Analysis of Air Transportation Systems

Fundamentals of Aircraft Performance (1)

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Introductory Remarks

Air vehicles are significant different than their ground vehicle counterparts in three aspects:

- Most aircraft require a prepared surface to operate from which affects the overall capability of the vehicle to carry useful payload
- Aircraft operate in a dynamic atmospheric environment where changes in temperature, density, and speed of sound are drastic and cannot be neglected
- Aircraft mass expenditures are significant and thus need to be accounted for in the air vehicle performance analysis. For example, a Boeing 747-400 can takeoff at near 390 metric tons and yet land at its destination at 220

metric tons thus making the fuel expenditure a significant factor in how the vehicle performs along the flight path

- The analysis of NAS performance is related to the performance of the vehicles operating in it (i.e., airport runway and airspace sector capacity depends on aircraft characteristics)
- The analysis of airline operations requires a careful examination of the aircraft performance that matches a specific route segment (i.e., DOC, travel time, seating capacity, etc.)

Aircraft Performance Basics (International Standard Atmosphere)

Assumptions of the International Standard Atmosphere

- Linear variation in temperature with altitude up to 11,000 meters (Troposphere)
- Constant temperature betwen 11,000 and 82,300 ft (25.1 kilometers) in the so-called stratosphere region
- Linearly increasing temperature from 82,300 ft. and above
- Most of the analysis we do in this class requires knowledge of temeperature variations up to 15,600 meters (51,000 ft.) thus only the first two layers of the atmosphere are of interest to us

Basic Relationships to Uderstand the Atmosphere

Equation of state:

$$p = \rho R T \tag{1}$$

where:

p is the air pressure (N/m^2), R is the universal gas constant (287 N-m/^{o}K), ρ is the air density (kg/m^3), and T is the absolute air temperature (^{o}K)

Basic Relationships (Hydrostatic Equation)

the hydrostatic equation that relates air pressure, density and height above sea level of a fluid is,

$$dp = -\rho g dh \tag{2}$$

where: dp is rate of change in air pressure, g is the gravity constant (9.81 m/s^2), ρ is the air density (kg/m^3), and h is the altitude of the fluid element above sea level conditions (m)

Note: For derivations of these equations consult any fluid dynamics textbook or aerodynamics text

Atmosphere with Constant Temperature

Using equations (1) and (2),

$$\frac{dp}{p} = \frac{-g\,dh}{RT} \tag{3}$$

This equation can be integrated to obtain a basic relationship between atmospheric pressures at various layers in the atmosphere as a function of altitude

$$\int_{p_0}^{p} \frac{dp}{p} = \int_{h_0}^{h} \frac{-g\,dh}{RT} \tag{4}$$

where the subindex 0 denotes a reference condition.

Atmosphere with Constant Temperature

$$\frac{p}{p_0} = e^{-\left(\frac{g}{RT}\right)(h-h_0)} \tag{5}$$

and

$$\frac{\rho}{\rho_0} = e^{-\left(\frac{g}{RT}\right)(h-h_0)} \tag{6}$$

if the temperature is constant - isothermal layer (only true in the stratosphere).

In this analysis we have assumed a constant value for the gravity constant. This is a good approximation in the tropopause and stratosphere.

Atmosphere with Linear Temperature Variation

According to the International Standard Atmosphere (ISA), the variation of temperature is linear up to 11,000 meters. Then,

$$T = T_o + \lambda(h - h_o) = T_o + \frac{dT}{dh}(h - h_o) \qquad (7)$$

where: $\lambda = \dfrac{dT}{dh}$ is the temperature lapse rate with altitude (i.e., rate of change in temperature with altitude)

and T_0 is the reference temperature (typically sea level)

Atmosphere with Linear Temperature Variation

Since $dh = \dfrac{dT}{\lambda}$ using the equation (4) we find an expression to relate the change in pressure with altitude in a non-isothermal layer of the atmosphere,

$$\int_{p_0}^{p} \frac{dp}{p} = \int_{h_0}^{h} \frac{-g}{R\lambda} \frac{dT}{T} \tag{8}$$

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{-\left(\frac{g}{R\lambda}\right)} \tag{9}$$

Atmosphere with Linear Temperature Variation

Using the equation of state for two refence points (sea level denoted by subidex zero and at altitude denoted by a function of altitude:

$$\frac{p}{p_0} = \frac{\rho}{\rho_0}\left(\frac{T}{T_o}\right) \tag{10}$$

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{-\left[\left(\frac{g}{R\lambda}\right)-1\right]} \tag{11}$$

Reference Values of Interest at ISA Conditions

Constant	Value
T_0 reference temperature	273.2 ^{o}K
λ temperature lapse rate	-0.0065 ^{o}K per meter
ρ_o air density	1.225 kg/m^3
p_o air pressure	101,325 N/m^2
a speed of sound	340.3 m/s
R universal gas constant	287 N-m/^{o}K

International Standard Atmosphere

Characteristics of the International Standard Atmosphere.

Geopotential Altitude (m.)	Temperature (^oK) T	Density (kg/m^3) ρ	Speed of Sound (m/s) a
0	288.2	1.225	340.3
1000	281.7	1.112	336.4
2000	275.2	1.007	332.5
3000	268.7	0.909	328.6
4000	262.2	0.819	324.6
5000	255.7	0.736	320.5
6000	249.2	0.660	316.4
7000	242.7	0.589	312.3
8000	236.2	0.525	308.1

Characteristics of the International Standard Atmosphere.

Geopotential Altitude (m.)	Temperature (^oK) T	Density (kg/m^3) ρ	Speed of Sound (m/s) a
9000	229.7	0.466	303.8
10000	223.2	0.413	299.5
11000	216.7	0.364	295.1
12000	216.7	0.311	295.1
13000	216.7	0.266	295.1
14000	216.7	0.227	295.1
15000	216.7	0.194	295.1
16000	216.7	0.169	295.1

Important Aircraft Speed Terms to Know

Indicated Airspeed (IAS) - is the speed registered in the cockpit instrument

True Airspeed (TAS) - is the actual speed of the vehicle with respect of the mass of air surrounding the aircraft (accounts for compressibility effects)

Calibrated Airspeed (CAS) - similar to IAS but corrected for instrument position errors (airflow problems outside the vehicle).

Ground speed (GS) - TAS corrected for wind

Stalling Speed (V_{stall}) - minimum speed for safe flight

Mach Number - ratio of the aircraft speed to the speed of sound, a (note a varies with altitude)

Mach number can be easily computed using the following equation,

$$a = \sqrt{\gamma R T} \tag{12}$$

where: R is the universal gas constant (287 N-m/^{o}K), T is the air temperature (^{o}K) and γ is the ratio of specific heat at constant volume ($\gamma = 1.4$ for air)

Air Compressibility Effects

A mathematical expression to estimate true airspeed (in terms of true Mach number) from CAS follows:

$$M_{true} = \sqrt{5\left[\left[\frac{\rho_0}{\rho}\left(\left[1 + 0.2\left(\frac{V_{CAS}}{661.5}\right)^2\right]^{3.5} - 1\right) + 1\right]^{0.286} - 1\right]} \qquad (13)$$

where: M_{true} is the true mach number, V_{CAS} is the calibrated airspeed in knots (CAS = IAS) in our analysis, ρ_0 is the atmospheric density at sea level, ρ is the density at the altitude the aircraft is flying, and the constants 0.2 and 661.5 account for the specific heat of the air and the speed of sound at sea level (in knots), respectively.

Defining true mach number (M_{true}) as the ratio of the true aircraft speed (V_{TAS}) and the speed of sound (a) at the flight level in question we have,

$$V_{TAS} = aM_{true} \tag{14}$$

Example Computation

Boeing 737-300 (a medium size jet transport) flies at 250 knots (IAS) at an altitude of 5.0 km. in a standard atmosphere. What is TAS?

A quick glance at the ISA Table reveals that air density at 5.0 km. is about 0.736 kg/m^3 thus resulting in a true mach number of 0.4824 (use Equation 10).

Since the speed of sound at that altitude is 320.5 m/s (see Table) then the true airspeed of the aircraft is 154.62 m/s or 300.56 knots.

Sample Computation (continuation)

Note that in this case there is a difference of 50.56 knots between IAS and TAS.

As the aircraft climbs the value of TAS increases even if IAS remains constant. True Airspeed (TAS) is needed to estimate Ground Speed (GS).

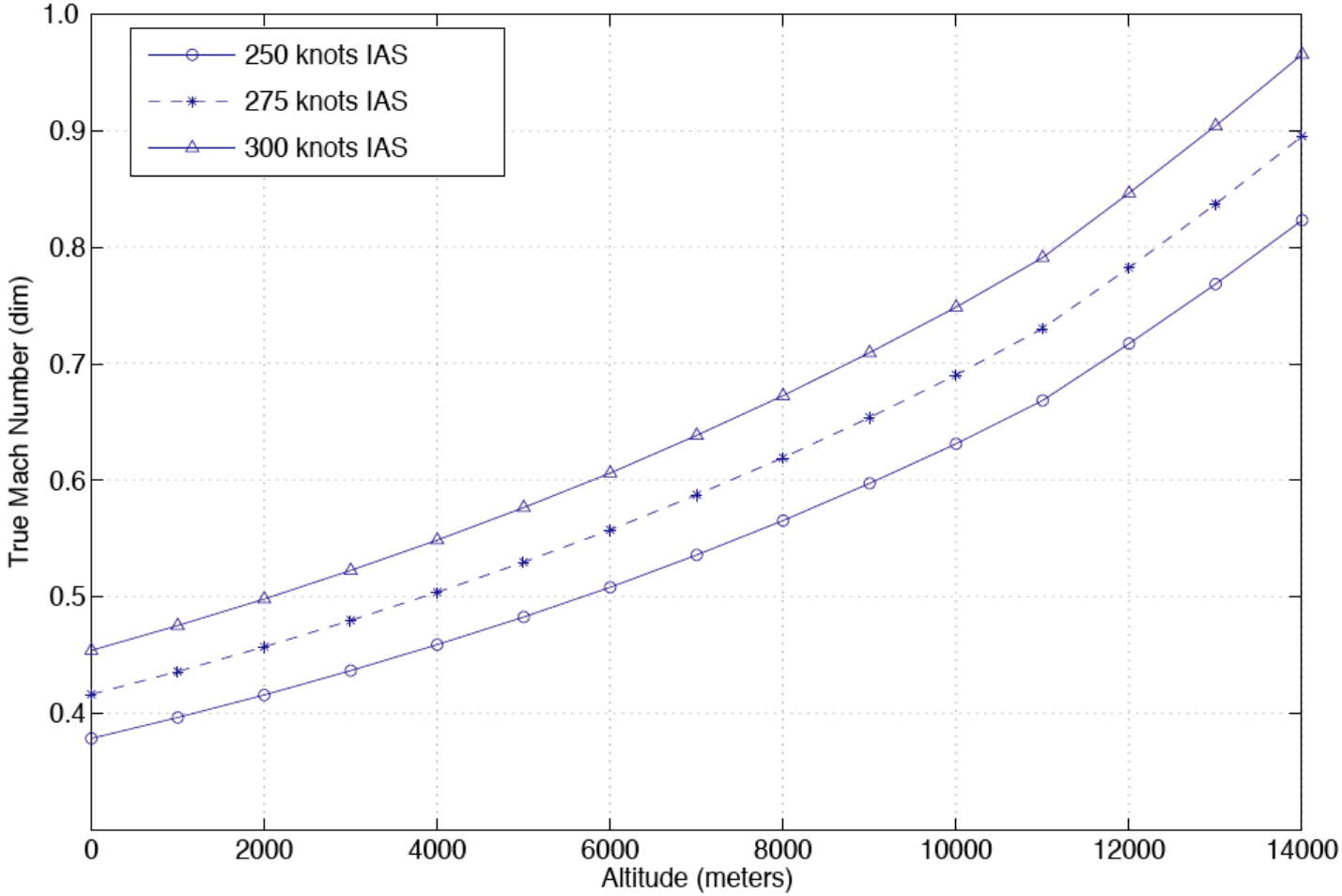
GS is ultimately responsible for the travel time between airports and thus it is important to learn how to estimate TAS for any feasible flight condition.

