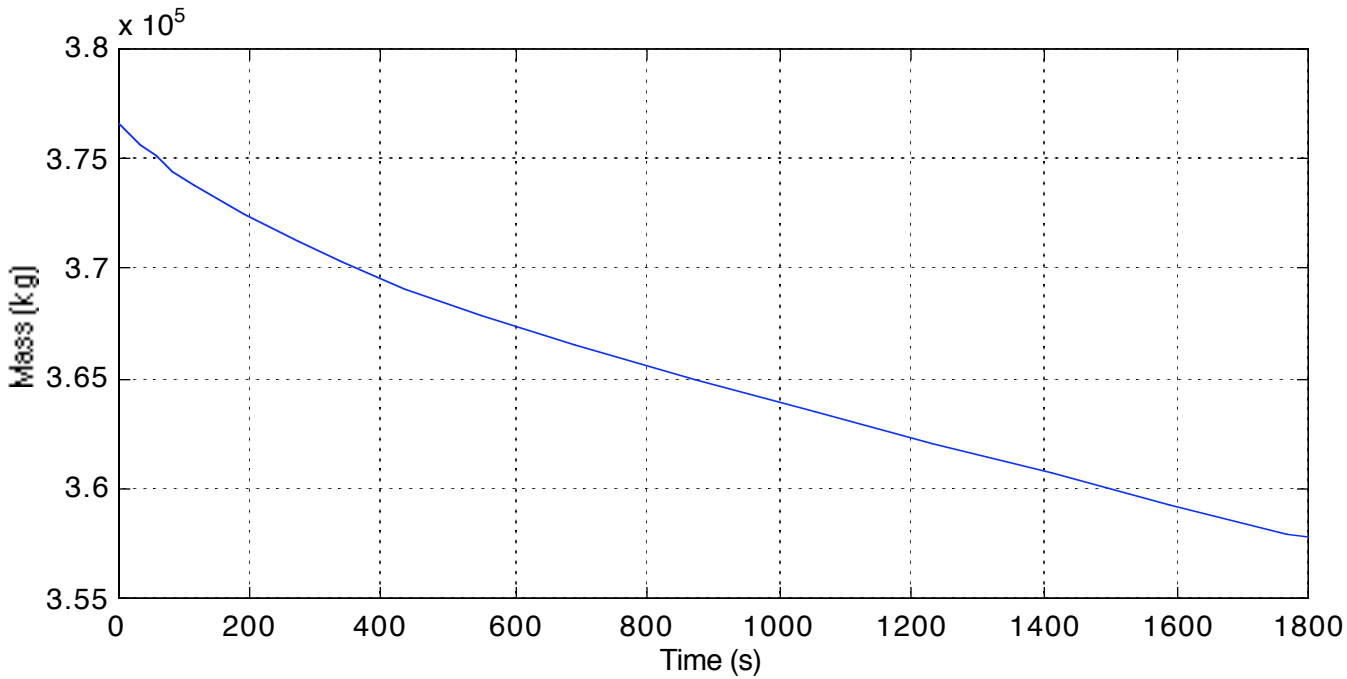
- Several engineering packages can perform these computations quickly and easily (Matlab, Mathematica, Mathcad, etc.)
- All of them have differential equation solvers that can be used in this analysis
- The source code to solve this problem is presented in Matlab at the course web site: http://128.173.204.63/courses/cee5614/syllabus_ce_5614.html
- The process can also be implemented in a standard Spreadsheet application like Excel

Computational Results

- Note that the aircraft takes about 25 minutes to climb to 10,000 m. and that the rate of climb is near zero at that altitude.
- The time solution for fuel consumption indicates that this aircraft consumes about 20 metric tons in the climb segment as shown in the figure.
- Note that this amount is reasonable considering that a the four engine aircraft carries up to 175 metric tons of fuel.

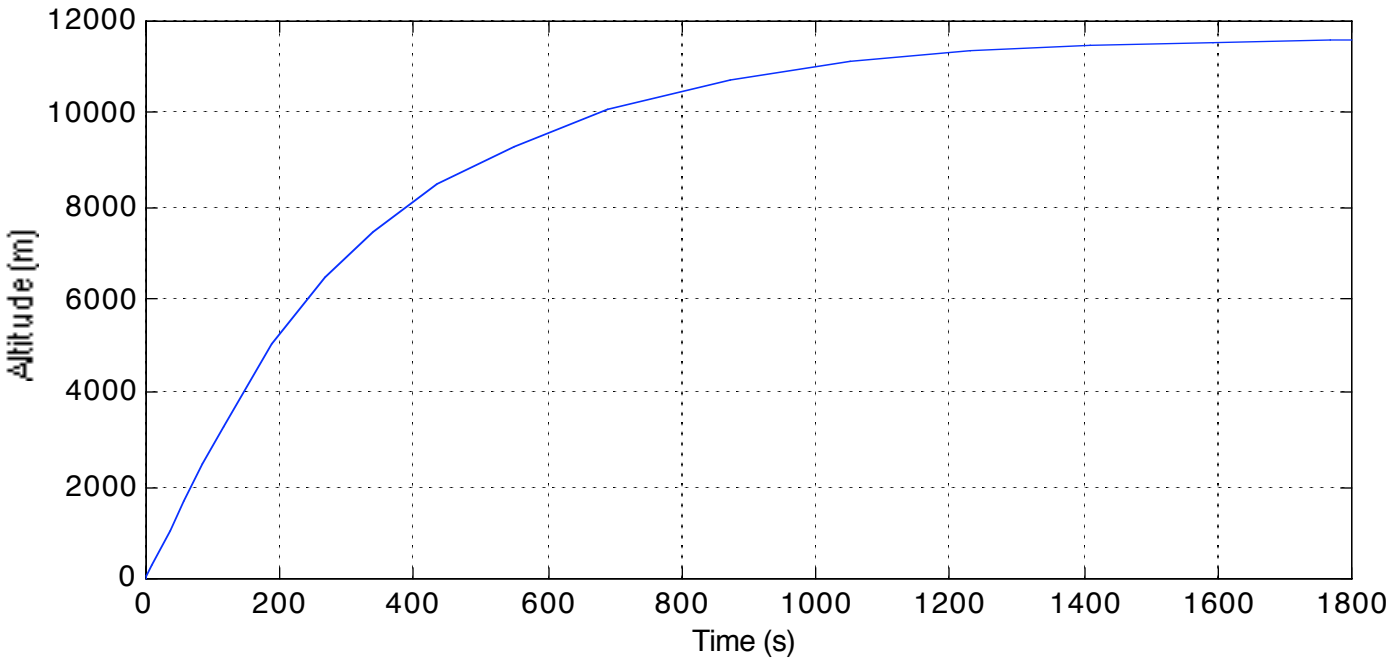
Climb Performance Estimation Results

The diagram illustrates the changes to aircraft mass as a function of time (dW/dt) for the hypothetical four-engine transport aircraft modeled



Climb Performance Estimation Results

The diagram illustrates the changes to aircraft altitude as a function of time for the hypothetical four-engine transport aircraft modeled

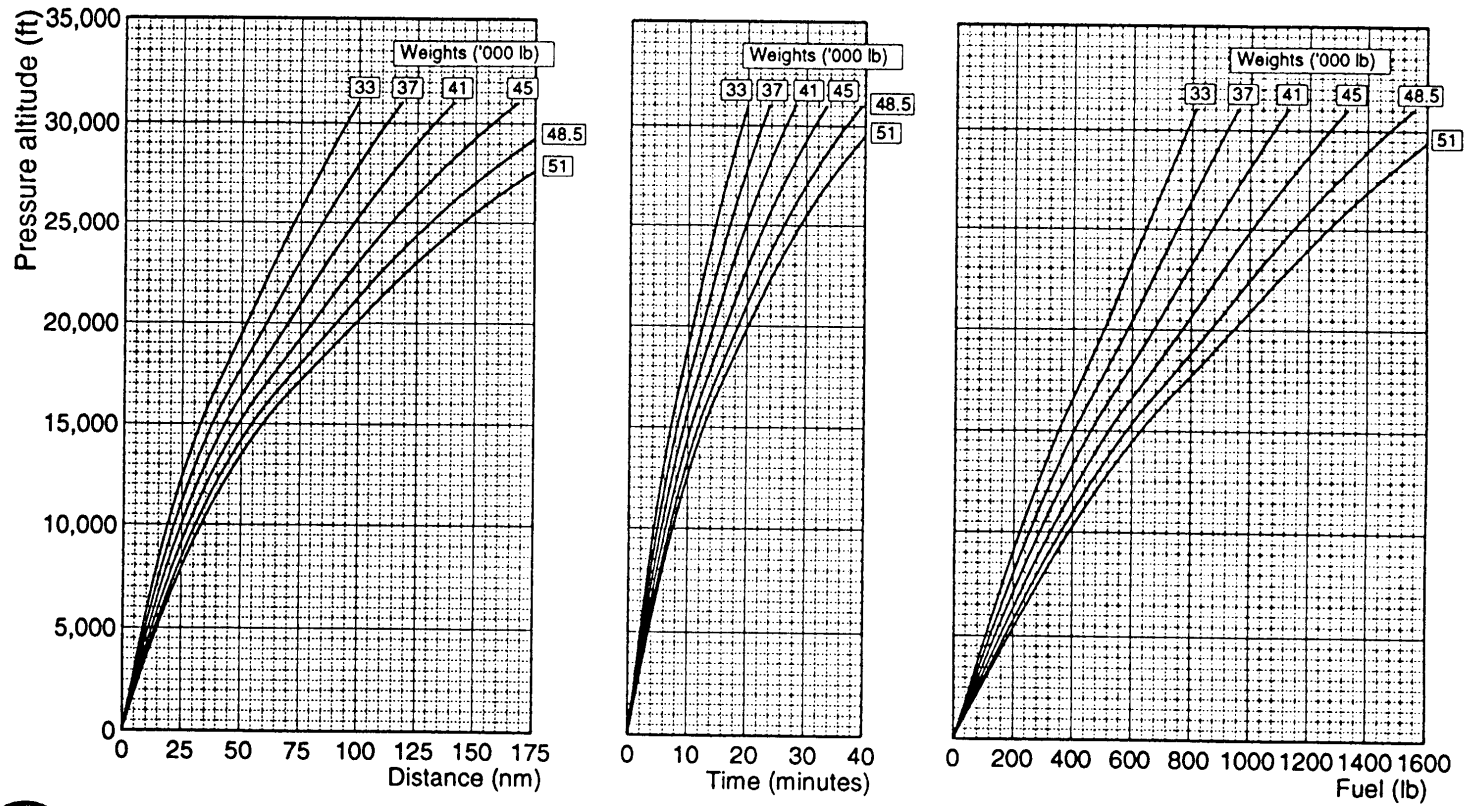


Climb Performance Presentation Charts

The previous discussion presented the foundations of the theoretical climb performance. In practice aircraft manufacturers and airlines present climb performance in graphical and tabular format. The figure below presents climb information for a Swedish-made Saab 2000 - a commuter aircraft powered by two turbo-propeller driven engines.

Sample Climb Estimation Presentation Charts

ISA + 15^0 C
240 Knots IAS / 0.5 Mach
Anti-Ice Off

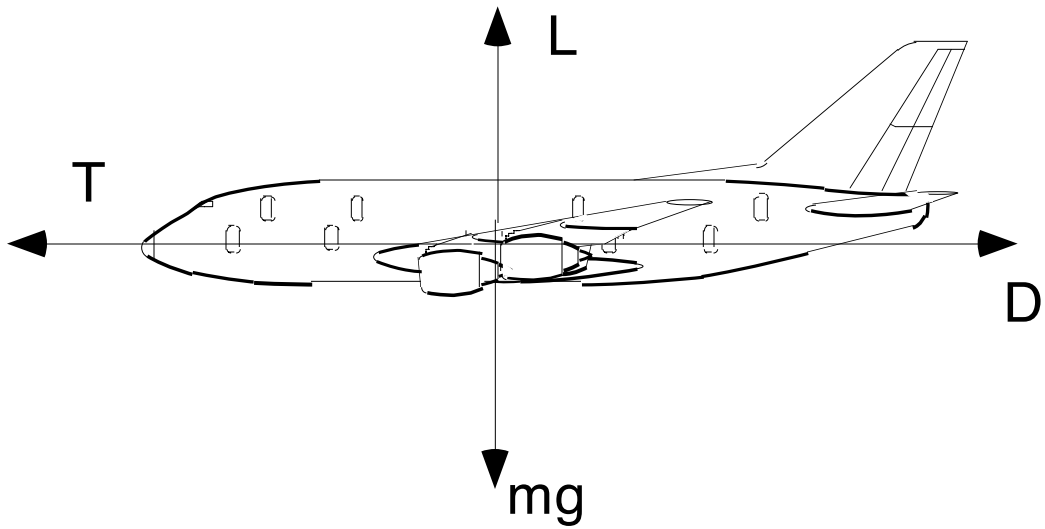


Cruise Performance Analysis

- One without doubt this is the most important phase of flight
- Except for short-range commuter operations, this is the phase of flight were most of the fuel is consumed (i.e., cost implications)
- Simpler analysis than climb profile (due to near steady-state situation)

Cruise Analysis

The forces acting on the air vehicle during cruising flight are shown below. Note that drag generated by the aircraft and the thrust supplied by the engine are equal for steady and level flight. Similarly, the lift and weight are equal.



Cruise Analysis

Lift and drag can be computed according to the well known aerodynamic equations re-stated below.

$$L = \frac{1}{2}\rho S C_L V^2 \tag{15}$$

$$D = \frac{1}{2}\rho S C_D V^2 \tag{16}$$

$$C_D = C_{D0} + C_{Di} = C_{D0} + \frac{C_L^2}{\pi A R e} \tag{30}$$

$$C_L = \frac{2mg}{\rho S V^2} \tag{15B}$$

Cruise Analysis

For typical subsonic aircraft ($M < 0.8$) the drag rise beyond the so-called critical mach number (M_{crit}) is quite severe and this produces a well defined maximum speed capability dictated by the rapid rise in the C_{D0} term in Equation 30.

A drag divergence mach number exists for every aircraft. The drag divergence mach number is characterized by a fast rise in drag coefficient due to wave drag and parasite/ friction drag effects at high speed.

Cruise Range Estimation

- An important consideration in assessing air vehicle performance is the range of the aircraft.
- Range is the maximum distance that an aircraft flies without refueling. Several range alternatives arise operationally for aircraft as will be shown in this section.
- The range represents a trade-off of how far and how much payload (i.e., the amount of passengers, cargo, or a combination of the two) an aircraft carries.