Later analysis will introduce more details on how to estimate travel times between Origin-Destination airports

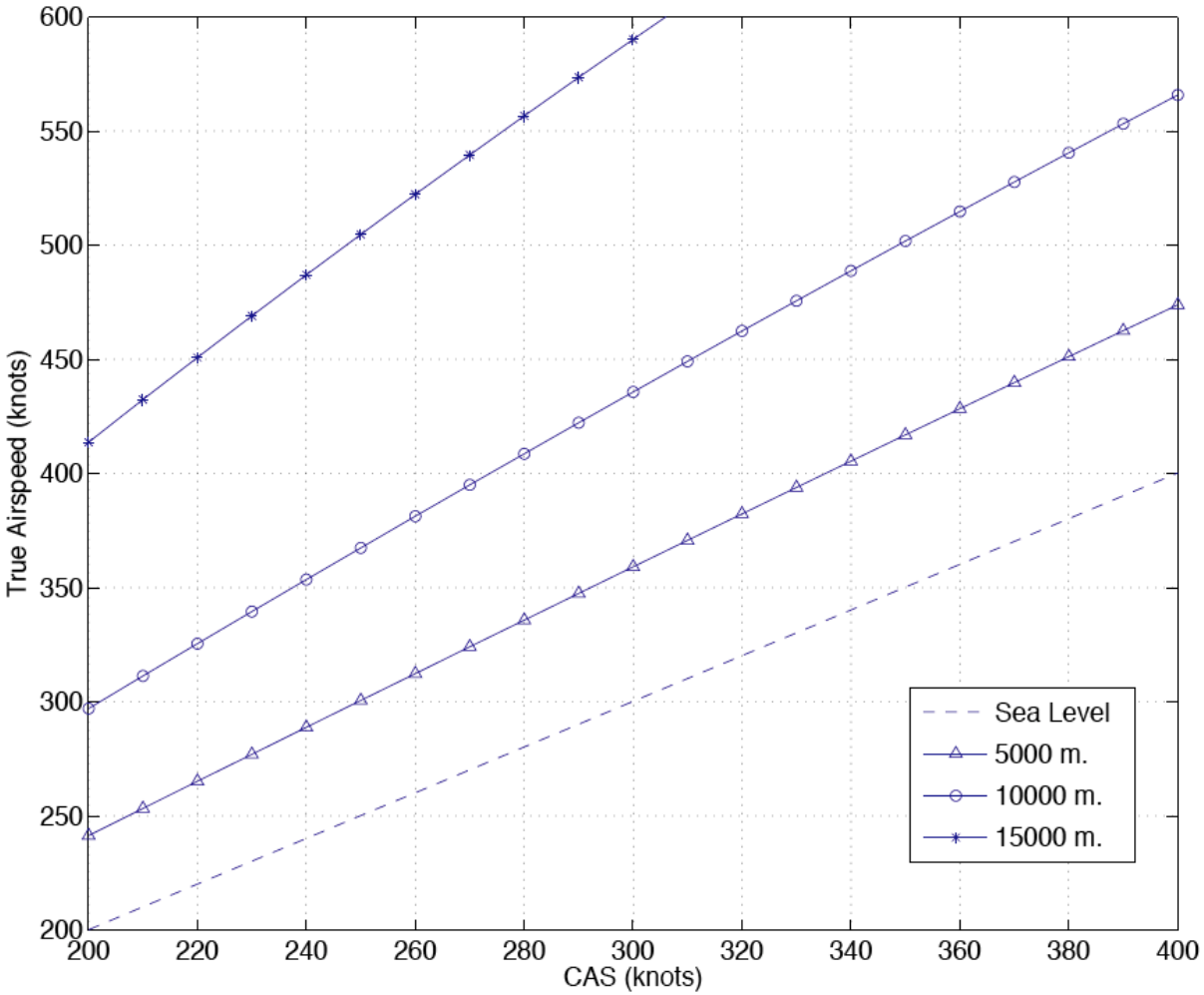
Sample Matlab Code Used (ISAM.m)

```
% Function to estimate: aircraft true mach number, air density, speed of sound
% and temperature given
% alt = altitude (m)
% ias = indicated airspeed (knots)

function [mtrue,a_alt,rho,temp] = isam(alt,ias)

rho_zero = 1.225;          % density at sea level (kg/m-m-m)
load atmosphere;           % loads ISA atmospheric tables

h = atmosphere(:,1);       % vector with values of altitude
t = atmosphere(:,2);       % vector with values of temperature
r = atmosphere(:,3);       % vector with values of density
a = atmosphere(:,4);       % vector with values of speed

rho     = interp1(h,r,alt,'cubic'); % interpolates to get density
mtrue   = sqrt(5 * ((rho_zero./rho .*((1 + 0.2 .* (ias./661.5).^2)...
          .^3.5 -1) + 1).^0.286 -1));
a_alt   = interp1(h,a,alt,'cubic'); % gets speed of sound
temp    = interp1(h,t,alt,'cubic'); % gets temperature
```

Plot of True Mach Number vs. Altitude



Plot of CAS vs. TAS (Subsonic Aircraft)



Aircraft Performance Estimation (Runway Length)

Aircraft Runway Length Performance Estimation

Critical issue in airport engineering and planning (errors in runway length are costly to the operator and perhaps unsafe)

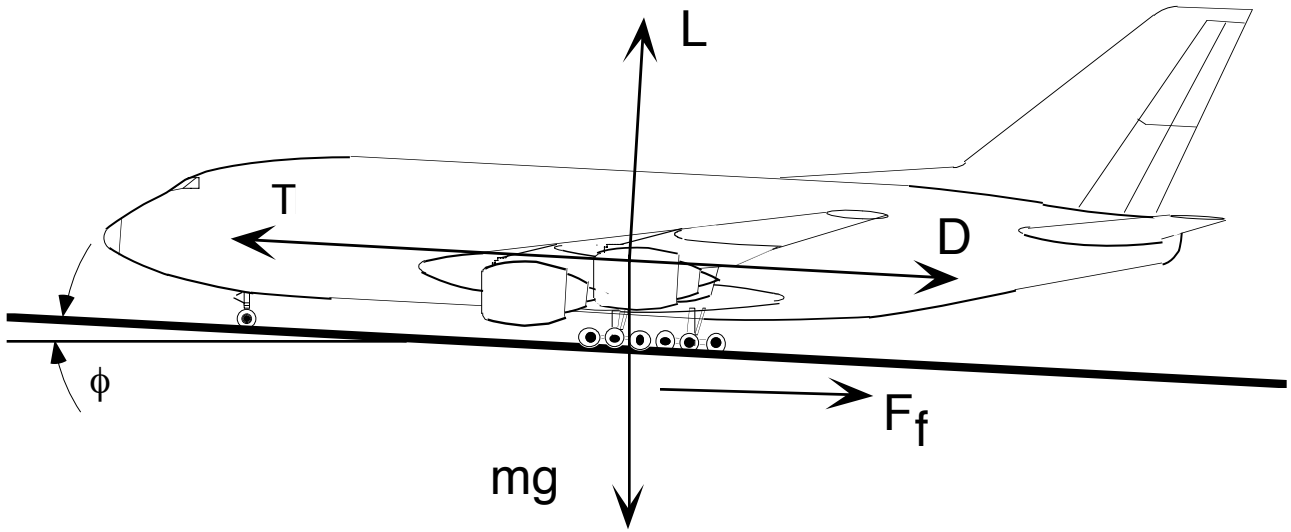


Figure 1. Forces Acting in the Aircraft During Takeoff.

Nomenclature

T - thrust force (also called tractive effort) provided by the vehicle powerplant

L - lifting force provided by the wing-body of the vehicle

D - drag force to the vehicle body, nacelle(s), landing gears, etc.,

F_f - friction force due to rolling resistance

The functional form of these forces has been derived from dimensional analysis (review your math course notes) and from extensive knowledge of fluid mechanics (wind tunnels and water tank experiments)

Functional Forms of the Forces

The functional form of these forces is as follows:

$$L = \frac{1}{2}\rho V^2 S C_L \tag{15}$$

$$D = \frac{1}{2}\rho V^2 S C_D \tag{16}$$

$$T = f(V, \rho) \tag{17}$$

$$F_f = (mg\cos\phi - L) f_{roll} \tag{18}$$

V is the vehicle speed (TAS), ρ is the air density (kg/m^3), S is the aircraft gross wing area, C_L is the lift coefficient (nondimensional), C_D is the drag coefficient (nondimensional), f_{roll} is the rolling friction coefficient (nondimensional), T is the engine thrust iin Newtons and

ϕ is the angle comprised between the runway plane and the horizontal

Notes on Various Parameters

1) C_L and C_D are specific to each airframe-flap configuration

2) f_{roll} is usually a function of runway surface conditions and aircraft speed

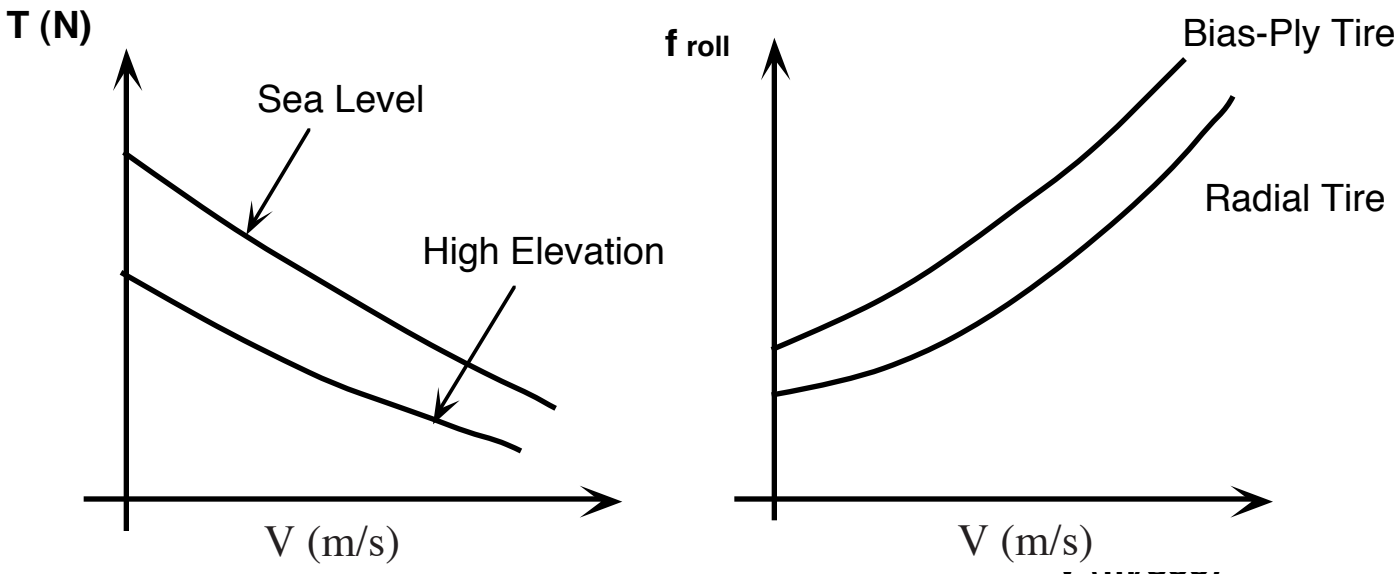


Figure 2. Typical Variations of T and f_{roll} with Aircraft Speed.

Estimating Runway Acceleration

Using Newton's second law and summing forces in the horizontal direction of motion (x),

$$ma_x = T(V, \rho) - D - (mg\cos\phi - L)f_{roll} - mg\sin\phi \tag{19}$$

linear variations of T (tractive effort or thrust) and f_{roll} can be assumed to be linear with respect to airspeed for the range of speed values encountered in practice. For small angles this equation can be expressed as,

$$ma_x = T(V, \rho) - D - (mg - L)f_{roll} \tag{20}$$

$$ma_x = T(V, \rho) - \frac{1}{2}\rho V^2 s c_D - \left(mg - \frac{1}{2}\rho V^2 S C_L\right) f_{roll} \tag{21}$$

$$a_x = \frac{1}{m}(T(V, \rho) + \frac{1}{2}\rho V^2 S(C_L f_{roll} - C_D) - mgf_{roll} \tag{22}$$

Remarks About the Aircraft Acceleration Equation

- The acceleration capability of the aircraft decreases as speed is gained during the takeoff roll due to a reduction in the thrust produced by the engines
- If Eq. 22 is **integrated twice** between an initial speed, V_0 and the lift-off speed, V_{lo} the distance traversed during the takeoff roll can be found
- Usually this requires a computer simulation since many parameters such as T and f_{roll} vary with speed (time varying) making the coefficient of the differential equation of motion time dependent.

Aerodynamic Coefficients

- The flap setting affects C_D and C_L and hence affects acceleration and runway length required for a takeoff. Typical variations of C_D with flap angle are shown below

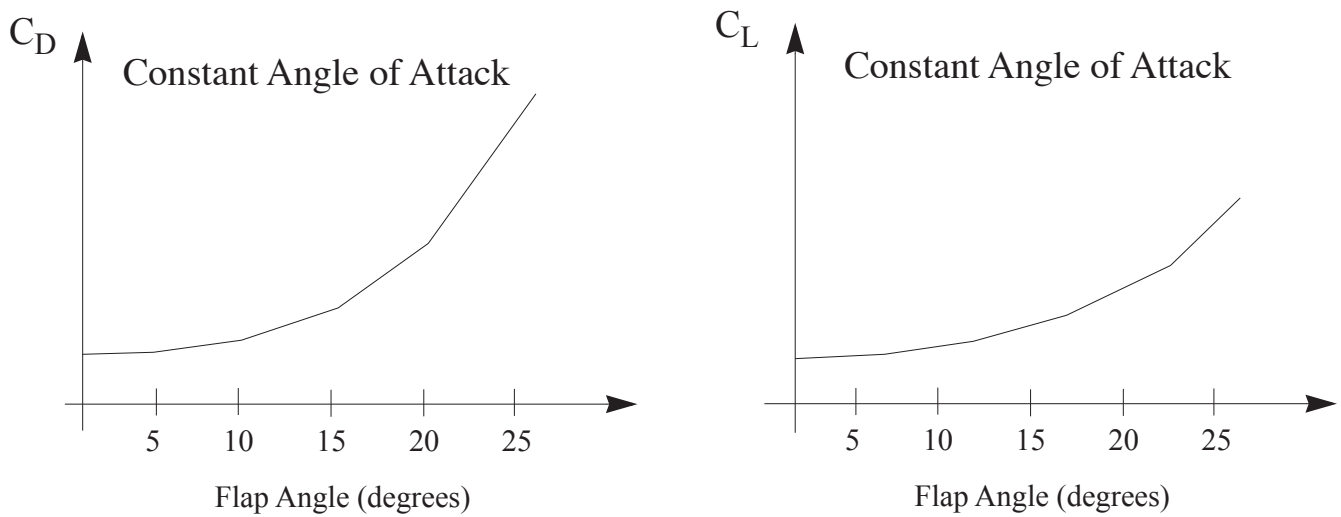
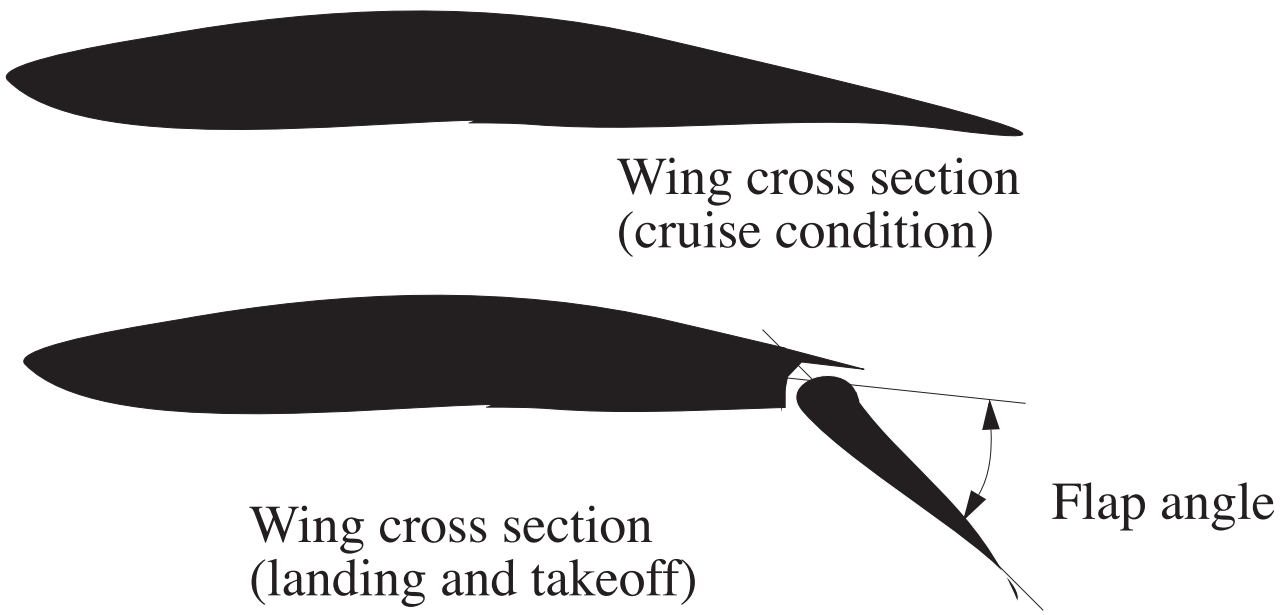


Figure 3. Typical Variations of C_D and C_L with Aircraft Wing Flap Angle.

Flap Angle

- Angle formed between the flap chord and the wing chord
- Flaps are used to increase lift (but they increase drag too) during takeoff and landing maneuvers
- Flaps reduce the stalling speed of the aircraft



Remarks About Aerodynamic Coefficients

- An increase in flap angle increases both C_L and C_D. However, these increments are not linear and consequently are more difficult to interpret
- Increasing the flap angle (δ_f) increases C_L and thus reduces the lift-off speed required for takeoff due to an increase in the lifting force generated.
- Increments in flap angle increases the value of C_D more rapidly which tends to reduce more drastically the acceleration of the aircraft on the runway thus increasing the runway length necessary to reach the lift off speed

Remarks

- The mass of the aircraft affects its acceleration (according to Newton's second law).
  - + Larger takeoff masses produce corresponding increments in the runway length requirement.
- The density of the air, ρ decreases with altitude
  - + Lower thrust generation capability at high airfield elevations
  - + The runway length increases as the field elevation increases
  - + The density also affects the second and third terms in Equation 2.10 (less drag at higher altitude)

Aircraft Operational Practices (Takeoff)

- At **small flap settings** (i.e., 5 or 10 degrees) the takeoff runway length is increased due to small gains in C_L (little increase in the lifting force). Useful for high-hot takeoff conditions.
- At **medium flap angle settings** (15-25 degrees) the gains in lift usually override those of the drag force. These are the flap settings typically used for takeoff **except** under extremely abnormal airport environments such as high elevation, hot temperature airport conditions and high aircraft weights or a combination of both. Note that the **maximum allowable takeoff weight (MTOW)** increases as the takeoff flap setting is reduced.

- At **large flap angles** (> 25 degrees) C_D **is excessive** and the airplane requires unreasonable large takeoff runway lengths. These flap settings are only used for landing since pilots want to land at the lowest speed possible thus reducing runway length.

Application of Equations of Motion to Takeoff Runway Length Requirements

- Equation 22 describes the motion of an air vehicle as it accelerates on a runway from an initial speed V_o to a final liftoff speed V_{lof}
- This equation can be integrated twice with respect to time to obtain the distance traveled from a starting point to the point of liftoff
- With a little more effort we could also predict the distance required to clear a 35 ft. obstacle as required by Federal Aviation Regulations Part 25 or 23 that sets airworthiness criteria for aircraft in the U.S.
- Airport engineers use tabular or graphical data derived from this integration procedure