Range Estimation Methodology

The differential distance (or range), *dR*, traveled at speed V over a small interval of time *dt* is,

$$dR = Vdt \tag{34}$$

Since the aircraft only looses weight due to fuel expenditure we can define the rate of change of the weight over time as the product of the specific fuel consumption (TSFC) and the tractive force required to move the vehicle at speed V (T),

$$\frac{dW}{dt} = (TSFC)T \tag{35}$$

Range Estimation Methodology

Define the Specific Air Range - SAR - (a measure of the efficiency of the aircraft) as the ratio of the distance flown per unit of fuel consumed,

$$\frac{dR}{dW} = \frac{(V)}{(TSFC)T} = SAR \tag{36}$$

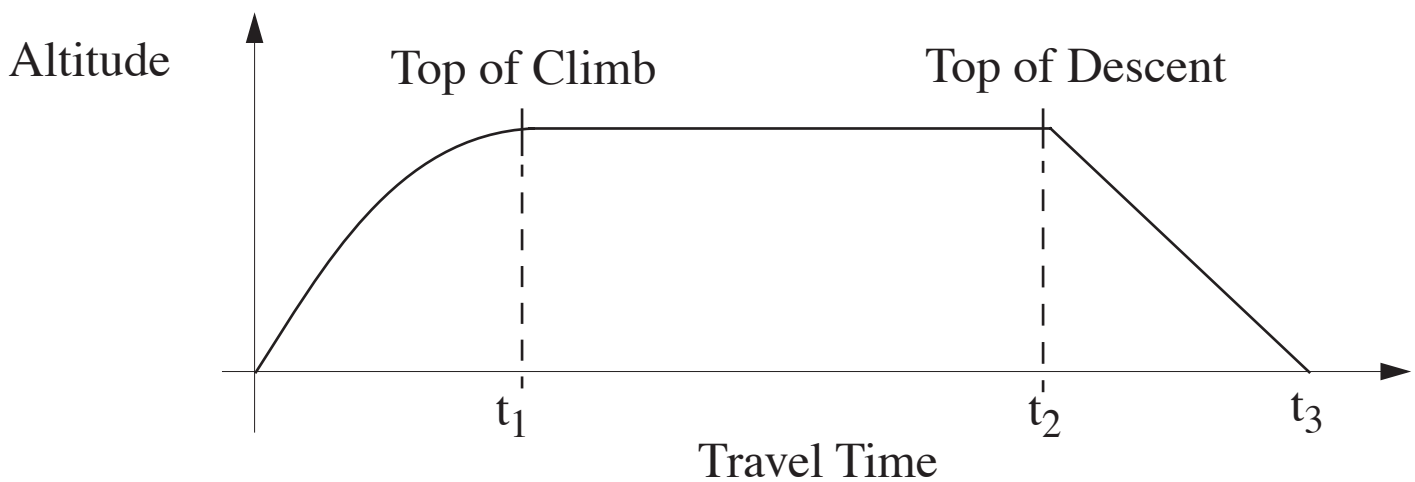
The typical units of TSFC are lb/hr/lbf (pounds per hour of fuel per pound of force produced) or kg/hr/kgf (kilograms per hour of fuel consumption per kilogram force produced). This parameter varies with altitude and speed.

Sample Use of TSFC

- The Pratt and Withney PW 4086 engine used in the Boeing 777 has a TSFC value of 0.6 lb/hr/lbf as the aircraft flies at mach 0.80 at 11,000 m. above mean sea level.
- If each engine produces 15,000 lb of thrust at that altitude to keep the aircraft flying straight and level then the average hourly fuel consumption would be (15,000 lb of thrust) (0.6 lb/hr/lbf) = 9,000 lb per hour (per engine).
- The solution to the so-called Breguet Range equation derived from SR is obtained if one separate variables and integrates over the weight expenditure of the vehicle from an initial weight, W_i to a final weight, W_f at the end of the cruising segment.

Cruise Range Analysis

In practical airline operations the initial and final cruising segment points are called Top of Climb (TOC) and Top of Descent (TOD), respectively.



Derivation of the Breguet Range Equation

Start with the basic equation of SAR (Equation 36),

$$\frac{dR}{dW} = \frac{(V)}{(TSFC)T} = SAR \tag{36}$$

Multiplying the right hand side of the previous equation by L/W and rearranging terms,

$$\frac{dR}{dW} = \frac{(V)}{(TSFC)TE}\left(\frac{L}{W}\right) = \frac{(V)}{(TSFC)W}\left(\frac{L}{D}\right) \tag{37}$$

Separating variables and integrating both sides,

$$\int_0^R dR = \int_{W_i}^{W_f} \frac{(V)}{(TSFC)}\left(\frac{L}{D}\right)\left(\frac{dW}{W}\right) \tag{38}$$

$$R = \frac{(V)}{(TSFC)}\left(\frac{L}{D}\right)\ln\left(\frac{W_i}{W_f}\right) \tag{39}$$

where: R is the aircraft range, $TSFC$ is the thrust specific fuel consumption, V is the cruise true airspeed, L is the lift, D is the drag produced while moving at speed V, and W_i and W_f are the initial and final weights of the aircraft at the top of climb and top of descent, respectively.

Note that for constant altitude cruise the term L/D is not constant because as the aircraft depletes its fuel and gets lighter over time. Consequently, the amount of lift needed to keep it flying at the same altitude will vary over time. The derivation of an approximate range equation can, nevertheless treat the term L/D as constant to give a first order approximation of the expected aircraft range.

Modifications to Breguet-Range Equation

In the range equation (Eq. 39) the term L/D can be alternatively substituted by C_L/C_D. To avoid problems the range expression for very long range aircraft can be subdivided into various cruising segments and then integrated using corresponding values of C_L/C_D for each segment. One approach to estimate with more precision the range is to integrate numerically the Specific Air Range equation (Eq. 36) considering variations in C_L/C_D using standard numerical methods.

Sample Jet Aircraft Performance Estimation

Suppose that we want to determine the range performance of a hypothetical twin-engine aircraft with the following characteristics:

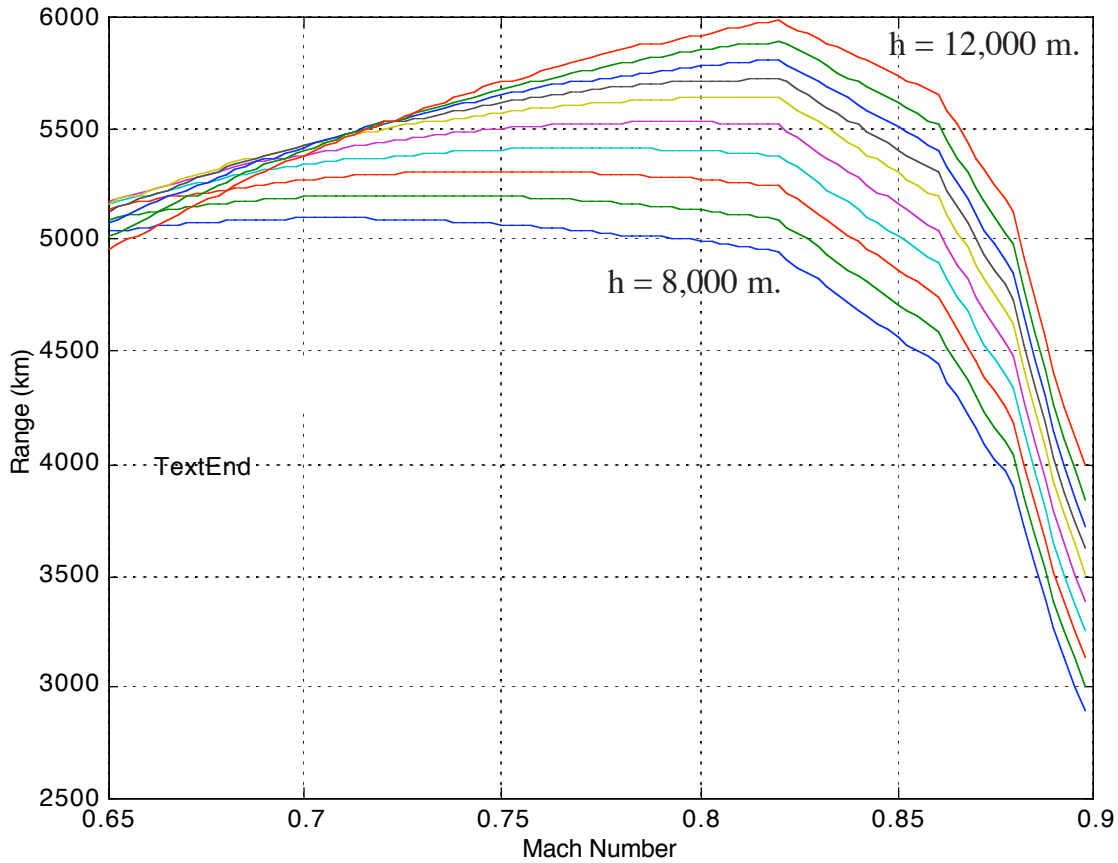
AR = 8.5 (aspect ratio), S=420 m^2, TSFC = 0.6 N/hr/N, mass at Top of Climb (TOC) = 224,000 kg., mass at Top of descent (TOD) = 178,000 kg., cruise altitude is 11,000 m., wing efficiency factor is 0.83,

Solution:

- The range solution also varies with mach number as seen in the following figure.
- There is an optimum mach number for long range cruise (0.81 for this aircraft at 12,000 m.).
- Also shown in our solution is the a Specific Air Range (SAR) chart for a high-speed commuter aircraft (Saab 2000).
- Note from this chart that SR peaks at intermediate speeds and that SR improves with increasing altitude due to gains in engine efficiency and reductions in aircraft drag (due to density effects).

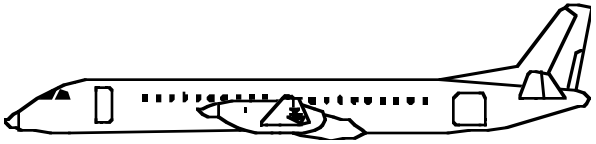
Twin-Engine Aircraft Cruise Performance

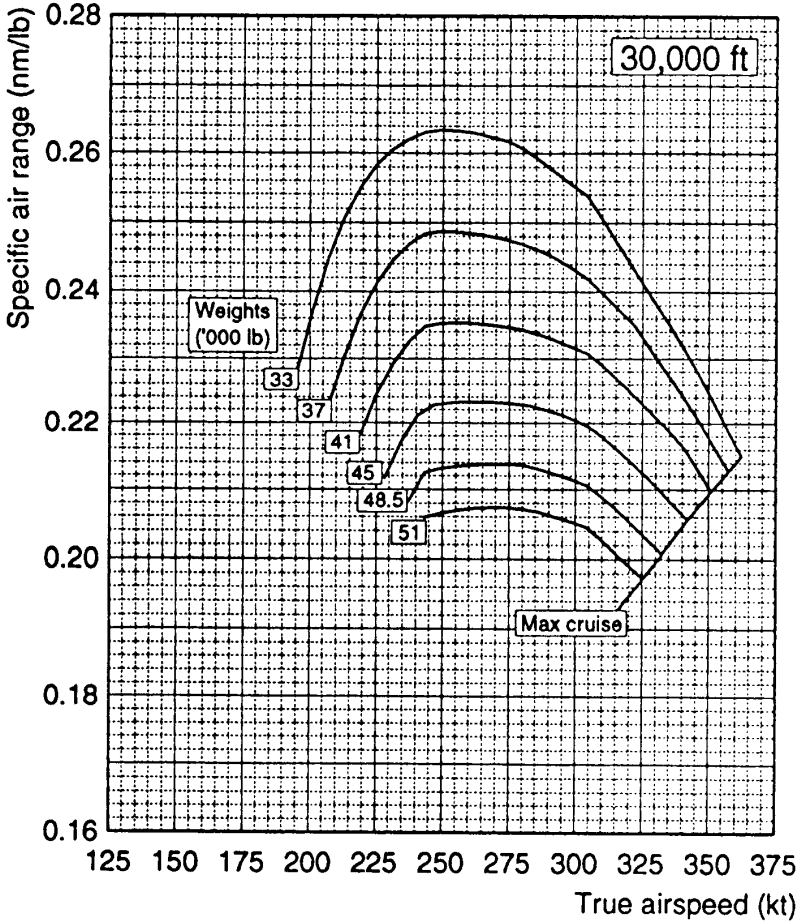
Plot of Mach Number vs. Range (km.)



Presentation of Cruise Information

ISA + 15^0 C
950 Propeller RPM
Anti-Ice Off





Descent Flight Operations

- Usually, transport aircraft descent at a rate of 900 m/min. (3,000 ft./min.) during the early stages of the descent segment.
- Below 3,000 m. (10,000 ft.) aircraft enter a dense terminal area and are usually required to maneuver around other air vehicles to establish coordinated arrival flows to runways
- In the U.S is customary to limit the indicated airspeed to 250 knots or lower below 3,000 m to avoid accidents in the rare event of a bird strike.

Descent Profile Operations

- Below 3,000 m. the descent rate typically decreases to 500 m/min. (1,500 ft./min.) or less and the descent profile might follow a series of “steps” at designated altitudes in the final stages of flight.
- Aircraft manufacturers report typical fuel consumption vs. distance traveled curves similar to those shown in the figure below.

Manufacturers also include distance vs. altitude curves for the descent phase of aircraft operations.

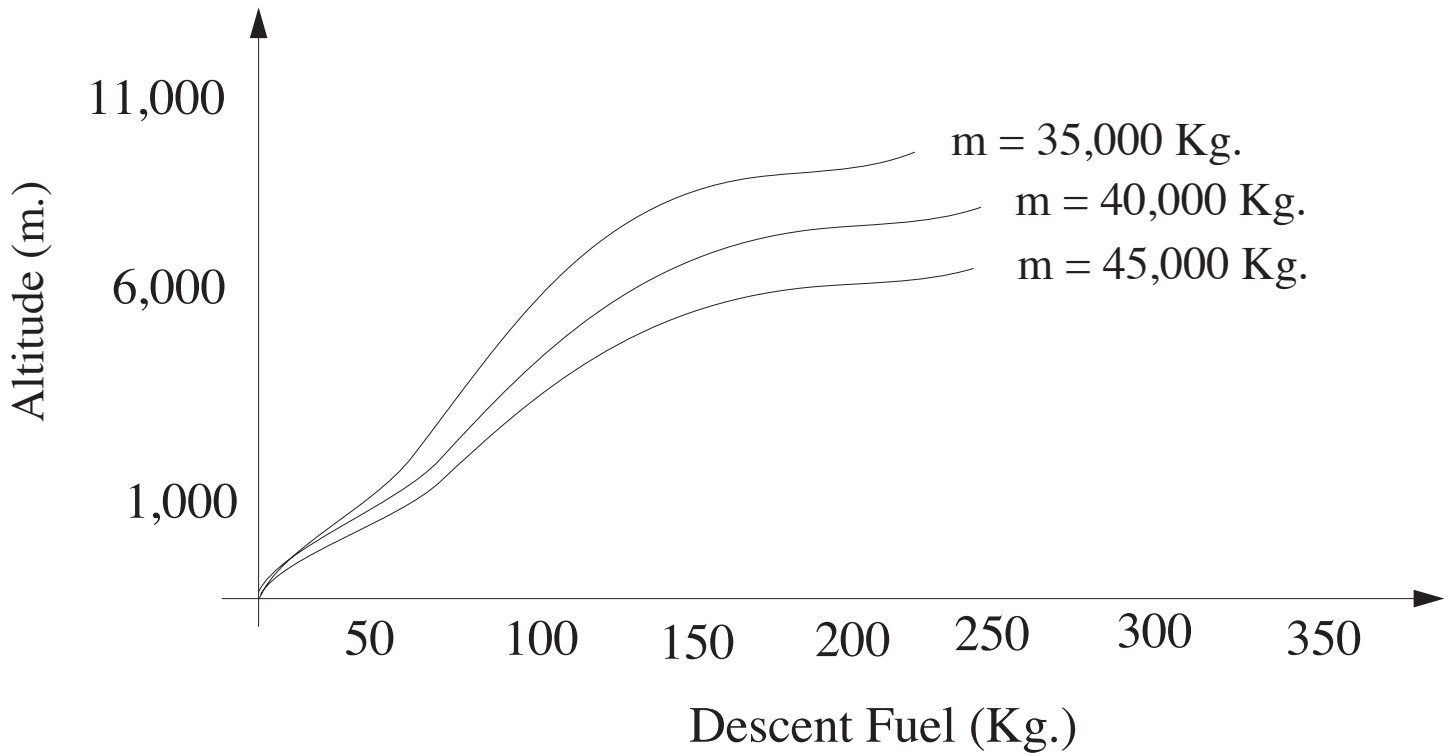
Descent Profile Operations

It must be emphasized that fuel consumption curves are very specific to aircraft-engine combinations and that you should consult with airline operators and ATC personnel before attempting to conduct airspace cost/benefit studies in the terminal area.

Air transportation analysts should carefully distinguish between vehicle differences before attempting further economic and travel demand analyses.

Presentation of Descent Profile Information

Sample fuel descent charts for Fokker F-100 aircraft.

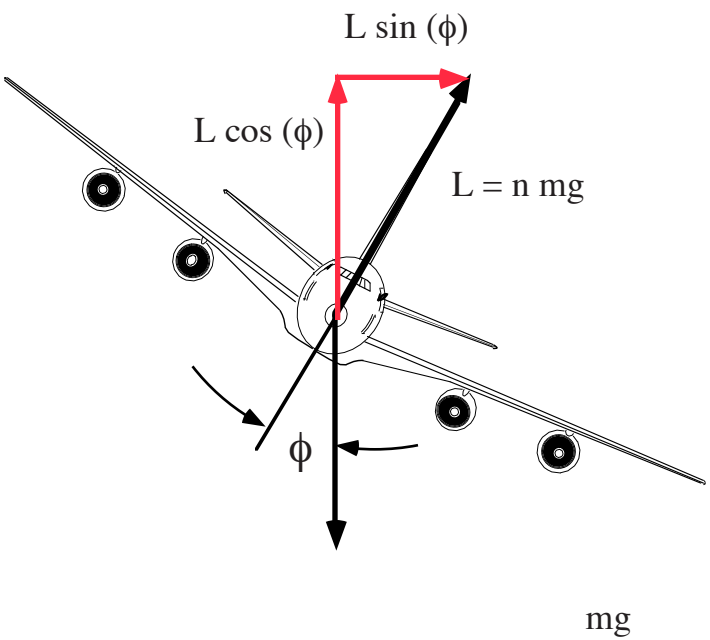


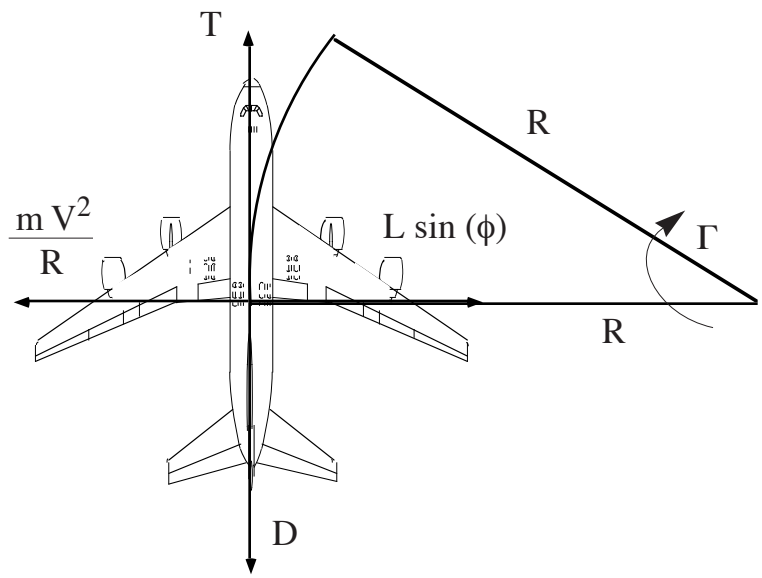
Adapted from Fokker 100 aircraft flight manual

Basic Turning Performance

- An important consideration in air transportation systems analysis (i.e., terminal areas operations and climb out procedures)
- Turning and climbing are the two most common maneuvers executed in the terminal area while an aircraft transitions from enroute airspace to the terminal and airport areas.

Basic Turning Performance Diagram





Turning Performance Analysis

- The basic forces acting on a turning aircraft that executes a steady level turn. It must realized that in many instances aircraft are instructed (or commanded by the pilot) to turn while climbing and descending. The equations of motion can be modified to include these three dimensional effects.

Balance of forces along the vertical axis (z-axis in aeronautical terms) yields,

$$L\cos\phi = mg \tag{40}$$

Turning Performance Basics

Similarly, balancing forces perpendicular to the turning motion,

$$L \sin\phi = \frac{mV^2}{R} \tag{41}$$

Note that along the flight path (x axis of the aircraft) thrust and drag are the same if the aircraft is in unaccelerated flight.

$$T = D = \frac{1}{2}\rho V^2 S C_D \tag{42}$$

Using the previous equations we can derive the radius of the turn, R for a given bank angle (ϕ) and airspeed (V), and the resulting turn rate (Γ).

Turning Radius and Rate of Turn Analysis

$$R = \frac{V^2}{g \tan\phi} \tag{43}$$

and,

$$\Gamma = \frac{V}{R} = \frac{g \tan\phi}{V} \tag{44}$$

In many instances you will find that pilots and engineers define the so-called load factor, n, as follows:

$$n = \frac{L}{mg} = \frac{1}{\cos\phi} \tag{45}$$

This parameter tells us how large the lift vector has to be to overcome the weight of the aircraft while turning.

Turning Performance

Note that as the bank angle (ϕ) increases so does L to maintain a coordinated, level turn.

Substituting n into equations 43 and 44 yields,

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} \tag{46}$$

and,

$$\Gamma = \frac{g\sqrt{n^2 - 1}}{V} \tag{47}$$

In practical airspace operations commercial aircraft seldom bank more than 30 degrees to keep passengers in comfort (this implies a load factor of 1.16 or less).

Standard Turn

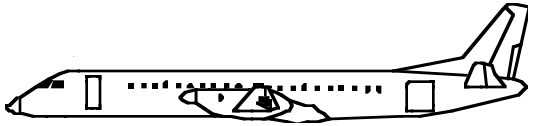
- It is also interesting to note that from the ATC perspective a standard turning maneuver is usually assumed in the design of terminal area flight paths using a three-degree per second turn rate.
- This implies that in a standard turn the aircraft takes one minute to complete a 180^0 maneuver.

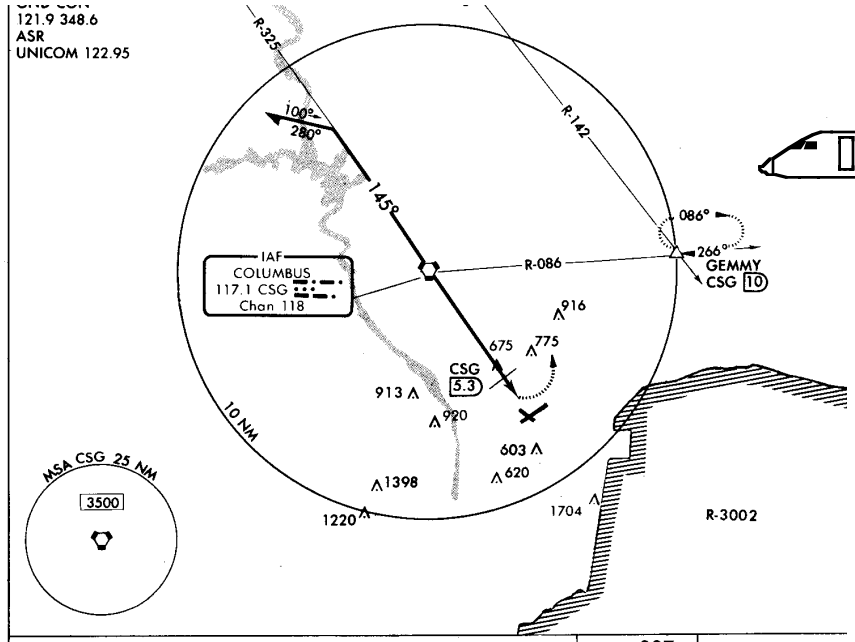
Example of Terminal Area Maneuvering

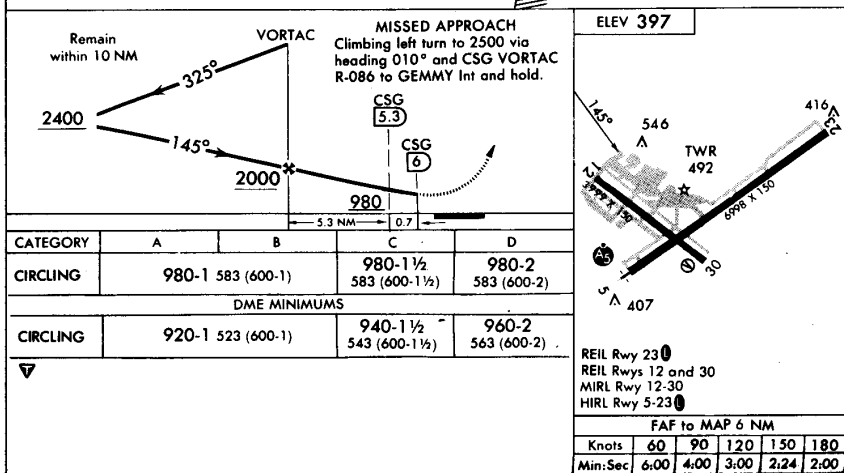
Suppose a Saab 2000 commuter aircraft approaches an airport and executes a VOR non-precision approach to Columbus, Georgia runway 12. This approach requires a flight outbound from the VORTAC and execute a procedure turn (225^0) before landing (see Figure).

If the pilot maneuvers the aircraft down to 150 knots (indicated airspeed) while executing the procedure turn. Find the bank angle, the load factor imposed on cargo and passengers, the turning time (t_t) and the radius of turn in the maneuver.

Approach Plate to Columbus, Georgia







CATEGORY	A	B	C	D
CIRCLING	980-1 583 (600-1)		980-1½ 583 (600-1½)	980-2 583 (600-2)
DME MINIMUMS				
CIRCLING	920-1 523 (600-1)		940-1½ 543 (600-1½)	960-2 563 (600-2)

FAF to MAP 6 NM					
Knots	60	90	120	150	180
Min:Sec	6:00	4:00	3:00	2:24	2:00

Solution

- Solving for the load factor in Equation 35 and substituting the corresponding values for g, V and Γ yields a bank angle of 9.35 degrees and a load factor of 1.0315.
- The resulting turn radius of the maneuver is 5,305 meters (2.86 nm). Note that the approach plate calls for the aircraft to stay within 10 nautical miles of the Columbus VOR (called initial approach fix).
- Note also that this aircraft is expected to complete the procedure turn while at 2,400 ft. above mean sea level (the airport is at 407 ft. MSL) and then descend to 2,000 ft. MSL at the VORTAC (see insert in left hand corner of

the approach plate) and then continue towards the airport.

Solution (cont.)

- The aircraft continues Missed Approach Point (MAP) altitude of 980 ft. MSL (the Saab 2000 is classified as a TERP B aircraft for ATC procedures).
- According to the approach chart this aircraft would take more than 2 minutes if flown at 180 knots from the Final Approach Fix (FAF) to the MAP. A more reasonable speed in final would be 120 knots for this aircraft.
- In the computation of turning radii true airspeeds should always be used. This is because the design of ATC procedures always looks at the topographical obstructions surrounding the airport facility to avoid collisions with terrain.

Final Note on Turning Performance Example

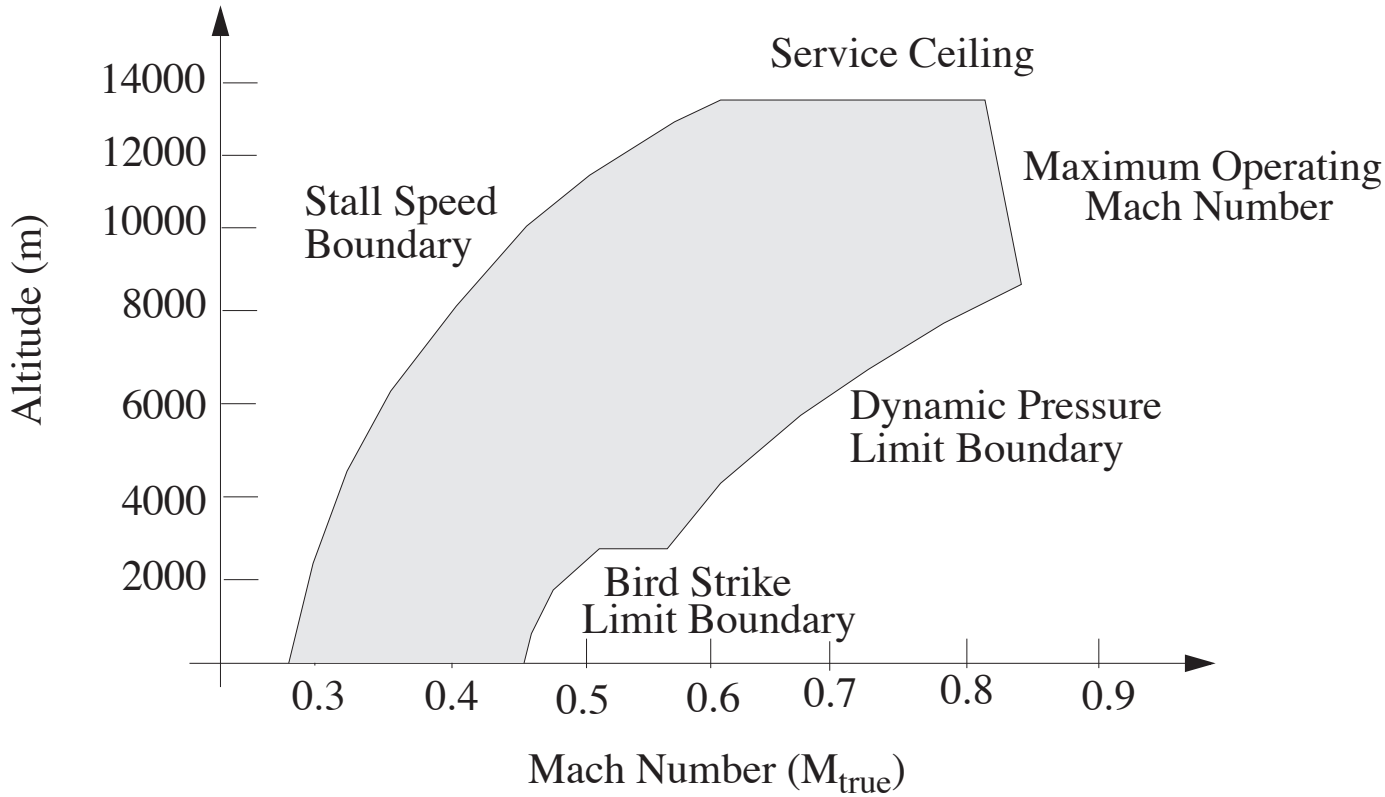
- The implication of using true airspeed is that TAS increases dramatically with altitude resulting in very large turning radii at moderate and high altitudes.
- For example an aircraft flying at an altitude of 5.0 km and 250 knots IAS - 300 knots true airspeed - would require a turning radius of 5.0 nautical miles while executing a standard turn.