A Word on Stalling and Lift-off Speeds

The stalling speed can be estimated from the basic lift equation

$$L = \frac{1}{2}\rho V^2 S C_L$$

Under steady flight conditions $L \cong mg$ so,

$$V = \sqrt{\frac{2mg}{\rho S C_L}}$$

define C_{Lmax} as the maximum attainable lift coefficient, then

$$V_{stall} = \sqrt{\frac{2mg}{\rho S C_{Lmax}}}$$

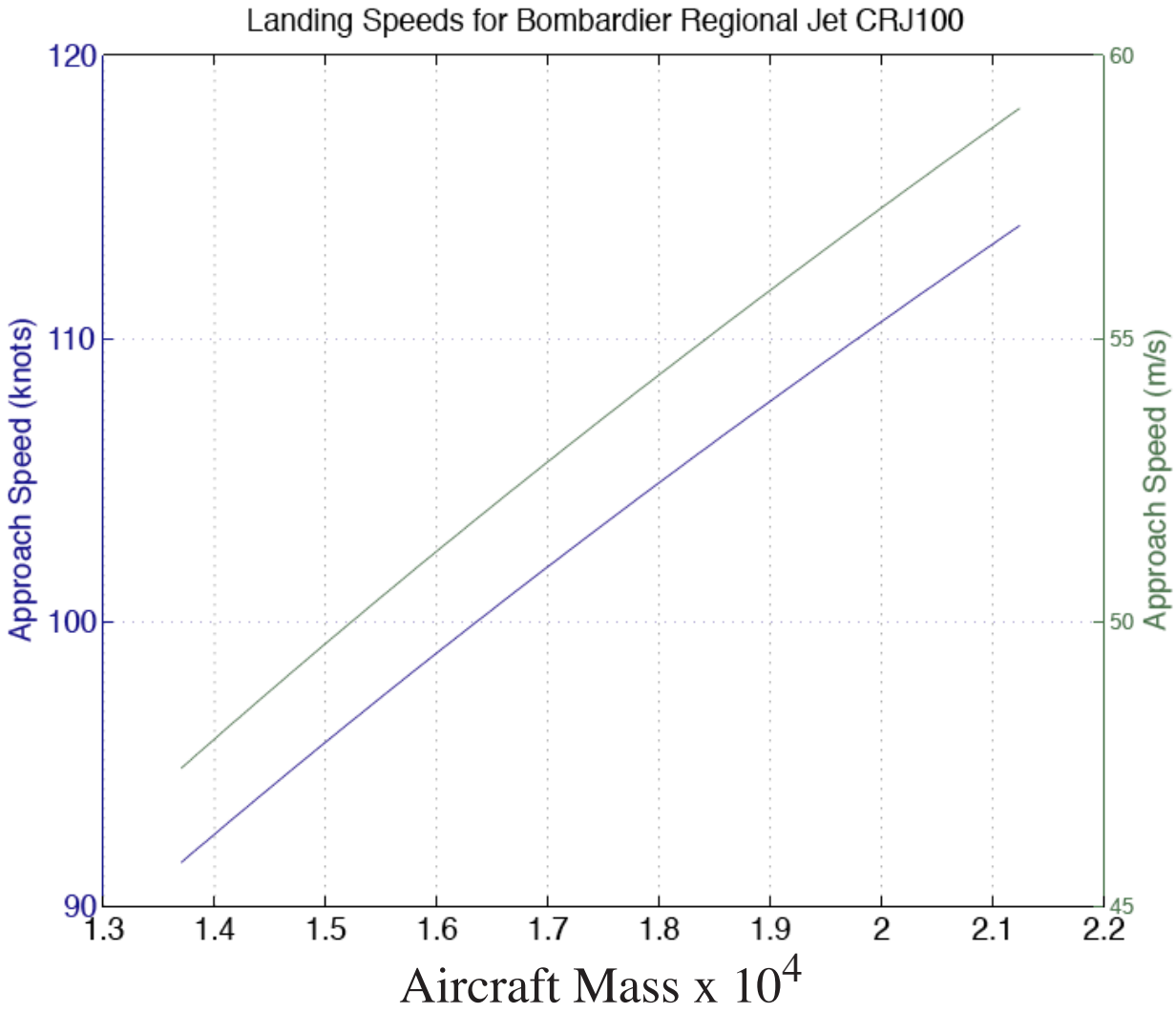
FAR Regulation Principles

Regulations (FAR 25) specify that:

- Aircraft should lift off at 10% above the stalling speed (V_{lof})
- Aircraft climb initially at 20% above the stalling speed (V_2)
- Aircraft speed during a regular approach be 30% above the stalling speed (V_{app})
- During takeoff aircraft should clear an imaginary 11 m (35 ft.) obstacle
- During landing aircraft should cross the runway threshold 15 m (50 ft.) above ground)

These considerations are necessary to estimate takeoff and landing distances (and thus size runway length)

Variation of Approach Speed with Aircraft Mass



Integration of Acceleration Equation

First obtain the aircraft speed at time t,

$$V_t = \int_{V_o}^{V_{lof}} \frac{1}{m}\left(T(V,\rho) + \frac{1}{2}\rho V^2 S(C_L f_{roll} - C_D) - mg f_{roll}\right)dt \qquad (23)$$

Now get the distance traveled, S_t

$$S_t = \int_{o}^{D_{lof}} V_t dt \qquad (24)$$

Sample Results (Boeing 727-200 Data)

The following results apply to a medium-size transport aircraft

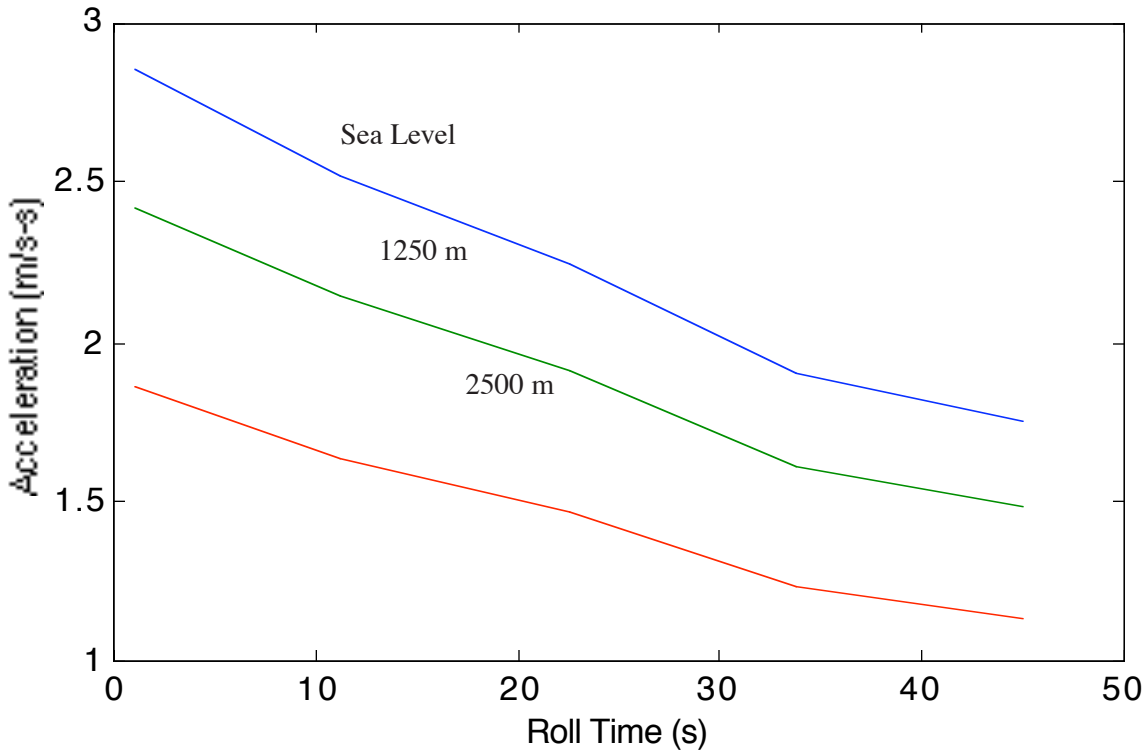


Figure 4. Sensitivity of Aircraft Acceleration vs. Field Elevation.

Aircraft Speed During Takeoff Roll

Note how speed increases at a nonlinear pace

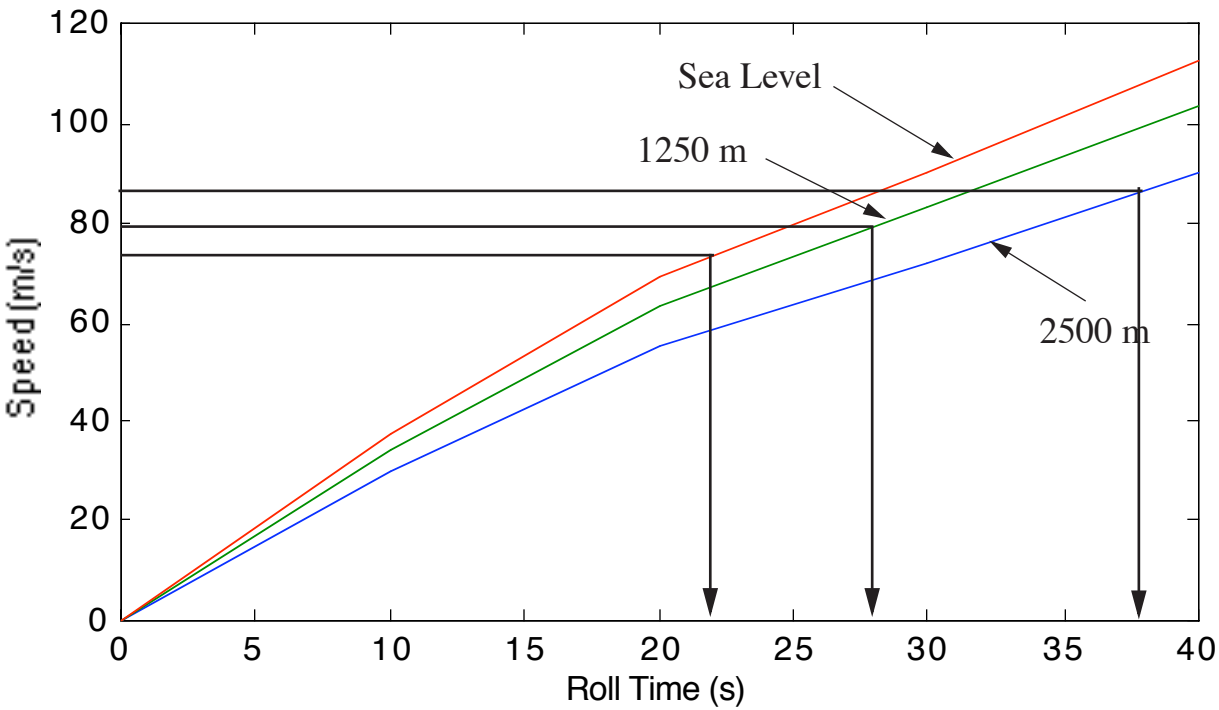


Figure 5. Sensitivity of Aircraft Speed vs. Field Elevation.

Distance Traveled During the Takeoff Roll

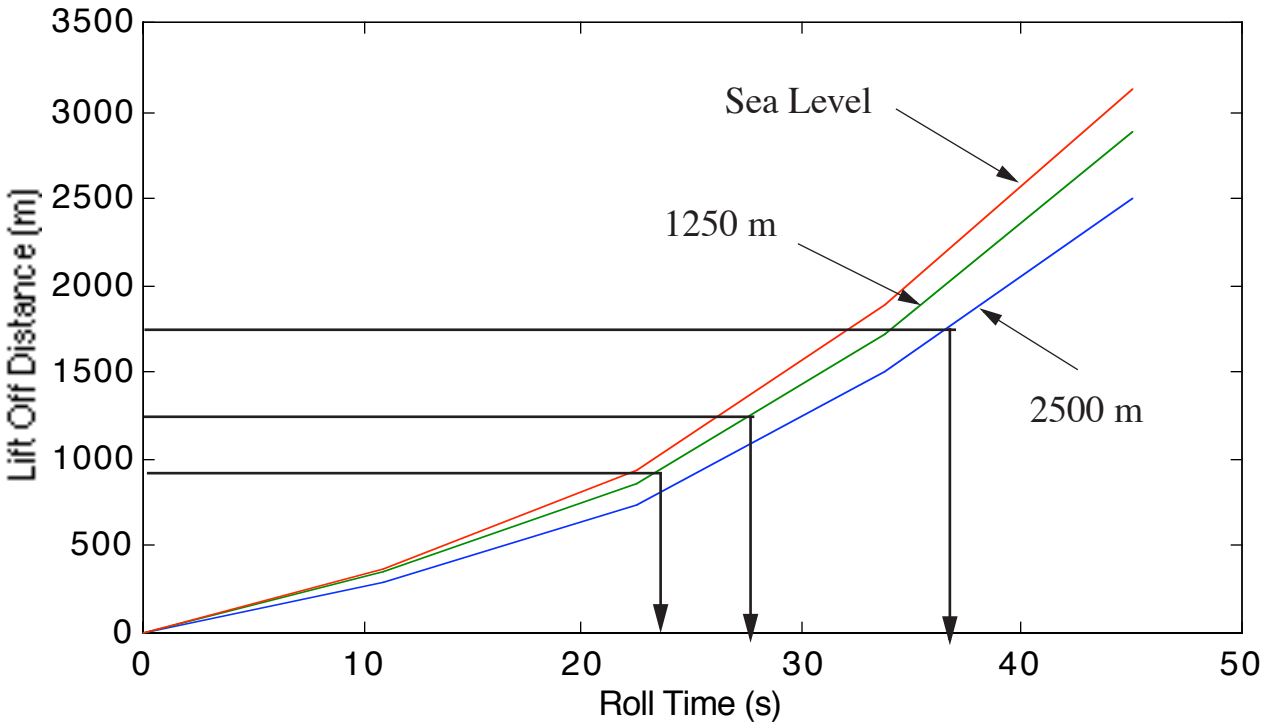


Figure 6. Lift-Off Distance vs. Field Elevation.