Aircraft Flight Envelope Characteristics

- Once the analysis of climb, cruise and descent trajectories has been made we are in the position to draw the typical boundaries that restrict the operation of an aircraft in flight.
- The following figure illustrates a typical flight envelope for a turbofan-powered, subsonic aircraft.

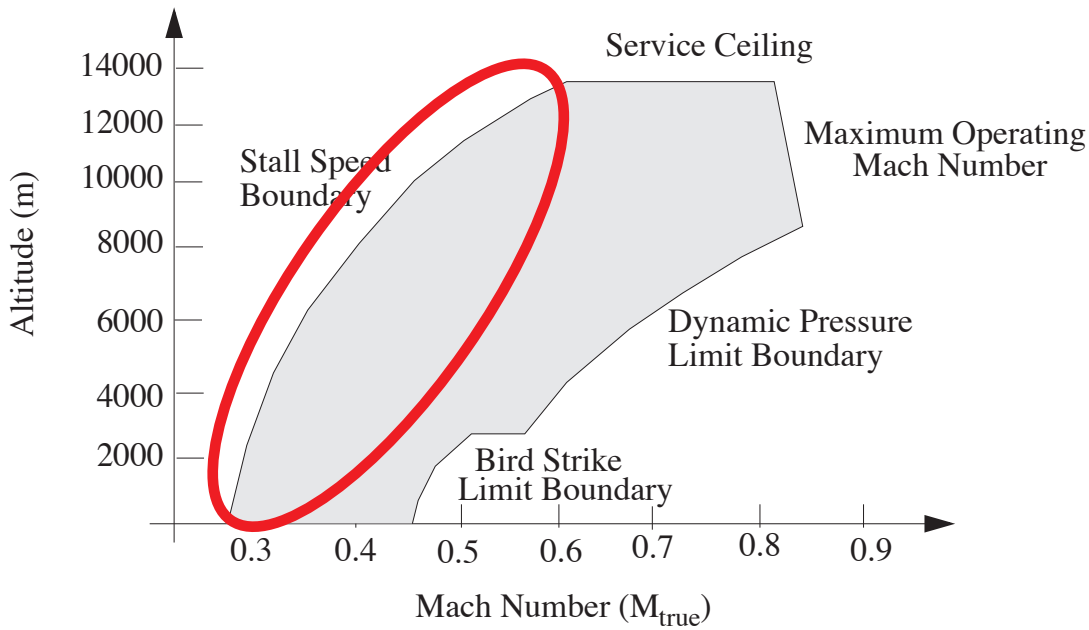
Aircraft Envelope Analysis

Typical subsonic aircraft envelope



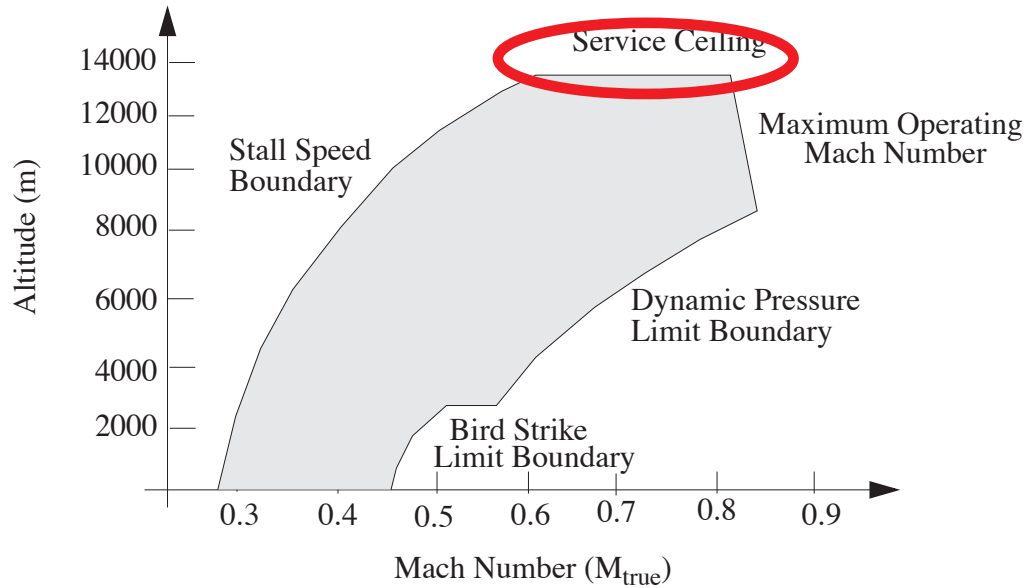
Low Speed Aircraft Envelope Boundary

- Low speed boundary indicating that the aircraft wing can only produce enough lift for a given speed at various altitudes.
- Stalling speed (in terms of true airspeed) increases with altitude



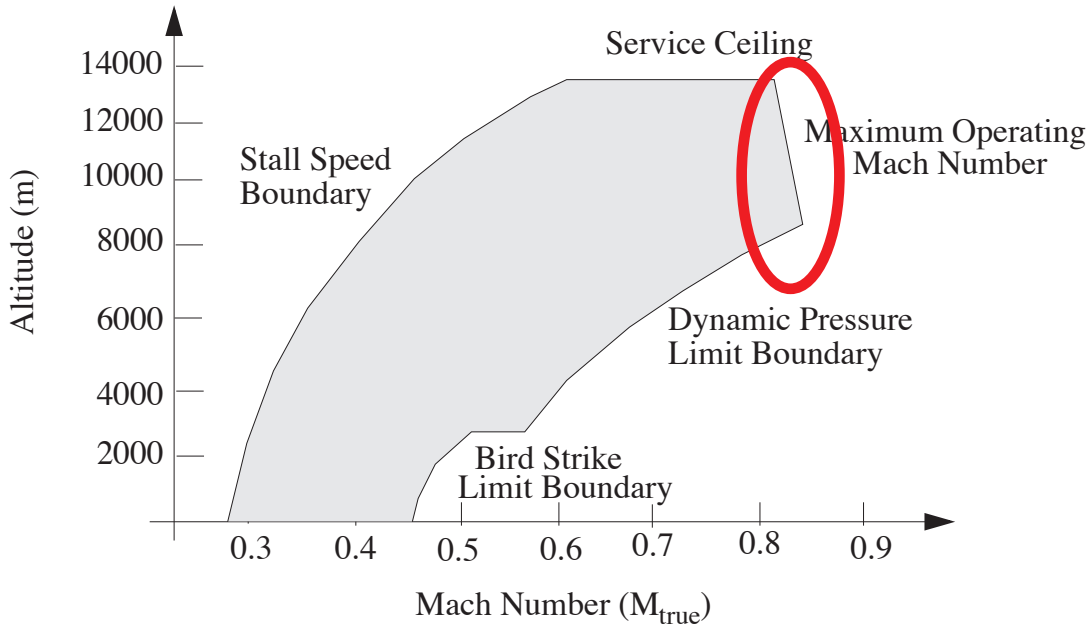
Service Ceiling Aircraft Envelope Boundary

- Is the maximum altitude that the aircraft attains while climbing at a very small climb rate (typically 100 ft./min. according to FAR Part 25 regulations). Modern transport aircraft such as the Boeing 757-200 and the Boeing 777 have been certified to fly up to 13,720 m (45,000 ft.) at moderate to light weights.



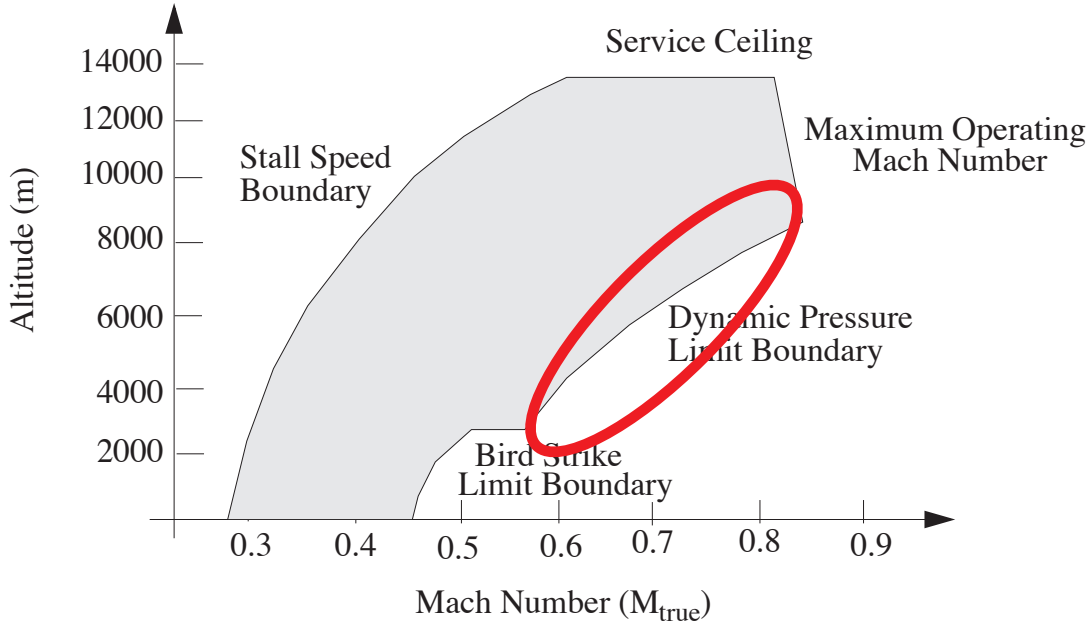
Maximum Speed Aircraft Envelope Boundary

- The aircraft reaches a region of flight where the drag produced increases sharply (i.e., drag divergence Mach number boundary) and thus the aircraft engines are incapable of producing enough thrust to accelerate the aircraft to faster speeds.



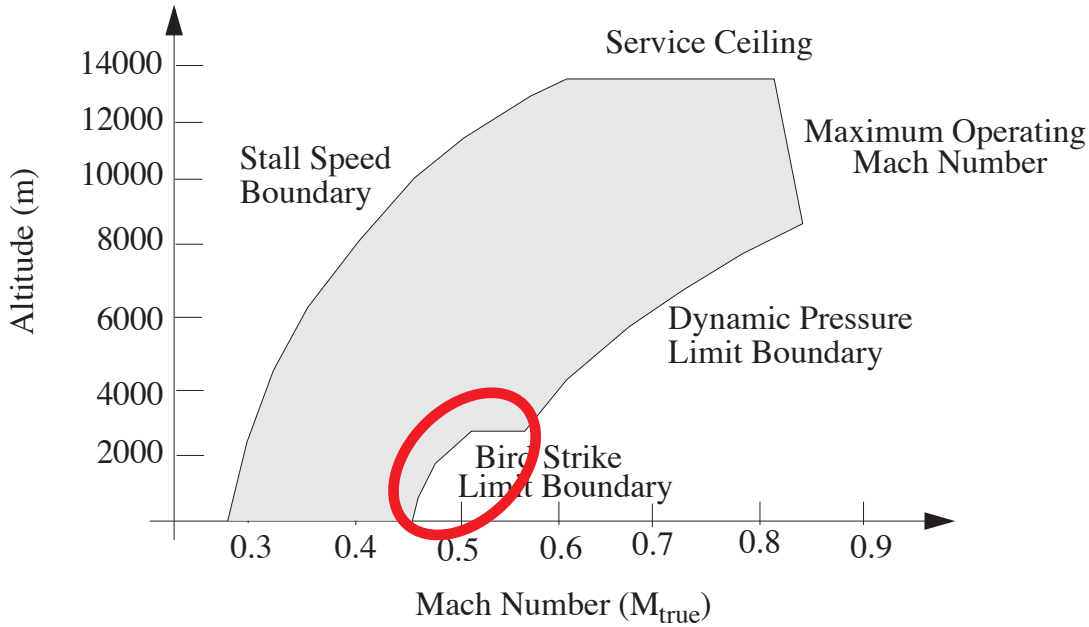
Dynamic Pressure Aircraft Envelope Boundary

- The design of all aircraft structures carries an assumption about the maximum loads that can be tolerated in flight. For our hypothetical aircraft a maximum dynamic pressure limit of 25,490 kg/m^2 has been used.



Bird Strike Aircraft Envelope Boundary

- Common sense and certification of flight deck windshields dictated a natural boundary below 10,000 ft. Traditionally this boundary has been set at 250 knots.



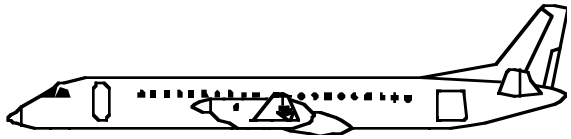
Fuel and Block Time Diagrams

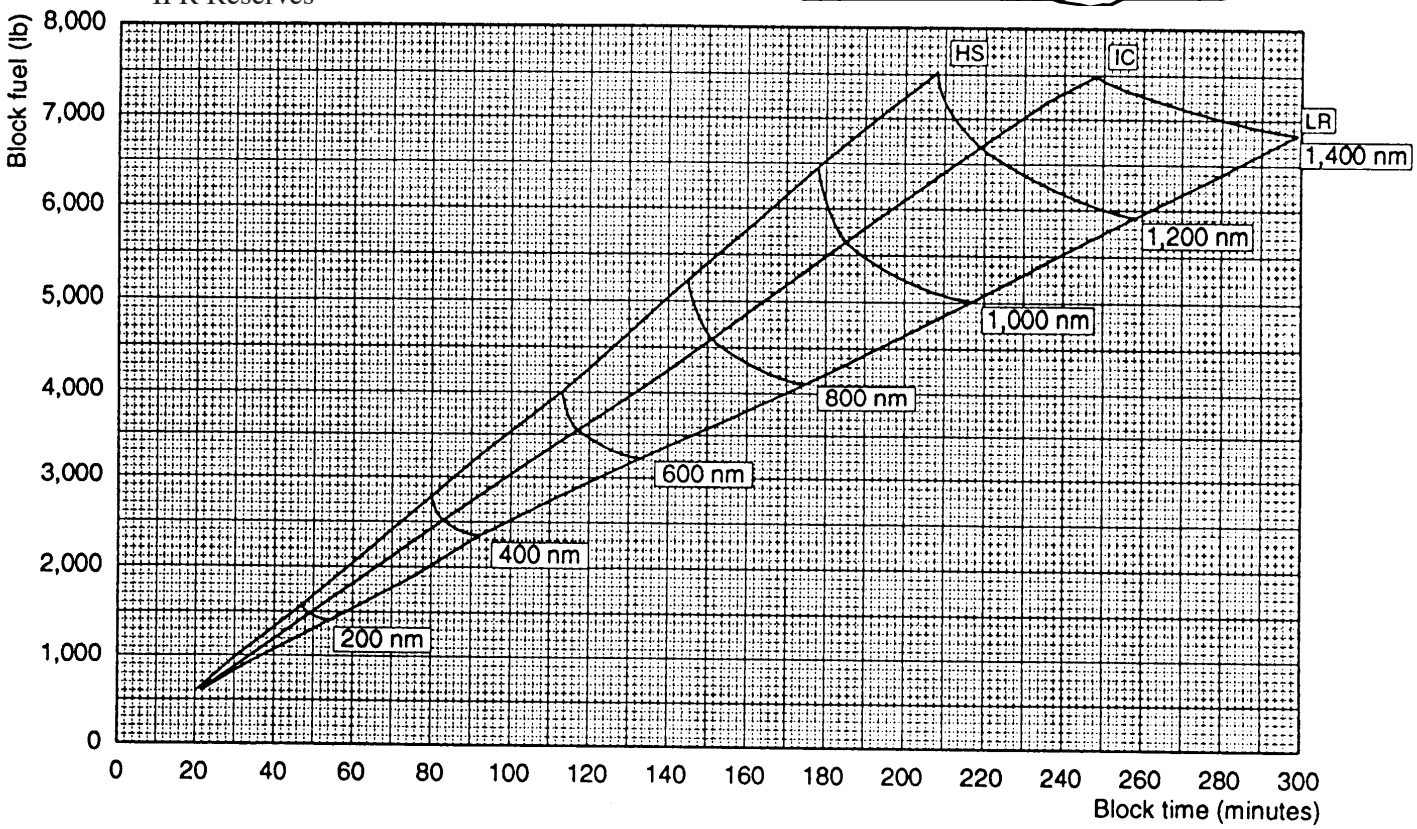
The complete understanding of a flight trajectory allows us to estimate block time and block fuel for an entire trip.