Takeoff Roll Distance vs. Aircraft Mass

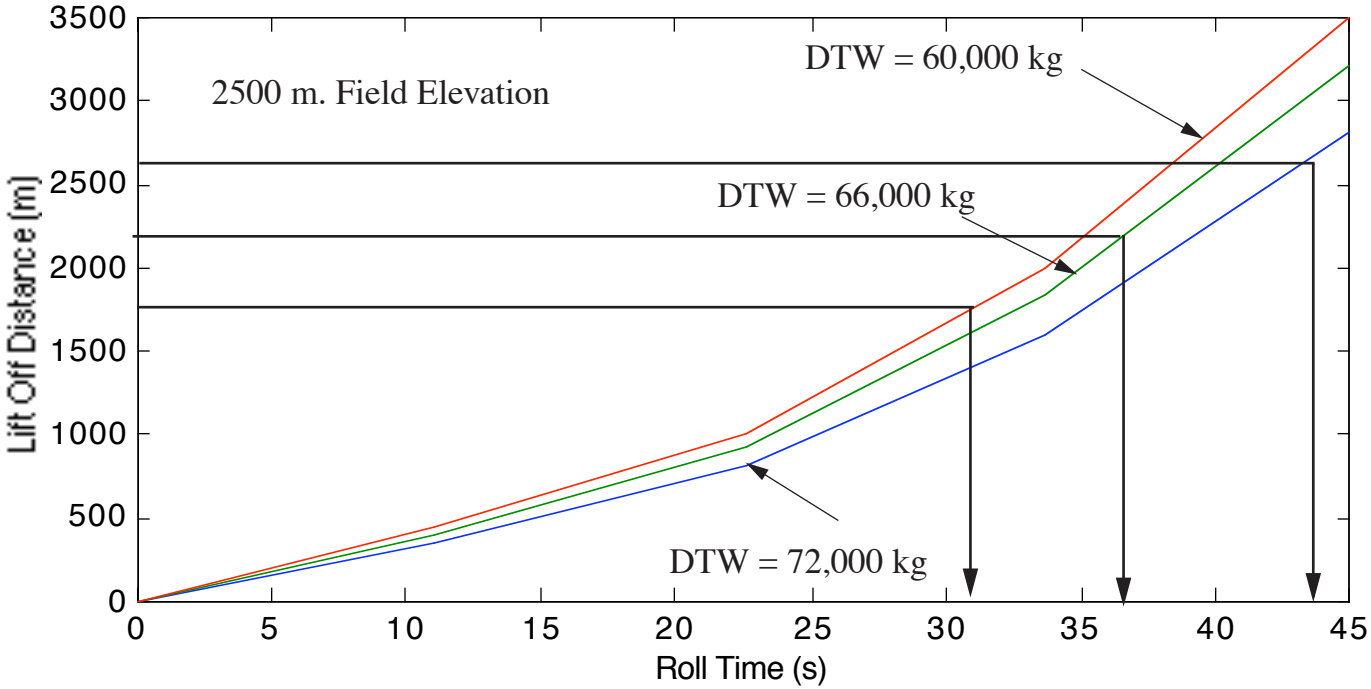


Figure 7. Lift-Off Distance vs. Aircraft Weight.

Regulatory Method to Estimate Runway Length at Airports

General Procedure for Runway Length Estimation (Runway Length Components)

Runways can have three basic components:

- Full strength pavement (FS)
- Clearways (CL)
- Stopways (SW)

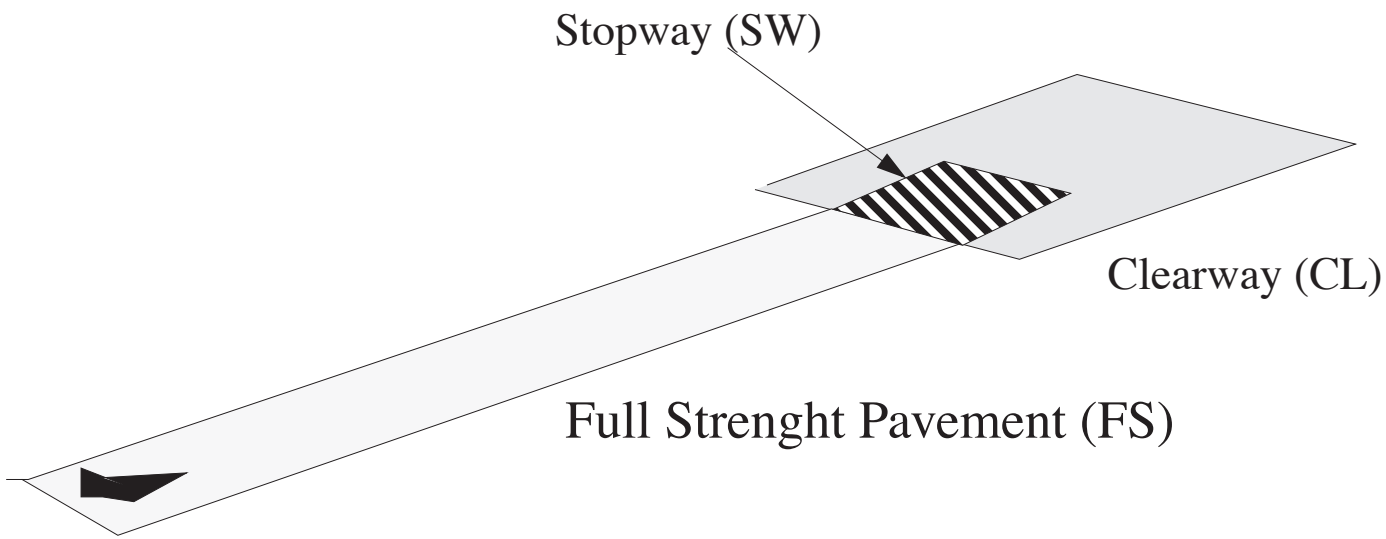
Full strength pavement should support the full weight of the aircraft

Clearway is a prepared area are beyond FS clear of obstacles (max slope is 1.5%) allowing the aircraft to climb safely to clear an imaginary 11 m (35’ obstacle)

Stopway is a paved surface that allows and aircraft overrun to take place without harming the vehicle structurally (cannot be used for takeoff)

Runway Components

Each runway end will have to be considered individually for runway length analysis



FAR Certification Procedures

FAR 25 (for turbojet and turbofan powered aircraft) consider three cases in the estimation of runway length performance

- Normal takeoff (all engines working fine)
- Engine-out takeoff condition
  - Continued takeoff
  - Aborted takeoff
- Landing

All these cases consider stochastic variations in piloting technique (usually very large for landings and smaller for takeoffs)

Regulations for piston aircraft do not include the normal takeoff case (an engine-out condition is more critical in piston-powered aircraft)

Nomenclature

FL = field length (total amount of runway needed)

FS = full strength pavement distance

CL = clearway distance

SW = stopway distance

LOD = lift off distance

TOR = takeoff run

TOD = takeoff distance

LD = landing distance

SD = stopping distance

D35 = distance to clear an 11 m (35 ft.) obstacle

Landing Distance Case

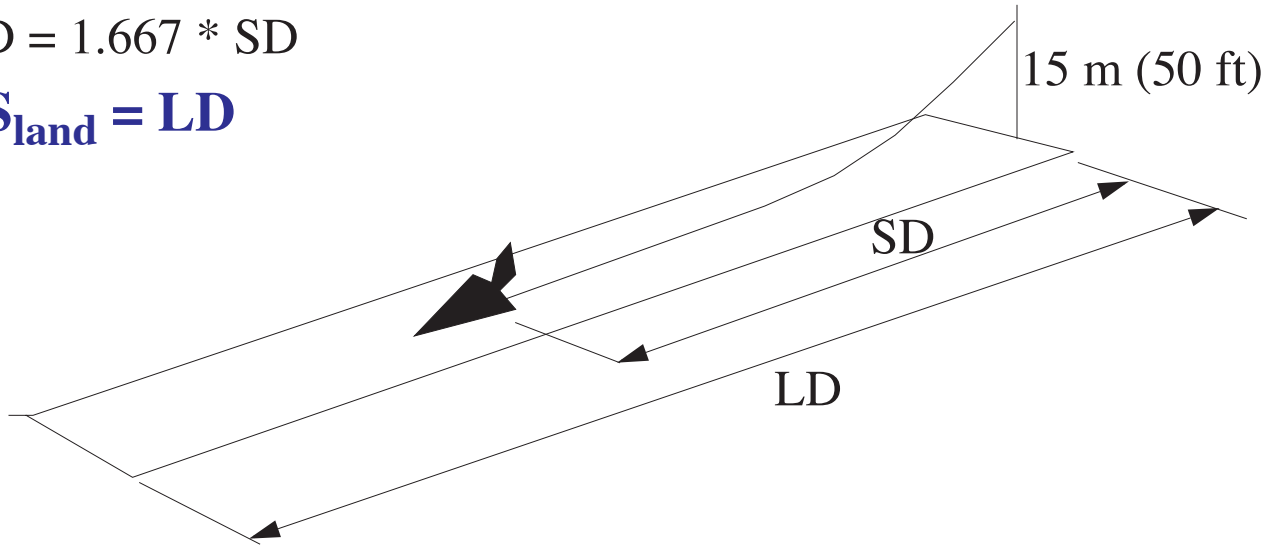
The landing distance should be 67% longer than the demonstrated distance to stop an aircraft

Large landing roll variations exist among pilots

Example touchdown point variations (μ=400 m, σ=125 m for Boeing 727-200 landing in Atlanta)