- Block time is defines as the time it takes an aircraft to complete its trip from gate to gate. This may include taxiing times and departure delay times that are common today in NAS operations.
- The figure illustrates a typical presentation of block fuel and block times for the Saab 2000 commuter aircraft. In this figure we identify three operating speed regimes: 1) high speed (HS), 2) typical cruise (TC) and 3) long-range (LR).

Sample Fuel and Block Time Diagram

ISA
Zero Wind
10 Minute Manuevering Time
IFR Reserves





Interpretation Fuel and Block Time Diagrams

- It is evident from this diagram that a clear trade-off exists between block speed and block fuel.
- For example, if the airline operator wants to fly this aircraft between two cities 800 nm apart it could use a long range speed profile taking 175 minutes and consuming 4,100 lb. of fuel.
- The same operator could use a high-speed profile using 5,250 lb. of fuel and taking 145 minutes for the same trip.
- One question that perhaps we should ask ourselves is whether or not a 30 minute block time savings is significant or not and at what operating cost.

Interpretation of Fuel and Block Time Diagram

- Saving 1,150 lb. of fuel could be significant considering that an aircraft of this type makes three to four trips per day. This could easily translate into several hundreds of thousand pounds of fuel saved in a year
- It is interesting to note that few airlines operate their aircraft at the most economical speed regime (i.e., long range) because in long trips the resulting block times could be quite high thus reducing the number total trips that a single aircraft completes in one day
- The operational cost is directly linked to the productivity of the aircraft in terms of the number of seat-miles offered. Therefore, faster block times could make a difference in the profit of the operator.

Sample Flight Performance Models

- BADA - Eurocontrol
  + Trajectory and fuel burn
- OPGEN - FAA and CSSI
  + Optimal trajectory and fuel burn
- ASAC - NAS and LMI
  + Optimal trajectory and fuel burn
- VPI Neural Network Fuel Burn and FTM Model
  + Fuel burn and flight trajectory (using BADA data)
- Most airspace and airport simulators have their own fuel burn models (Old SIMMOD fuel consumption model and TAAM fuel consumption table functions)

The BADA Performance Model

- Developed by Eurocontrol Experimental Centre (ECC) to model various Air Traffic Management (ATM) concepts
- 160 aircraft supported in BADA 3.3
- 67 aircraft models are supported directly
- 84 aircraft models are supported indirectly (equivalent models)
- Main outputs of the models are fuel consumption, aerodynamic and speed procedure parameters

BADA Data Organization

BADA 3.5 Model

Operations Performance File (OPF)

1) Aero coefficients
2) Mass coefficients
3) Flight envelope restrictions
4) Engine thrust
5) Fuel consumption

Airline Procedures File (APF)

Recommended speed procedures: climb, cruise and descent

Performance Table File (PTF)

Typical speed procedures for climb, cruise and descent at ISA conditions

Sample BADA 3.0 OPF File

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC A320__.OPF CCCCCCCCCCCCCC/
CC                                                                       /
CC                AIRCRAFT PERFORMANCE                                   /
CC                operational    files                                   /
CC                                                                       /
CC     BADA RCS File Id                                                  /
CC    File Name        Current Revision      Last Modification           /
CC                     revision    date      revision     date           /
CC    A320__.OPF           3.0   98/03/12    2.6.1.1    98/03/11         /
CC                                                                       /
CC   BADA Revision:                                                      /
CD   Rev  3.0                                                            /
CC====== Actype ==========================================================/
CD   A320__          2 engines    Jet                        M           /
CC   Airbus A320-111 with CFM56_5_A1 engines                wake         /
CC                 (source: AIR FRANCE OPS manual)                       /
CC====== Mass (t) ========================================================/
CC    reference      minimum      maximum     max payload  mass grad     /
CD     .62000E+02   .41800E+02   .73500E+02   .19220E+02   .32000E+00   /
CC====== Flight envelope =================================================/
CC     VMO(KCAS)       MMO        Max.Alt       Hmax       temp grad     /
CD     .35000E+03   .82000E+00   .39000E+05   .36500E+05  -.40000E+02   /
CC====== Aerodynamics ====================================================/
CC Wing Area and Buffet coefficients (SIM)                               /
CCndrst Surf(m2)     Clbo(M=0)       k           CM16                    /
CD 5   .12240E+03   .12100E+01   .47000E+00   .00000E+00                 /
CC   Configuration characteristics                                       /
CC n Phase  Name    Vstall(KCAS)    CD0          CD2        unused       /
CD 1 CR   Clean     .14700E+03   .20000E-01   .40000E-01   .00000E+00   /
CD 2 IC   1+F       .11700E+03   .00000E+00   .00000E+00   .00000E+00   /
CD 3 TO   1+F       .11700E+03   .00000E+00   .00000E+00   .00000E+00   /
```

Sample BADA 3.0 APF File

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC B767__.APF CCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CC
CC                AIRLINES PROCEDURES FILE
CC
CC    File Name        Current Revision            Last Modification
CC                     revision    date            revision     date
CC    B767__.APF           3.0   98/03/12          2.4.1.2    96/09/05
CC
CC   BADA Revision:
CD   Rev  3.0
CC
CC   LO= 90.00 to ---.--  /  AV= ---.-- to ---.--  /  HI= ---.-- to 181.40
CC
CC=============================================================================
CC  COM CO    Company name ------climb-------  --cruise--  -----descent------  --app
CC                   mass  lo  hi               lo  hi         hi  lo            (ur
CC    version engines  ma  cas cas mc xxxx xx  cas cas mc  mc cas cas xxxx xx  xxx x
CC===:=======:=======:==:===:===:==:====:==:===:===:==:==:===:===:====:==:===:
CD  *** **    Default Company
CD    300ER   PW4060   LO  290 290 78          310 310 80  78 290 290            0
CD    300ER   PW4060   AV  290 290 78          310 310 80  78 290 290            0
CD    300ER   PW4060   HI  290 290 78          310 310 80  78 290 290            0
CC===:=======:=======:==:===:===:==:====:==:===:===:==:==:===:===:====:==:===:
CC//////////////////////////////////  THE END   ////////////////////////////////
```

Sample BADA 3.0 PFT File

BADA PERFORMANCE FILE 98/03/12

AC/Type: F28___

Last BADA Revision: 3.0
Source OPF File: 3.0 98/03/12
Source APF file: 3.0 98/03/12

Speeds:	CAS(LO/HI)	Mach	Mass Levels [kg]		
climb	- 250/270	0.65	low	-	20880
cruise	- 250/300	0.70	nominal	-	24000
descent	- 250/280	0.70	high	-	33000

Temperature: ISA

Max Alt. [ft]: 35000

FL	CRUISE TAS [kts]	CRUISE fuel [kg/min] lo	CRUISE fuel [kg/min] nom	CRUISE fuel [kg/min] hi	CLIMB TAS [kts]	CLIMB ROCD [fpm] lo	CLIMB ROCD [fpm] nom	CLIMB ROCD [fpm] hi	CLIMB fuel [kg/min] nom	DESCENT TAS [kts]	DESCENT ROCD [fpm] nom	DESCENT fuel [kg/min] nom
0					127	2760	2860	2370	108.1	108	900	19.5
5					128	2710	2820	2330	106.4	108	910	19.5
10					129	2670	2780	2290	104.6	114	890	19.4
15					135	2800	2880	2360	103.3	125	850	19.4
20					136	2760	2830	2320	101.6	157	820	19.4
30	261	19.5	20.8	25.8	159	3270	3260	2650	100.0	230	1070	19.3
40	265	19.5	20.9	25.8	193	3960	3820	3070	99.3	233	1080	19.2
60	272	19.6	21.0	26.0	272	5170	4570	3410	98.3	240	1110	19.1

The BADA Performance Model

BADA uses a total energy model to derive aircraft performance.

$$mg\frac{dh}{dt} + mV\frac{dV}{dt} = V[T - D] \tag{48}$$

where:

$\frac{dh}{dt}$ is the rate of climb (m/s)

$\frac{dV}{dt}$ is the acceleration along the flight path (m/s^2)

h is the aircraft altitude (m)

m is the aircraft mass (kg)

V is the aircraft true airspeed (m/s)

g is the gravitational acceleration (9.81 m/s^2)

T is the aircraft thrust (N) and

D is the aircraft drag (N)

Computation of Aircraft Cruise Performance Parameters

Drag Coefficient:

$$C_D = C_{D0-CR} + C_{D2-CR} C_L^2 \tag{49}$$

where: C_D is the total aircraft drag coefficient (dim)

C_{D0-CR} is the zero lift drag coefficient in the cruise configuration (dim)

C_{D2-CR} is a lift-dependent coefficient (dim)

C_L is the aircraft lift coefficient (dim)

Estimation of Aircraft Lift Coefficient

The estimation of the aircraft drag coefficient requires knowledge of C_L. This non-dimensional coefficient is calculated assuming small flight path angles,

$$C_L = \frac{2mg}{\rho S V^2 \cos(\phi)} \tag{50}$$

where: S is the aircraft wing reference area (m^2)

ρ is the air density (kg/m^3)

m is the aircraft mass (kg)

$\cos(\phi)$ is the cosine of the bank angle (dim) and all other parameters as previously defined.

The total aircraft drag is then,

$$D = \frac{1}{2}\rho V^2 S C_D$$

where:

D is the total aircraft drag (N)

S is the aircraft wing reference area (m^2)

BADA 3.0 Fuel Consumption

The aircraft thrust specific fuel consumption (η) is estimated as follows:

$$\eta = C_{f1}(1 + V/C_{f2}) \tag{51}$$

where:

η is the aircraft thrust specific fuel consumption (kg/min./kN)

V is the aircraft true airspeed (knots)

C_{f1} and C_{f2} are model coefficients

BADA 3.0 Fuel Consumption

$$f_{nom} = \eta T \tag{52}$$

where:

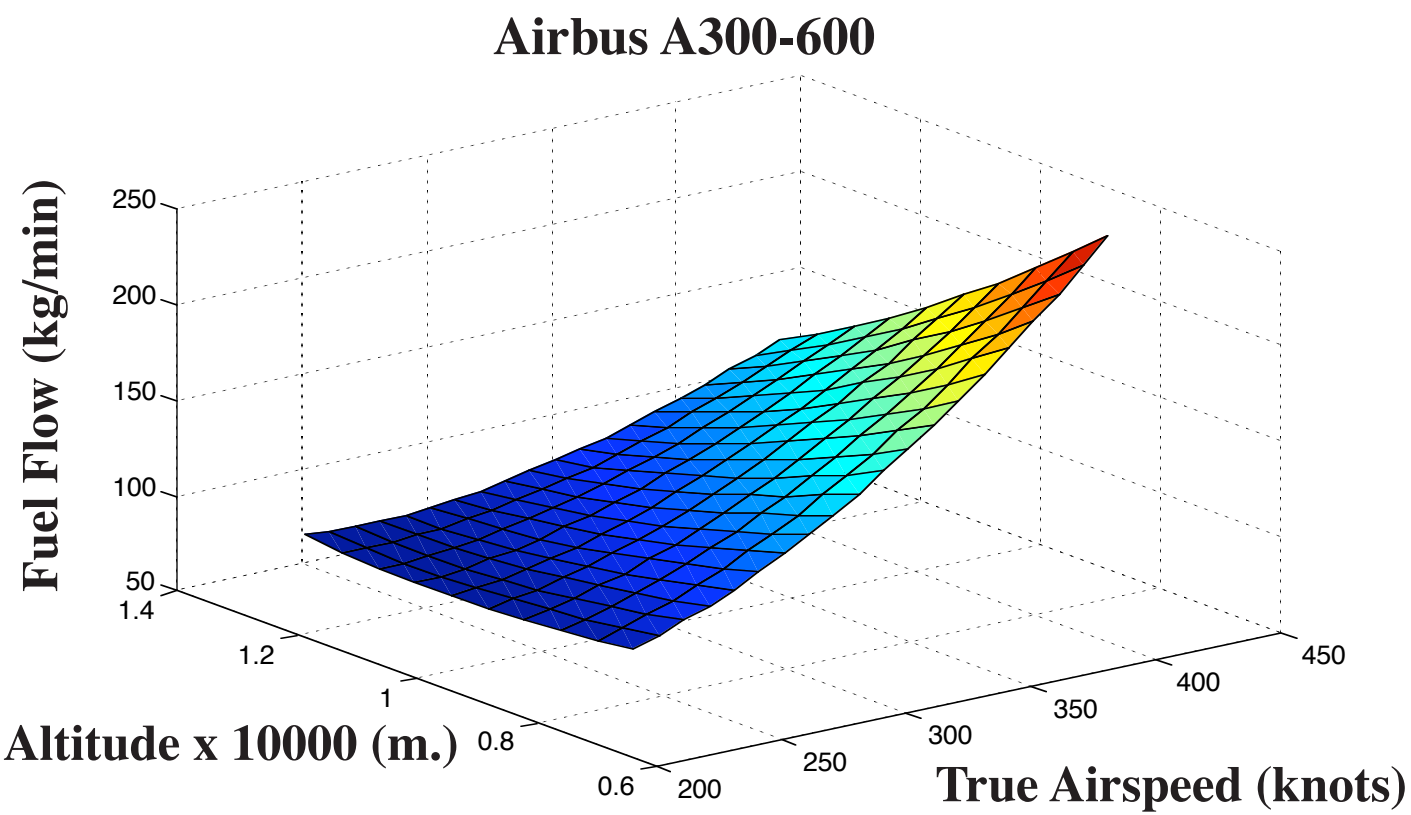
f_{nom} is the nominal aircraft fuel consumption (kg/min.)