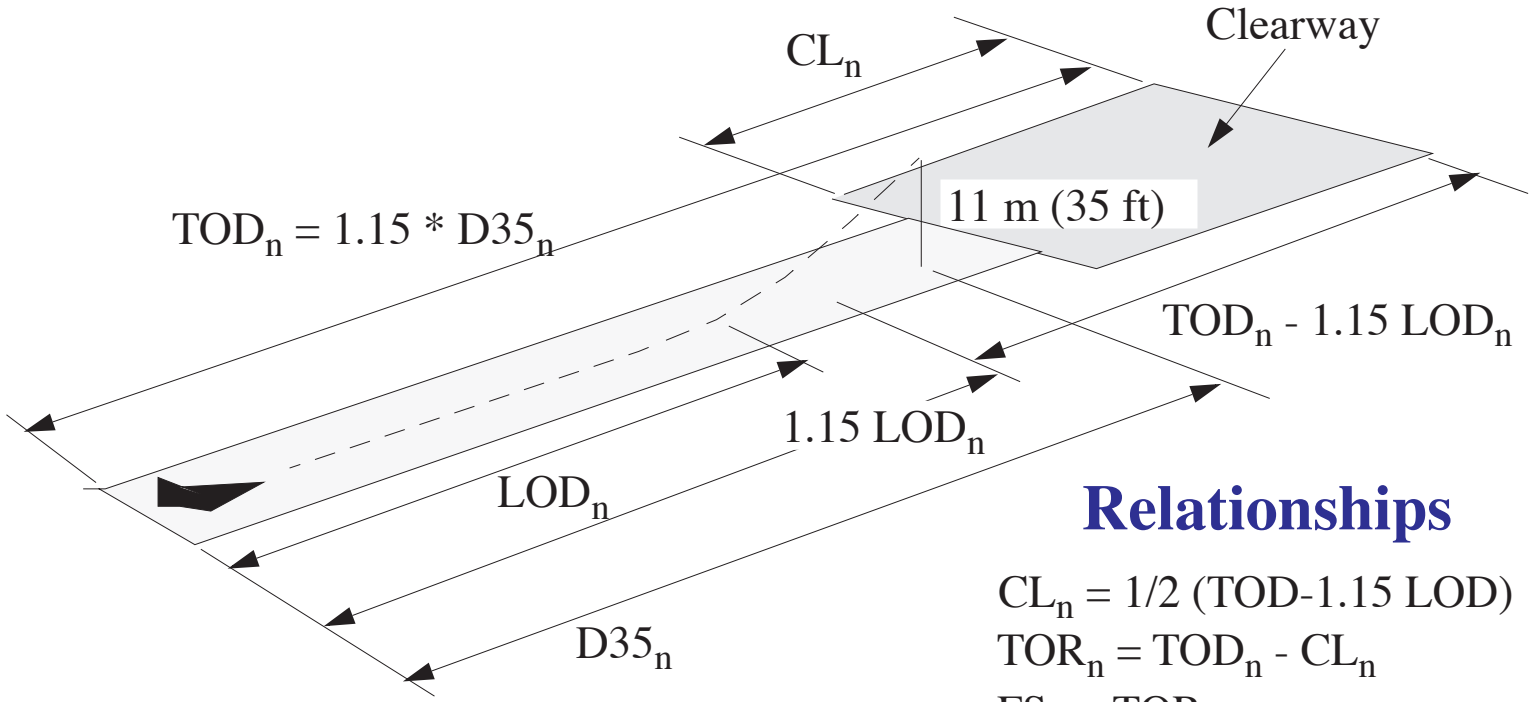
LD = 1.667 * SD

FS_{land} = LD



Normal Takeoff Case

The Takeoff Distance (TOD) should be 115% longer than the demonstrated Distance to Clear an 11m (35 ft.) obstacle (D35)



Relationships

$CL_n = 1/2\ (TOD-1.15\ LOD)$

$TOR_n = TOD_n - CL_n$

$FS_n = TOR_n$

$FL_n = FS_n + CL_n$

Engine-Out Takeoff Case

Dictated by two scenarios:

Continued takeoff subcase

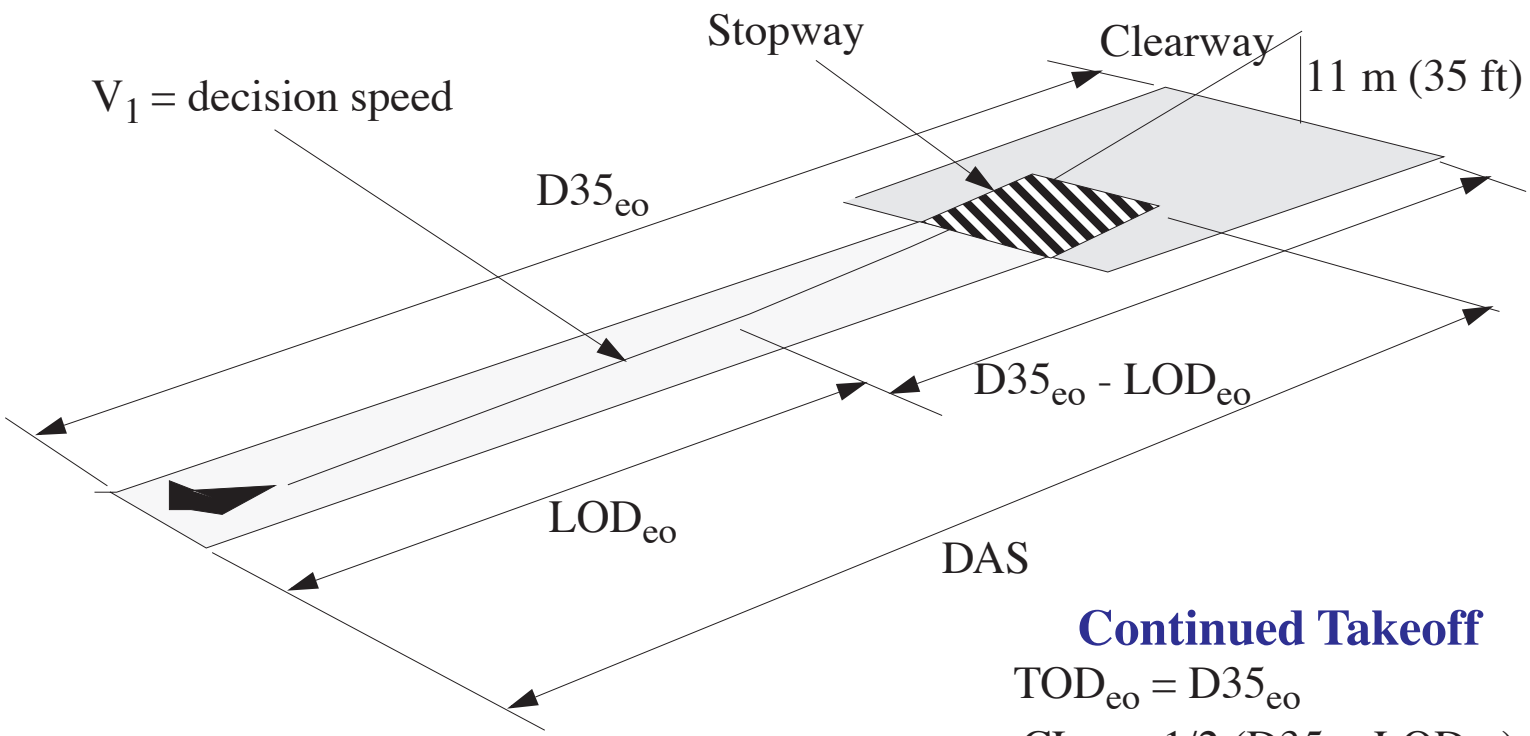
- Actual distance to clear an imaginary 11 m (35 ft.) obstacle D35 (with an engine-out)

Aborted or rejected takeoff subcase

- Distance to accelerate and stop (DAS)

Note: no correction is applied due to the rare nature of engine-out conditions in practice for turbofan/turbojet powered aircraft

Engine-Out Analysis



Aborted Takeoff

$FS_{eo\text{-}a} = DAS - SW$

$FL_{eo\text{-}a} = FS_{eo\text{-}a} + SW$

Continued Takeoff

$TOD_{eo} = D35_{eo}$

$CL_{eo} = 1/2\ (D35_{eo}\text{-}LOD_{eo})$

$TOR_{eo} = D35_{eo} - CL_{eo}$

$FS_{eo\text{-}c} = TOR_{eo}$

$FL_{eo\text{-}c} = FS_{eo\text{-}c} + CL_{eo}$

Runway Length Procedures (AC 150/5325-4)

Two different views of the problem:

- For aircraft with MTOW up to 27,200 kg (60,000 lb.) use the aircraft grouping procedure
  - If MTOW is less than 5,670 kg use Figures 2-1 and 2-2 in FAA AC 150/5325-4
  - If MTOW is > 5,670 kg but less than 27,200 kg use Figures 2-3 and 2-4 provided in Chapter 2 of the AC 150/5325-4
- For aircraft whose MTOW is more than 27,200 kg (60,000 lb.) use the critical aircraft concept
  - The critical aircraft is that one with the longest runway performance characteristics
  - This aircraft needs to be operated 250 times in the year from that airport

Review some examples

Advisory Circular 150/5325-4

U.S. Department of Transportation
Federal Aviation Administration

Advisory Circular

Subject: RUNWAY LENGTH REQUIREMENTS FOR AIRPORT DESIGN

Date: 1/29/90
Initiated by: AAS-110

AC No: 150/5325-4A
Change:

1. PURPOSE. This advisory circular (AC) provides design standards and guidelines for determining recommended runway lengths.

2. CANCELLATIONS. This advisory circular cancels the following documents:

a. AC 150/5325-3, Background Information on the Aircraft Performance Curves for Large Airplanes, dated November 14, 1967.

b. AC 150/5325-4, Runway Length Requirements for Airport Design, dated September 27, 1978.

3. APPLICATION. The standards and guidelines contained in this advisory circular are recommended by the Federal Aviation Administration (FAA) for use in the design of civil airports. For airport projects receiving Federal grant-in-aid assistance, the use of these standards is mandatory.