The Specific Air Range (SAR) a measure of aircraft fuel efficiency is,

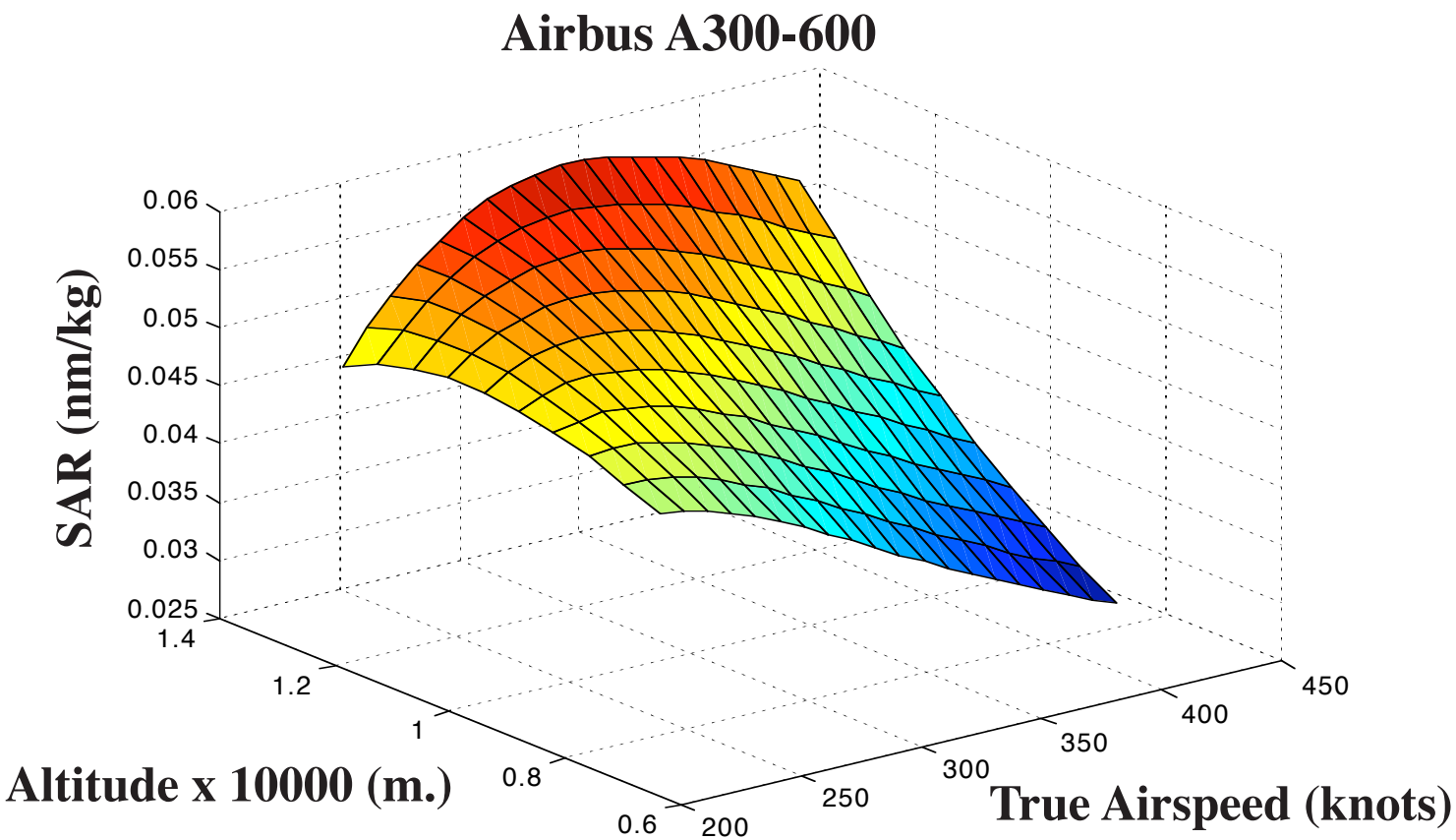
$$SAR = \frac{\Delta d}{\Delta f} \tag{53}$$

where: Δd is the change in position over time t and Δf is the fuel consumed traveling distance Δd.

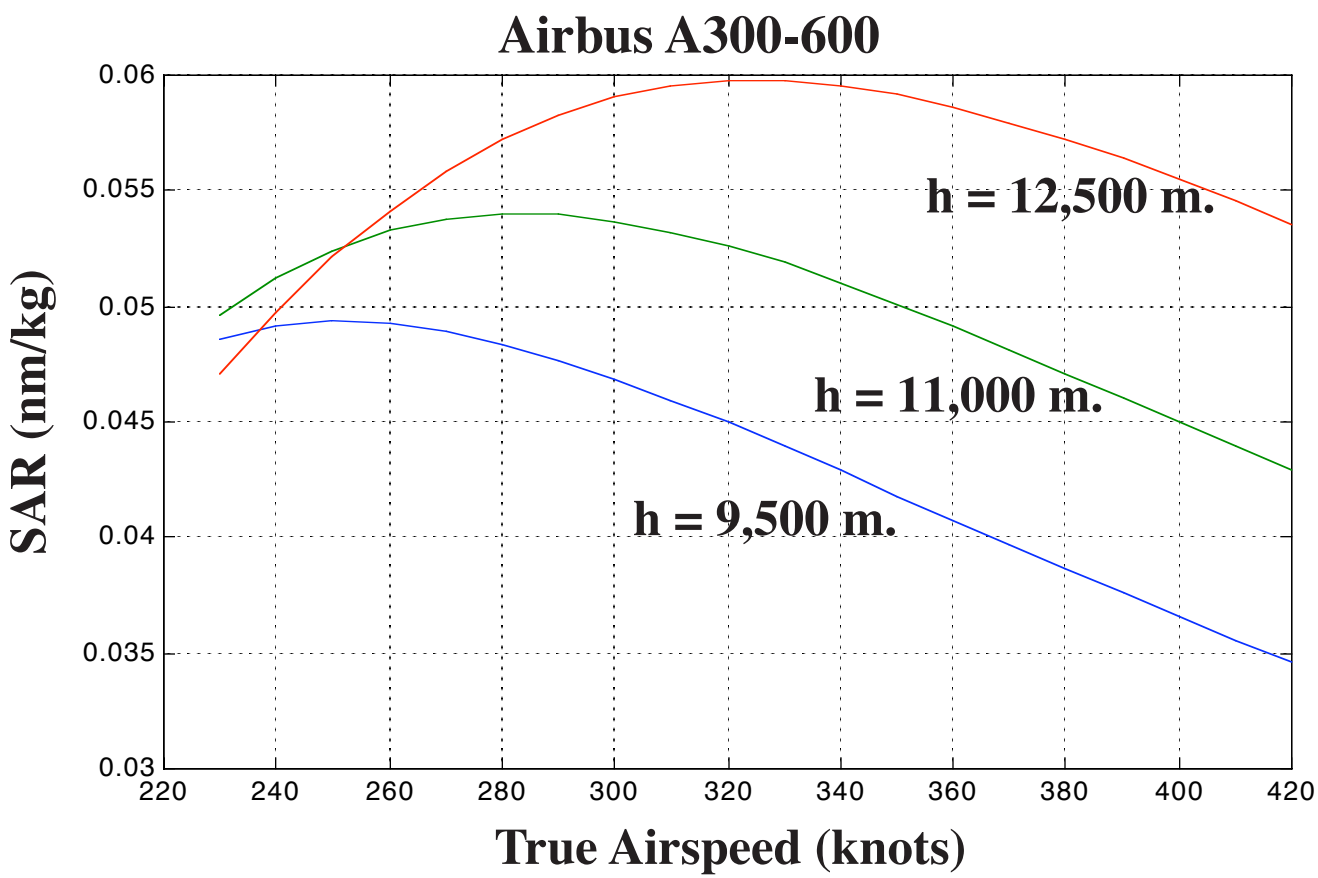
BADA 3.0 Results (Fuel Consumption)



BADA 3.0 Results (Specific Air Range - SAR)



BADA 3.0 Results (Specific Air Range)

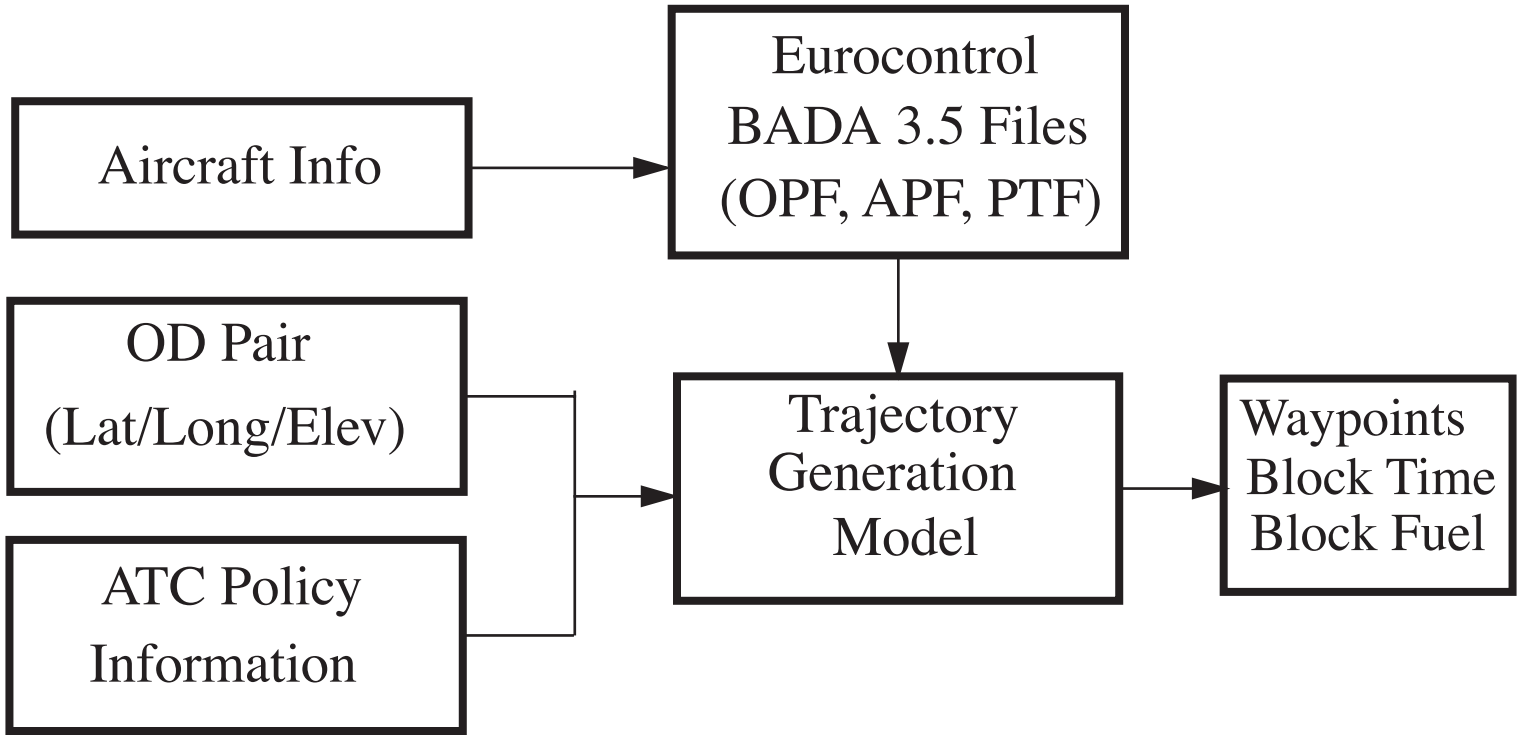


Interpretation of SAR - Specific Air Range

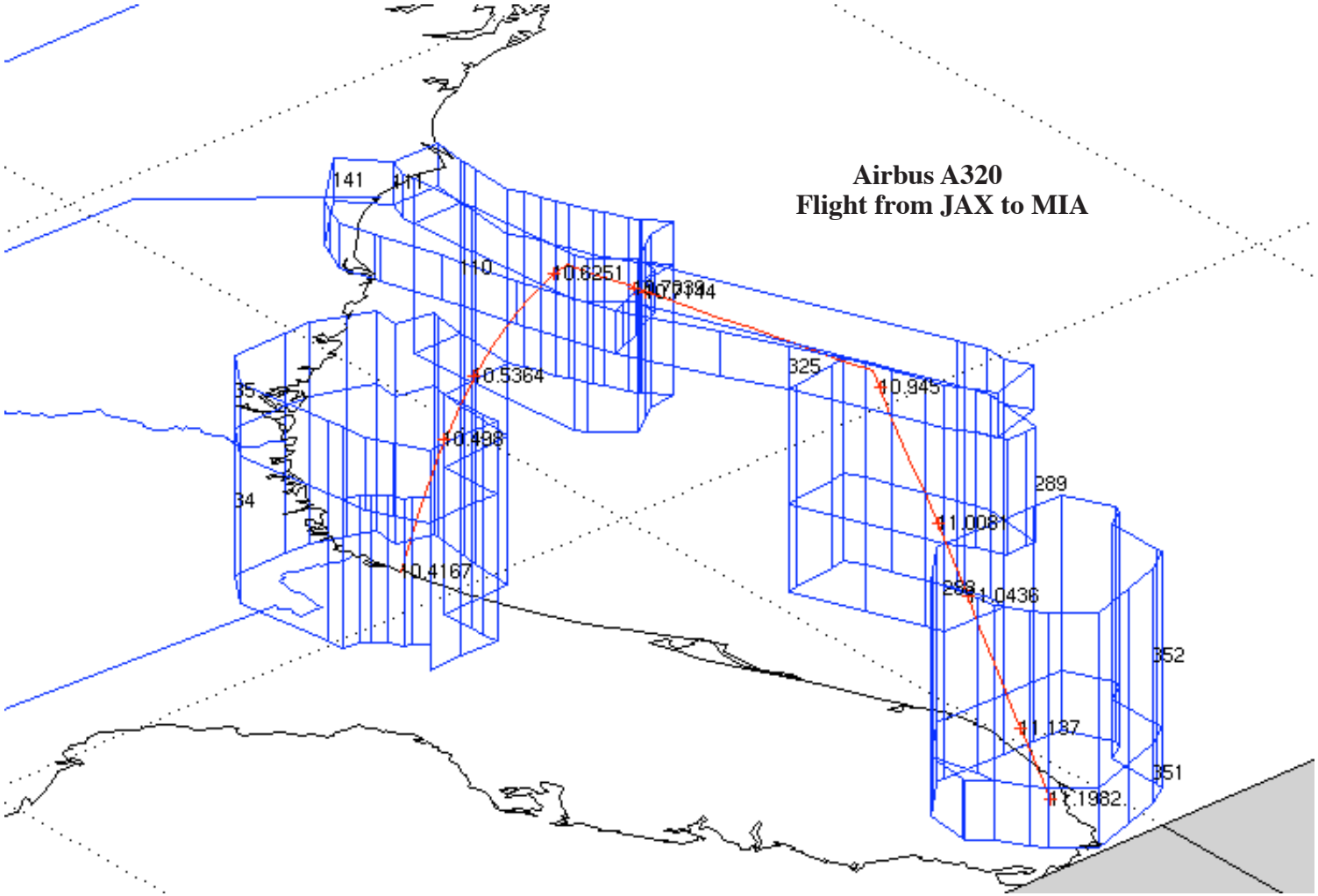
- SAR represents a measure of aircraft efficiency
- the higher the SAR parameter, the more fuel efficient the aircraft is
- For example: in the figure of page 20, the A300 has a maximum SAR value near 0.06. This implies that the aircraft covers 0.06 nautical miles per kg of fuel used.
- The aircraft is more fuel efficient when flying higher (at 12,500 m. instead of 9,500 m.).

Potential Implementation Scheme

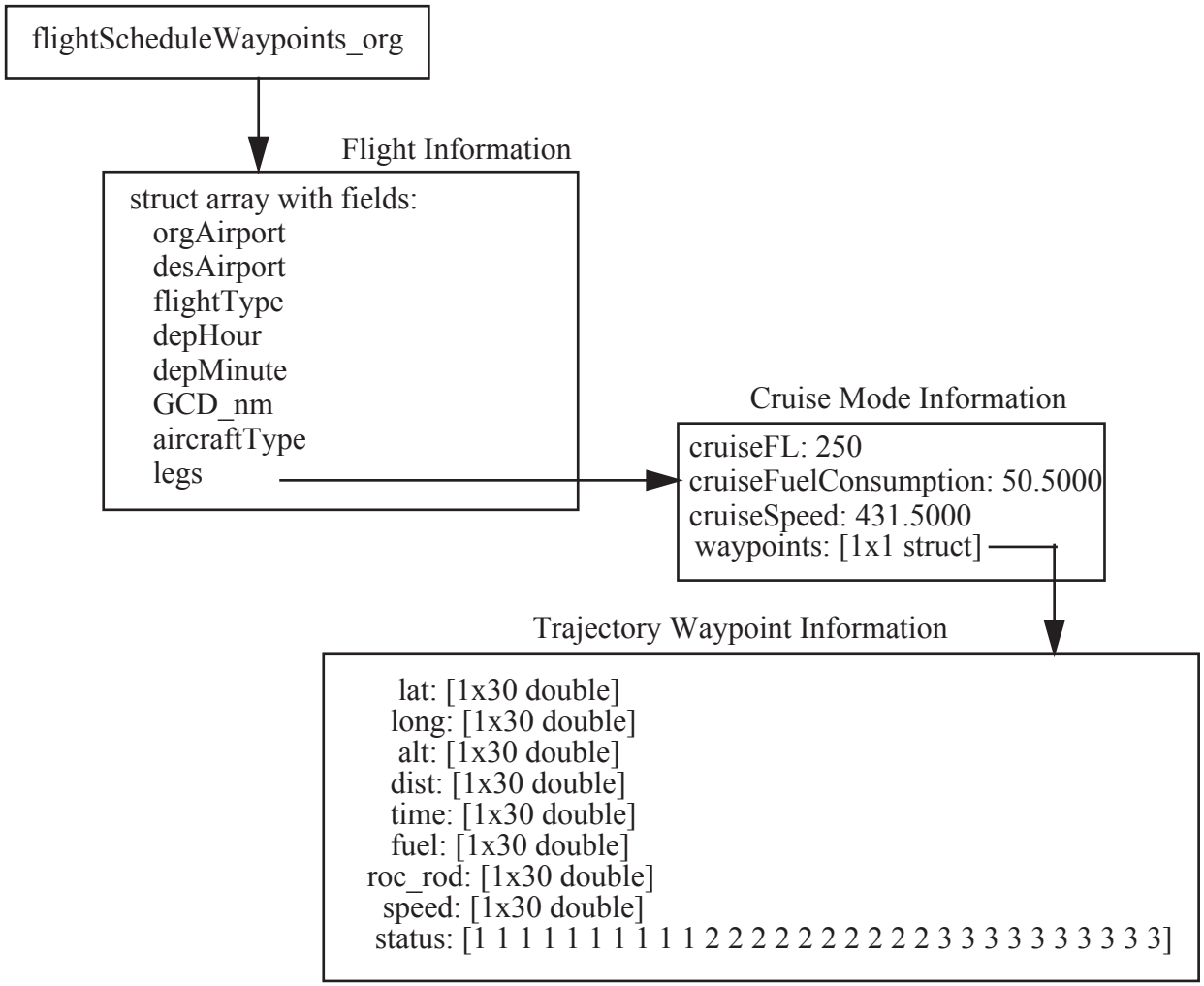
- Virginia Tech flight trajectory and fuel consumption implementation



Sample Implementation (VT Flight Trajectory Model)



VT Flight Trajectory Model Information
Data structure



Virginia Tech Neural Network-Based Fuel Burn Model (Description)

- Neural networks can accommodate and replace a variety of non-linear functions
- Fuel burn if a non-linear function with Mach number, temperature, aircraft weight, etc.
- Implement a neural network-based model to estimate fuel burn across an entire aircraft profile
- Investigate the possible use of the model in fast-time simulation airspace simulators (like SIMMOD or TAAM)

Virginia Tech Neural Network-Based Fuel Burn Model

Fuel burn evaluation of a particular aircraft for each mission is divided in six segments:

- Warm Up and Taxi
- Takeoff and Climb-Out
- Climb
- Cruise
- Descent
- Approach and Landing

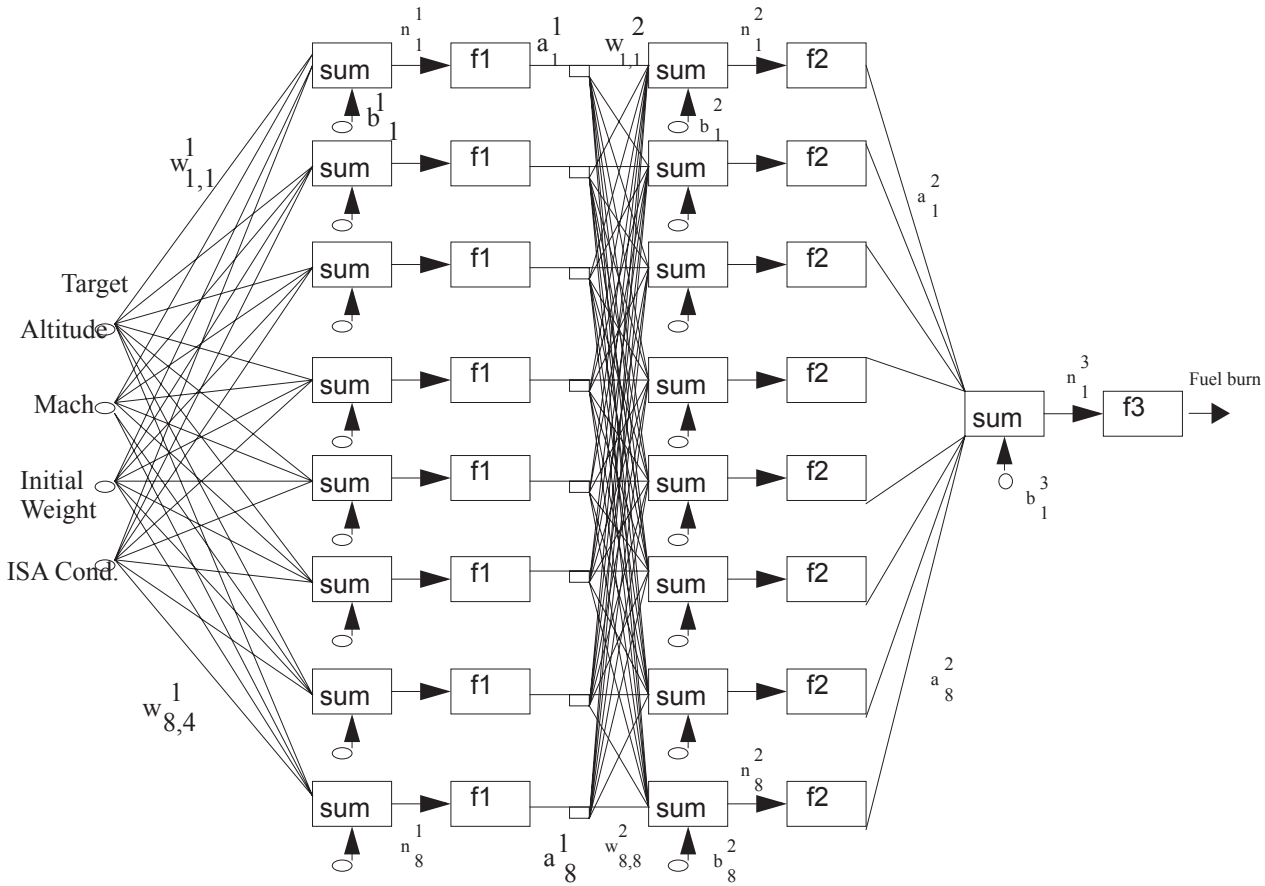
Topology Analysis

Number of layers	Number of neurons	Transfer functions	Mean Relative Error (%)	Standard Deviation of Error (%)	Floating point operations
2	4	tansig-purelin	0.611	0.0282	4.667E+08
2	6	tansig -purelin	0.606	0.0281	1.631E+09
2	8	tansig - purlin	0.628	0.0288	8.258E+08
3	4	logsig-tansig-pure-lin	0.617	0.0288	1.563E+10
3	6	logsig-tansig-pure-lin	0.610	0.0280	1.809E+10
3	8	logsig-tansig-pure-lin	0.604	0.0279	7.687E+08
3	10	logsig-tansig-pure-lin	0.656	0.0290	2.076E+09
3	12	logsig-tansig-pure-lin	0.667	0.0295	1.271E+09

Neural Network Approach

- For each segment of flight, there is a set of trained neural network weights and biases
- The raw data for training purposes is obtained from the flight manual of a particular aircraft and used as inputs to train the neural network
- Backpropagation scheme used to minimize weights and biases

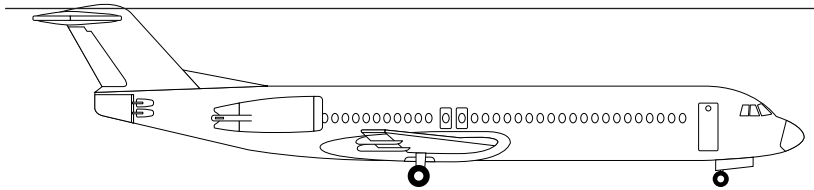
VT Fuel Burn Neural Network

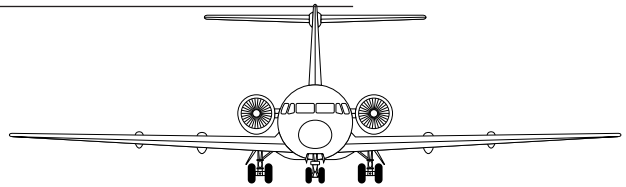


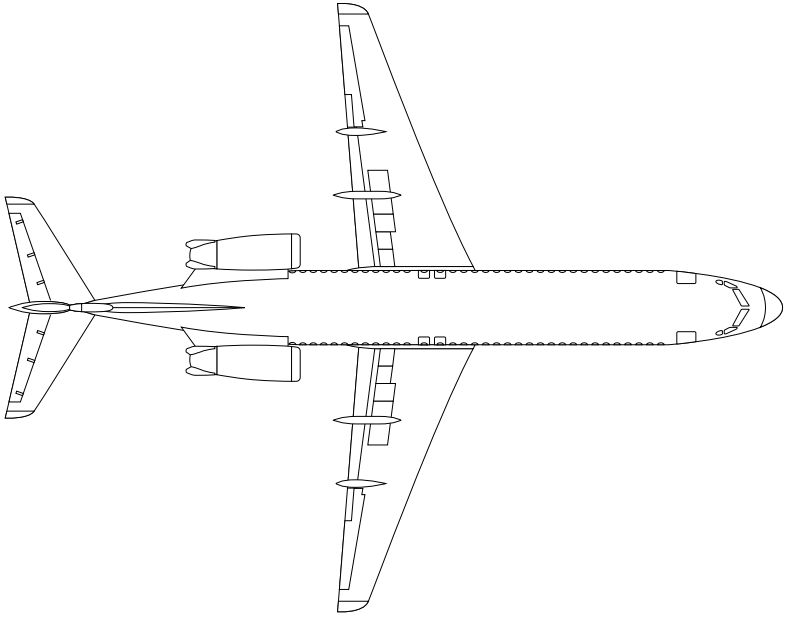
Where f1 - log-sigmoid transfer function
f2 - tansigmoid transfer function
f3 - purelin transfer function

Three layers (5 inputs and 1 output)

Aircraft Used to Demonstrate the Procedure

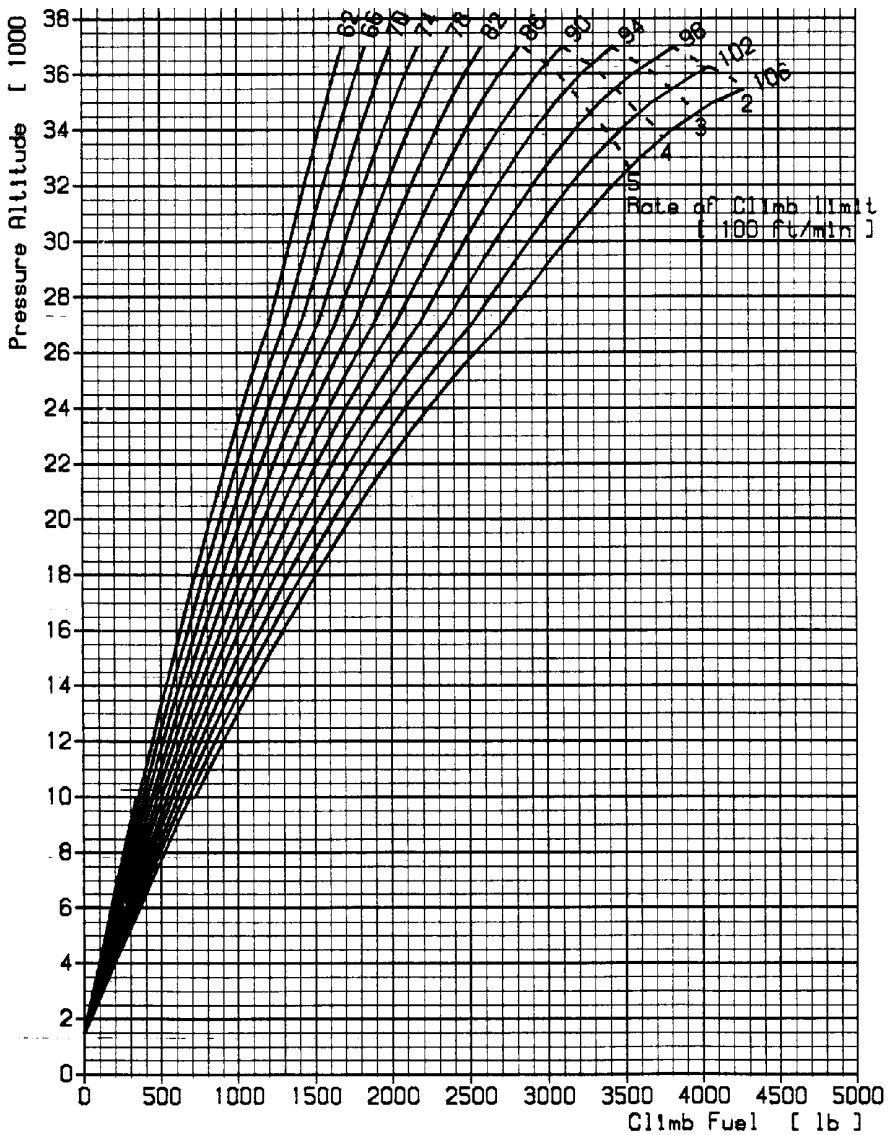






Aircraft Characteristic	Value
Wingspan	28.08 m
Length	35.33 m
Overall height	8.5 m
Cruising speed	Mach 0.75
Approach speed	64.3 m/s
Service ceiling	12,000 m
Landing field length	1,520 m
Takeoff field length	1,830 m
Range	3,100 km
Powerplant	2 x RR Tay 650