Contents of Advisory Circular 150/5325-4

Be familiar with all items contained in FAA AC 150/5325-4

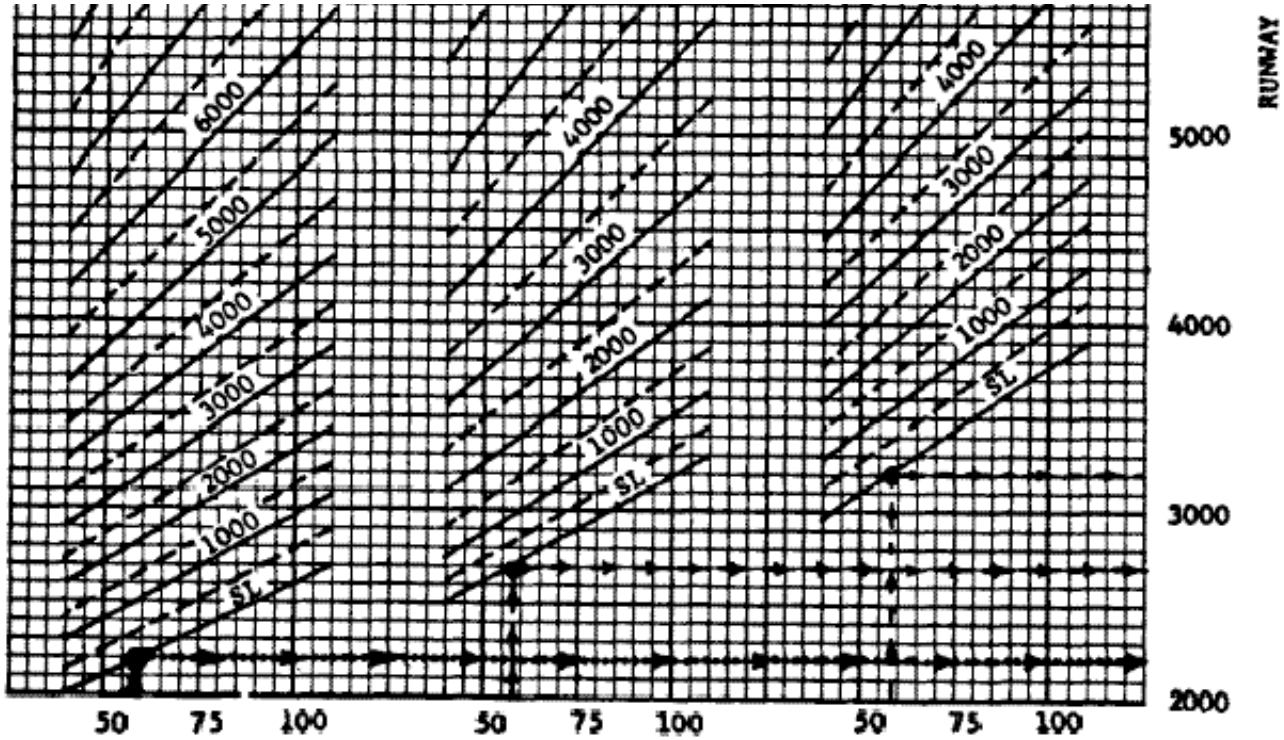
- Chapter 1 - Introduction (background)
- Chapter 2 - Runway length design based on aircraft groupings
- Chapter 3 - Runway length design for specific aircraft
  - Aircraft performance curves
  - Aircraft performance tables
- Chapter 4 - Design rationale
  - Airport temperature and elevation
  - Wind and runway surface
  - Difference in runway centerline elevations
- NOTE: The runway length procedure using **declared the distance concept** is outlined in FAA AC 150/5300-13

Advisory Circular 150/5325-4

The following examples illustrates the use of Figures 2-1 through 2-4 in AC 150/5325-4

- These procedures apply to a collection of aircraft (a group of aircraft)
- The process requires correction of runway length due to runway slope and wet pavement conditions
  - Wet pavement correction is critical for landing aircraft
  - Runway gradient (or slope) is critical for departing aircraft
  - Apply the largest correction possible

Groupings Method AC 150/5325-4 (Figure 2-1)



MEAN DAILY MAXIMUM TEMPERATURE (F"), HOTTEST MONTH OF YEAR

75% OF FLEET 95% OF FLEET 100% OF FLEET

Figure 2-1. Runway length to serve small airplanes having less than 10 passenger seats

Example # 1 AC 150/5325-4

Suppose we want to size a runway for a small general aviation airport **all** serving single engine aircraft (MTOW < 5,670 kg)

The airport is to be located on plateau 915 m. above sea level

The proposed airport site has a mean daily temperature of the hottest month of 24 ^{o}C (75 ^{o}F)

Solution:

Consulting Figure 2-1 in AC 150/5325-4 we obtain:

RL = 4,600 ft. (or 1,403 m.)

Sample Calculations (FAA AC 150-5325-4 Individual Aircraft)

Runway Length Requirements Using AC/150-5325-4

Outline of runway length requirement procedures from FAA Advisory AC 150/5325-4.

The method considers takeoff and landing phases as independent events. This method already factors for a takeoff engine failure and wet runways in the solution. Three intermediate computations are:

NOTE: the new advisory circular has eliminated a large number of aircraft tables. The procedure advocated by the new advisory circular is to use design charts provided by each aircraft manufacturer for aircraft above 60,000 lb. (MTOW)

I) Landing Analysis

Estimate the maximum allowable landing weight (MALW) at the given airport conditions.

Compare this MALW with that of the desired weight (DTW).

a) If MALW > DTW use DTW in your computations

b) If MALW < DTW use MALW for in your computations.

FAA method - Estimate the runway length required and use the <u>shortest</u> Landing Runway Length of all possible flap configurations allowed.

NOTE: This method is dangerous because if an engine failure occurs after takeoff and the pilot would want to come back and land the aircraft it would have to dump large amounts of fuel. Using the lowest flap setting provides a safer design and thus can be used instead.

II) Takeoff Analysis

a) Estimate the desired takeoff weight (DTW) for payload/range data provided.

DTW = OEW + FW + PYL

where:

OEW = Operating empty weight

PYL = Payload

FW = Fuel weight (be sure to include reserve fuel)

b) Estimate the maximum allowable takeoff weight for each flap setting (allowable)

NOTE: Discard those flap settings that do not allow a takeoff at DTW.

c) For each flap setting/aircraft operation combination find reference factors and required takeoff distances.

NOTE: This procedure is executed in two steps.

d) Select the shortest runway length from step (c) as this will be the pilot's choice from an operational point of view since pilots would like to depart using the shortest takeoff roll possible.

e) Adjust the takeoff runway length as needed for effective gradient.

III) Landing and Takeoff Runway Length Reconciliation

Once the previous computations have been done select the longest runway length of this method.

Sample Computation (Old Tabular Method)

- Boeing 727-200 with Pratt and Whitney JTD8-15 engines
- 20° mean daily maximum temperature of the hottest month
- 1000 m. field elevation
- 1,200 statute mile stage length
- 150 passengers
- Maximum difference in elevation of runway centerlines is 30 ft.

I) Landing Analysis

a) Estimate the Desired Takeoff Weight (DTW) as a preliminary step

DTW = OEW + FW + PAY

OEW = 54,325 kg. (from table AC 150/5325-4)

FW = (6.2 kg/km) (1200) (1.609) = 11,970.96 kg.

PAY = (150 Pax/kg) (100 kg) = 15,000 kg.

DTW = 54,325 + 15,000 + 11,970.96 + 0

<u>DTW = 81,296 kg.</u> OK, below MTOW (maximum allowable structural weight)

For several flap settings (θ_f) the following numbers were obtained. Note that using a flap setting of 40 degrees will exceed the MALW for this flap setting and thus 40 degrees is not recommended for a landing in case of an engine failure and return to the airfield. In this

case the small flap setting is used because the aircraft is not capable of landing at the departing airport even after all the fuel has been expended.

θ_f (Deg.)	MALW (kg)	ELW [a](kg.)	Runway (m)	Check
40	64,600	69,325*		
30	72,500	69,325	1,777.50	NO

a.ELW stands for emergency landing weight (assuming all fuel is dumped after takeoff to return for a landing.

Simple interpolation for a flap angle of 30 degrees yields a runway length needed of 1,777 meters thus is rounded off to the nearest largest integer, say 1,800 m.

Weight (kg.)	Elevation = 1000 mts.	
72,000	1,765	$R_{0_{f=30f}} = 1777.50\,m$
(72,500)	(?)	$R_{L_{0_{S=30}}} = 1,800\,m$
74,000	1,815	

II) Takeoff Analysis

a) Recall the desired takeoff weight DTW = 82,396 kg.

b) Verify all flap settings that will allow the aircraft to execute a safe takeoff.

The following table illustrates all possible flap angle takeoff configurations for the Boeing 727-200.

θ_f (Deg.)	MTOW (kg)	Check
25	76,500	NO[a]
20	81,800	OK
15	86,900	OK
5	89,400	OK

a.DTW is greater than the maximum allowable takeoff weight so do not consider.

Note that the aircraft can use 3 flap settings at this elevation/payload combination. Therefore, 3 options for runway length requirements need to be investigated.

c) Compute the takeoff runway length requirements for all permissible flap angle configurations.

θ_f (Deg.)	"R" Factor	RL_{TO} (mts.)	"Round Off" (m)
20	70.60	2,696	2,700
15	74.30	2,862	2,900
5	84.50	3,303	3,300

d) The values of runway length were obtained using double linear interpolation from each table (see AC 150/5325-4 for a sample computation). The optimum flap angle for these conditions is $\theta_f = 20^0$ and the one that should be used by pilots and airport engineers to size the runway. the resulting runway length is 2,800 m.

e) Correct for runway gradient. For a 30 ft. change in elevation is equivalent to 9.146 m. For a 2,700 m. runway this implies a 0.339% effective gradient Increase 10% for every 1% in effective gradient

$$R_{L_{TO}} = (2,700)(1.0339) = 2,787.40$$

rounding off we get 2,800 m of runway needed for takeoff.

III) Landing and Takeoff Runway Length Reconciliation

Since the runway needed for takeoff is larger than that required to land select the takeoff runway length as this is the critical dimension.

RL = 2,800 meters

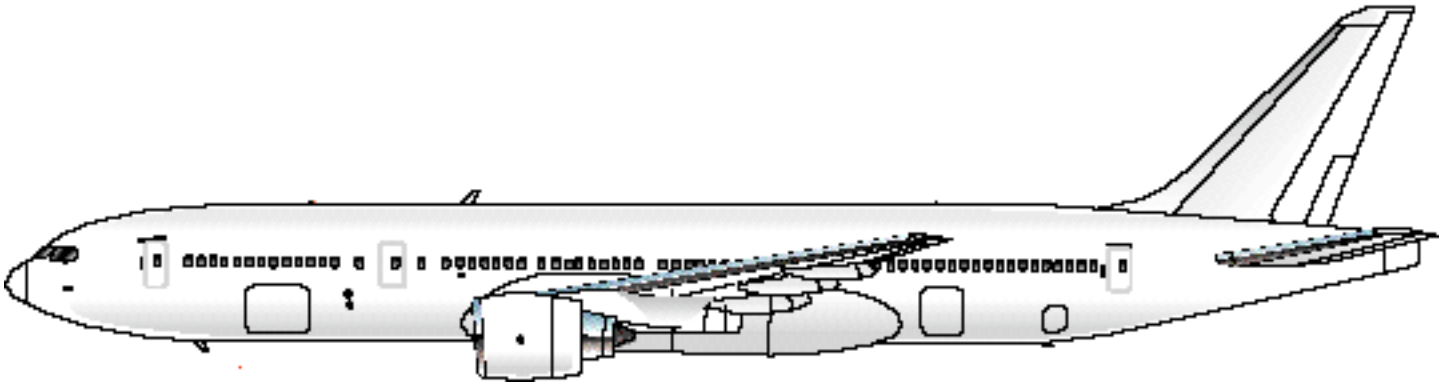
NOTE: This procedure **accounts for wet runways** so no further correction is needed.