Fokker 100 Flight Manual Information

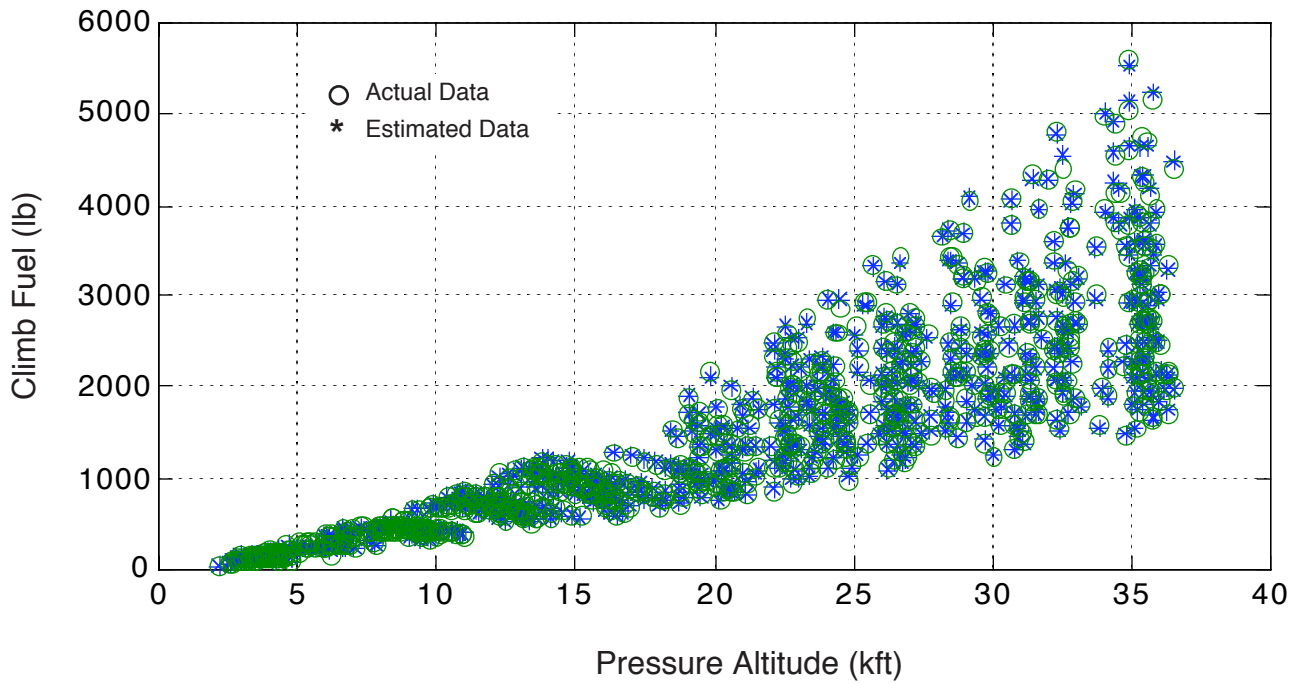


Neural Network Data Sets

Flight Phase	Number of Training Points	Number of testing points	Input Parameters	Output
Takeoff and Climb-Out	8	N/A	a) ISA Cond. b) Initial Weight (1000 lb.)	Fuel Burn Rate (lb/min)
Climb to Cruise Altitude	852 (Fuel) 854 (Distance) Total 1706	852 (Fuel) 854 (Distance) Total 1706	a) Initial Weight (1000 lb.) b) ISA Cond c) Mach Number d) Target Altitude (1000 ft)	a) Fuel Burn (lb.) b) Distance to Climb (nm)
Cruise	805	805	a) Cruise Mach Number b) Cruise Weight (1000 lb) c) Cruise Altitude (1000 ft)	Specific Air Range (nm/lb)
Descent	1210 (Fuel) 288 (Distance) Total 1498	1210 (Fuel) 288 (Distance) Total 498	a) Initial Weight (1000 lb) b) ISA Cond c) Mach Number d) Target Altitude (1000 ft.)	a) Fuel Burn (lb) b) Descent Distance (nm)

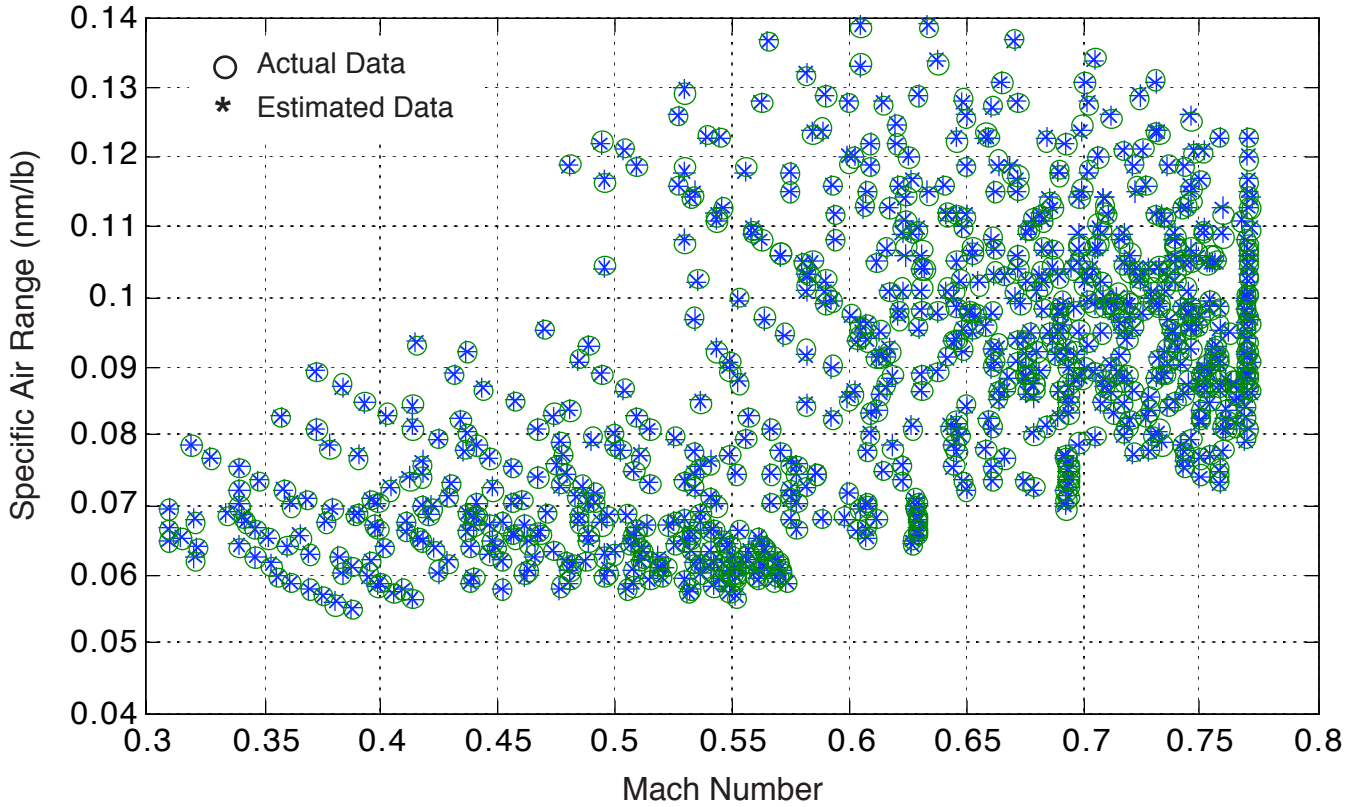
Sample Results (Climb)

The estimated climb fuel burn values show good agreement with the flight manual data



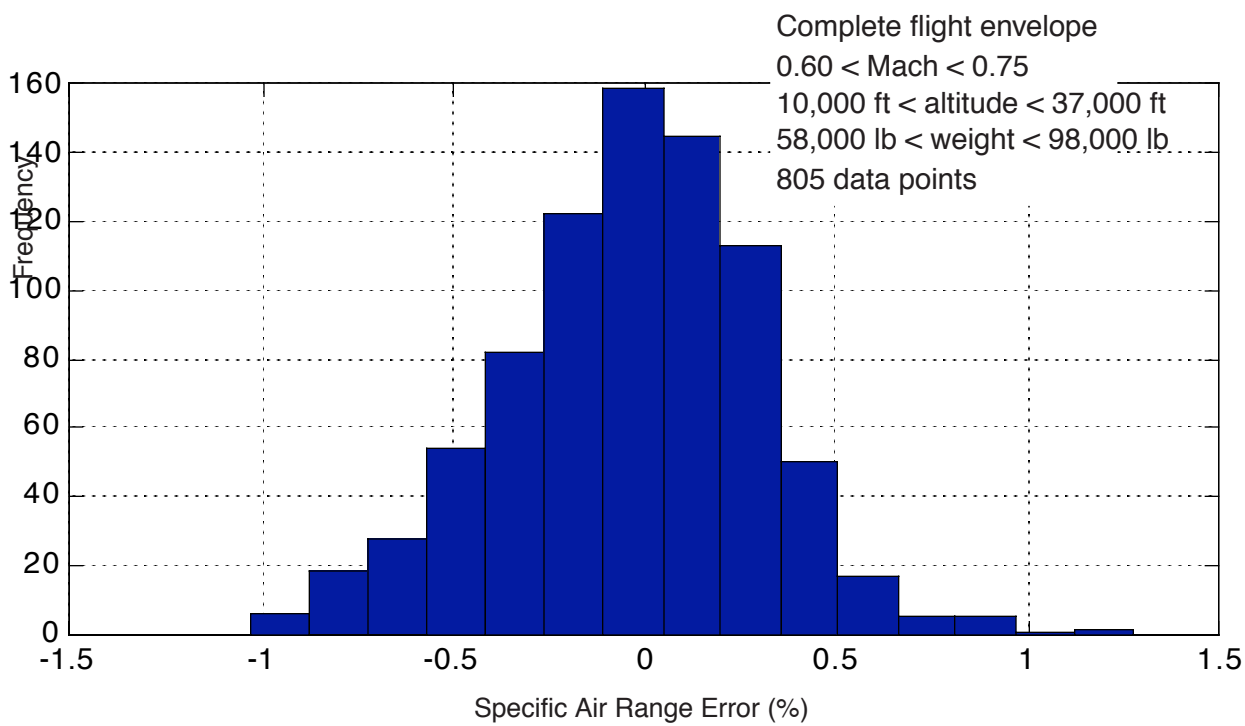
Sample Results (Calibration Step)

Comparison of actual vs. estimated SAR (Specific Range) values (using random values after training)



Sample Results (SAR)

The figure illustrates the errors in Specific Air Range (SAR) obtained using the neural network model



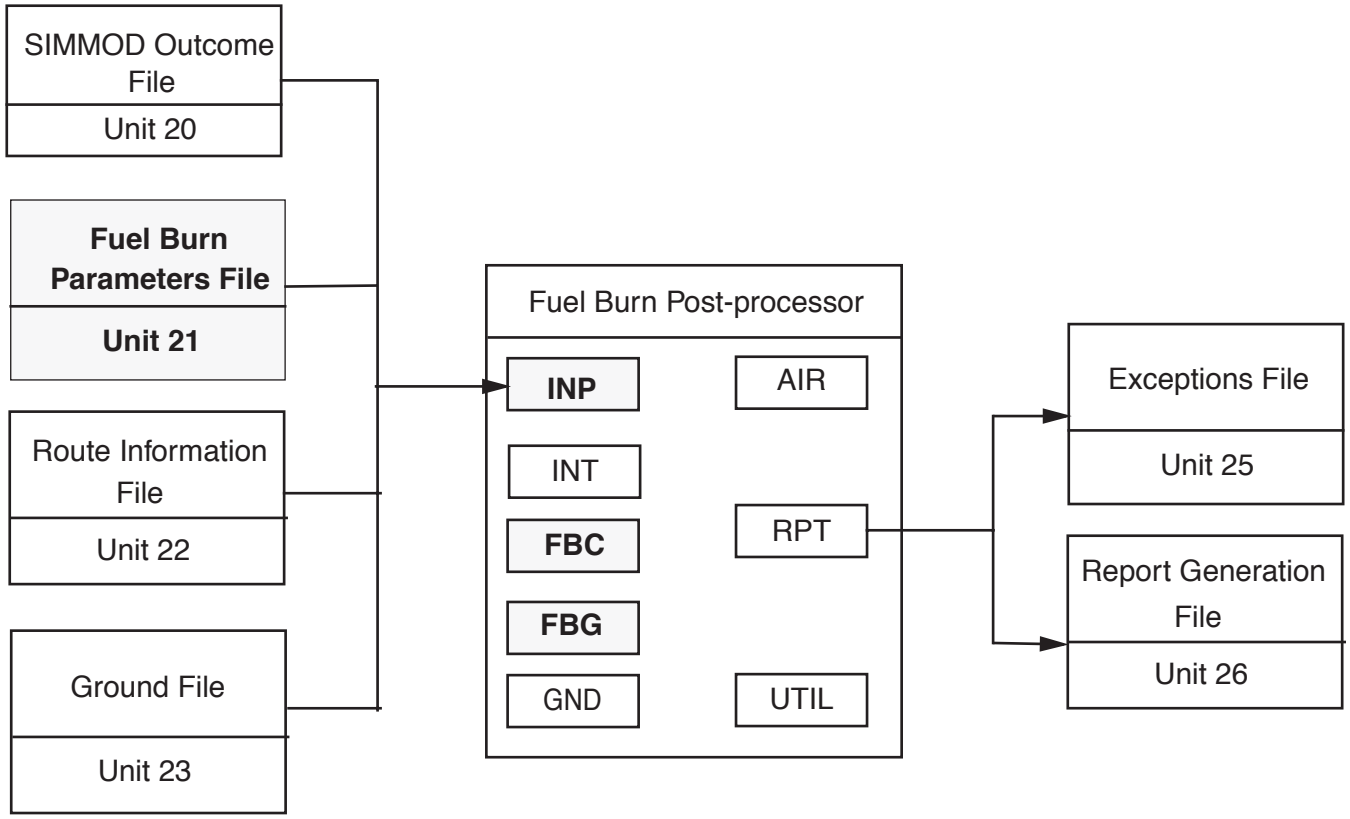
Sample Statistical Analysis of NN Calibration

Flight Phase	Mean Error (%)	Standard Deviation (%)	Hypothesis Test ($\alpha = 0.01$)
Climb			
• Distance	0.377	0.305	Accept
• Fuel	1.026	0.190	Accept
Cruise Specific Air Range	-0.034	0.334	Accept
Descent			
• Distance	1.760	1.860	Accept
• Fuel	1.423	1.177	Accept

Sample Results Using Fokker 100 across Various Routes in NAS

Flight	Cruise Flight Level (FL)	Distance (nm) / Time (hr.)	Flight Manual Fuel Burn (lb.)	Neural Net Fuel Burn (lb.)	Percent Difference (%)
ROA[a]-MDW[b]	280	448 / 1:08	6,457	6,546	1.37
	310	448 / 1:10	6,360	6,330	0.46
MIA[c]-DFW[d]	310	972 / 2:24	11,851	11,865	0.12
	350	972 / 2:13	11,510	11,544	0.29
ROA-LGA[e]	290	352 / 0:57	5,298	5,260	0.71
	330	352 / 0:58	5,343	5,429	1.61
ATL[f]-MIA	290	518 / 1:20	6,990	7,047	0.80
	330	518 / 1:21	7,009	7,082	1.04
ATL-DCA[g]	290	475 / 1:13	6,549	6,584	0.54
	330	475 / 1:14	6,590	6,654	0.97

a. ROA - Roanoke Regional Airport (Virginia)
b. MDW - Midway Airport (Illinois)
c. MIA - Miami International (Florida)
d. DFW - Dallas-Forth Worth International (Texas)
e. LGA - Laguardia Airport (New York)
f. ATL - Atlanta Hartsfield International Airport (Georgia)
g. DCA - National Airport (Virginia)

Implementation with SIMMOD

A method was developed to integrated the VT Neural Network Model with SIMMOD.