Runway Length Analysis using Aircraft Manufacturer Data for Airport Design

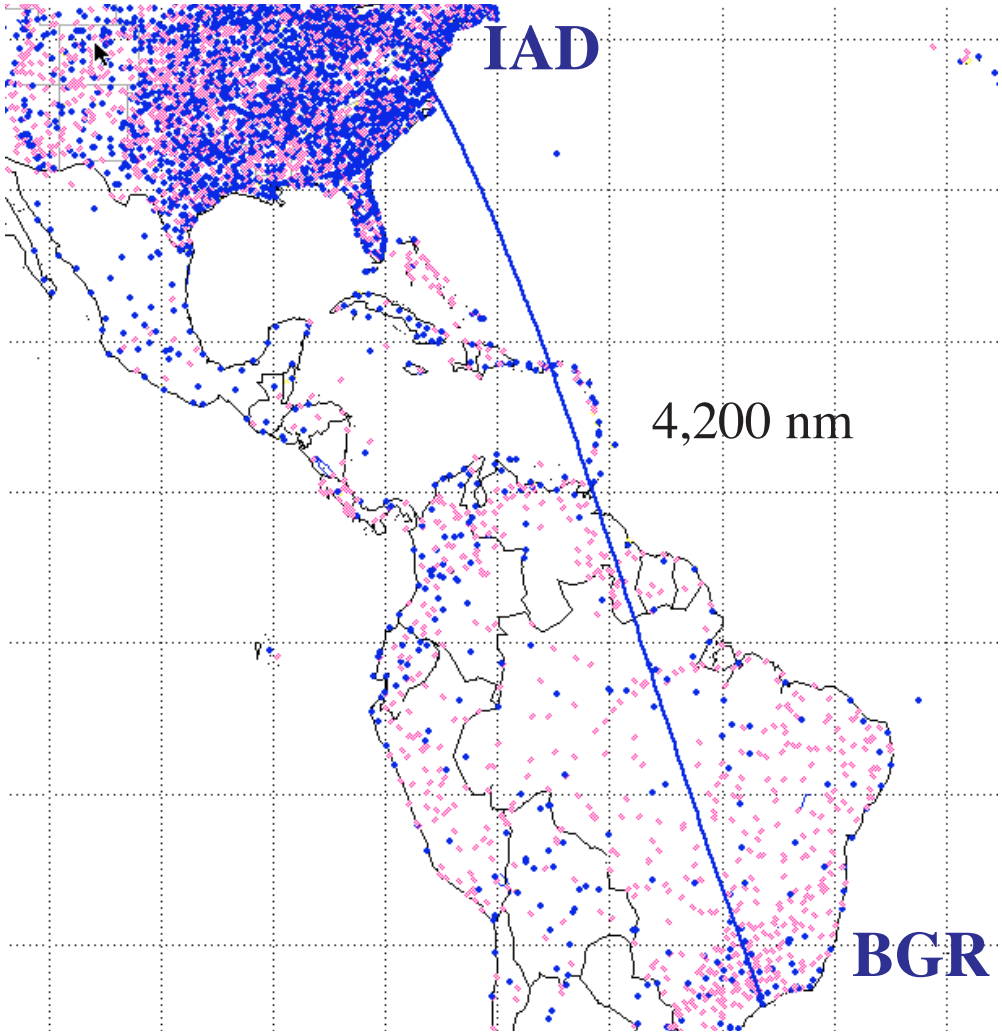
Boeing 777-200 High Gross Weight

Estimate the runway length to operate a Boeing 777-200 High Gross Weight (HGW) from Washington Dulles to Sao Paulo Guarulhos airport in Brasil (a stage length of 4,200 nm) at Mach .84.

After consultation with the airline you learned that their B777s have a gross weight of 592,000 lb. (HGW option) and have a standard three-class seating arrangement.The airline has B 777-200 HGW with General Electric engines. Assume hot day conditions.



IAD-BGR Trip



Discussion of Computations

1) Estimation of Desired Takeoff Weight (DTW)

$$DTW = PYL + OEW + FW$$

where:

PYL is the payload carried (passengers and cargo)

OEW is the operating empty weight

FW is the fuel weight to be carried (usually includes reserve fuel)

Note: PYL and OEW can be easily computed

Boeing 777-200 (GE Engines)

CHARACTERISTICS	UNITS	BASELINE AIRPLANE			HIGH GROSS WEIGHT OPTION		
MAX DESIGN	POUNDS	508,000	517,000	537,000	582,000	592,000	634,500
TAXI WEIGHT	KILOGRAMS	230,450	234,500	243,500	263,640	268,480	287,800
MAX DESIGN	POUNDS	506,000	515,000	535,000	580,000	590,000	632,500
TAKEOFF WEIGHT	KILOGRAMS	229,500	233,600	242,630	263,030	267,500	286,900
MAX DESIGN	POUNDS	441,000	445,000	445,000	460,000	460,000	460,000
LANDING WEIGHT	KILOGRAMS	200,050	201,800	201,800	208,700	208,700	208,700
MAX DESIGN ZERO	POUNDS	420,000	420,000	420,000	430,000	430,000	430,000
FUEL WEIGHT	KILOGRAMS	190,470	190,470	190,470	195,000	195,000	195,000
SPEC OPERATING	POUNDS	298,900	298,900	299,550	304,500	304,500	304,500
EMPTY WEIGHT (1)	KILOGRAMS	135,550	135,550	135,850	138,100	138,100	138,100
MAX STRUCTURAL	POUNDS	121,100	121,100	120,450	125,550	125,550	125,550
PAYLOAD	KILOGRAMS	54,920	54,920	54,620	56,940	56,940	56,940
SEATING	TWO-CLASS	375 - 30 FIRST + 345 ECONOMY					
CAPACITY (1)	THREE-CLASS	305 - 24 FIRST + 54 BUSINESS + 227 ECONOMY					
MAX CARGO	CUBIC FEET	5,656(2)	5,656(2)	5,656(2)	5,656(2)	5,656( )	5,656(2)
- LOWER DECK	CUBIC METERS	160.3 (2)	160.3 (2)	160.3 (2)	160.3 (2)	160.3 (2)	160.3 (2)
USABLE FUEL	US GALLONS	31,000	31,000	31,000	45,220	45,220	45,220
	LITERS	117,300	117,300	117,300	171,100	171,100	171,100
	POUNDS	207,700	207,700	207,700	302,270	302,270	302,270
	KILOGRAMS	94,240	94,240	94,240	137,460	137,460	137,460

Computation of Payload and OEW

Just look at the tables for the specific aircraft:

OEW = 592,000 lb. = 138,100 kg.

PYL = (305 passengers)(200 lb. /passengers) = 61,000 lb. = 27,727 kg.

OEW + PYL = 165,827 kg. = 364,820 lb.

The fuel weight requires knowledge of fuel consumption rates during the flight. These can be extracted from the Payload-Range diagram next.

Computation of Fuel Weight

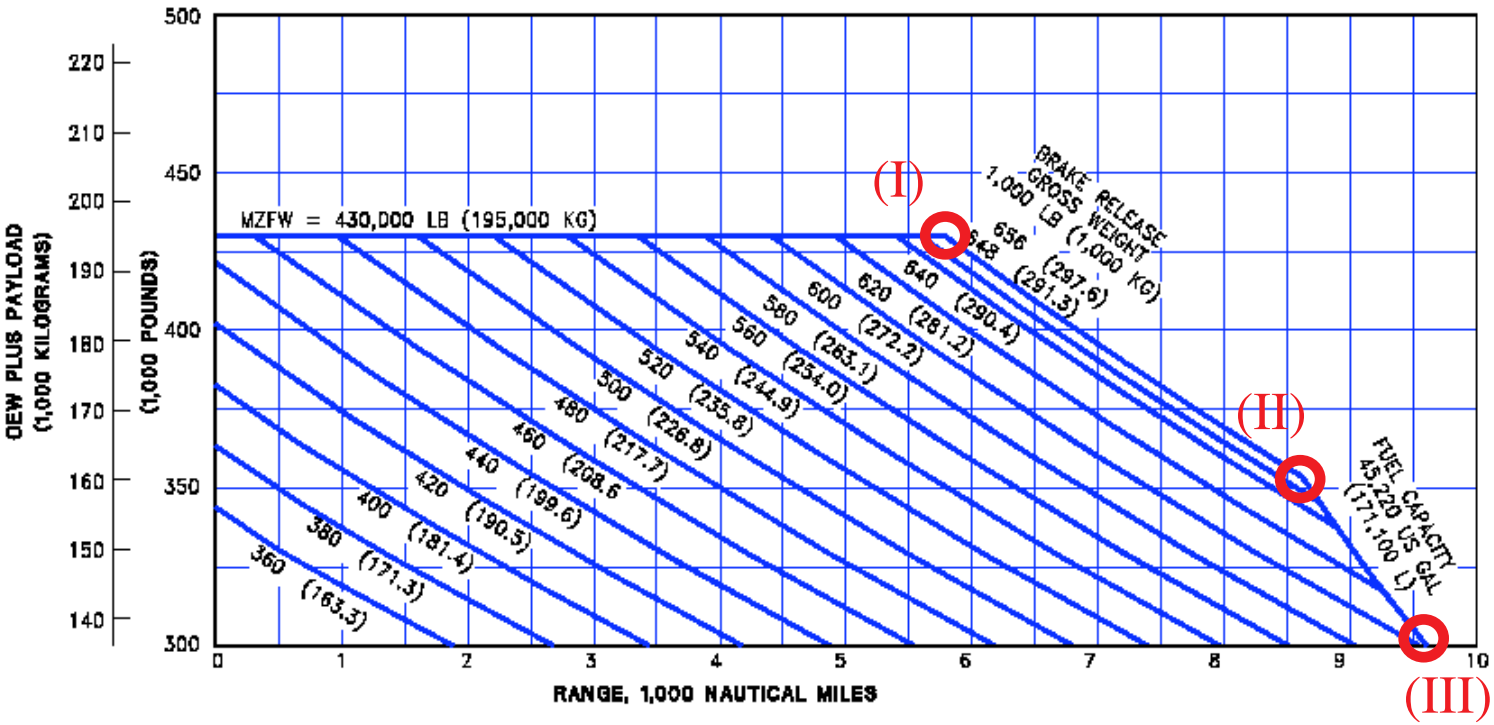
This analysis requires information on fuel consumption for this aircraft flying at a specific cruising condition. Use the payload range diagram of the aircraft to estimate the average fuel consumption in the trip.

The Payload-Range Diagram is a composite plot that shows the operational trade-off to carry fuel and payload.

- As the payload carried increases the amount of fuel to conduct a flight might be decreased thus reducing the actual range (distance) of the mission
- P-R diagrams consider operational weight limits such as MZFW, MTOW and MSPL

Range-Payload Diagram for Boeing 777-200

NOTES:
* STANDARD DAY, ZERO WIND
* 0.84 MACH STEP CRUISE
* TYPICAL MISSION RULES
* NORMAL POWER EXTRACTION AND AIR CONDITIONING BLEED
* CONSULT USING AIRLINE FOR SPECIFIC OPERATING PROCEDURE AND OEW PRIOR TO FACILITY DESIGN



Explanation of Payload-Range Diagram Boundaries

From this diagram three corner points representing combinations of range and payload are labeled with roman numerals (I-III). An explanation of these points follows.

Operating point (I) represents an operational point where the aircraft carries its maximum payload at departs the origin airport at maximum takeoff gross weight (note the brake release gross weight boundary) of 297.6 metric tons.

The corresponding range for condition (I) is a little less than 5,900 nautical miles. Note that under this conditions the aircraft can carry its maximum useful payload limit of 56,900 kg (subtract 195,000 kg. from 138,100 kg. which is the OEW for this aircraft).

Payload-Range Diagrams Explanations

Operating Point (II) illustrates a range-payload compromise when the fuel tanks of the aircraft are full (note the fuel capacity limit boundary).

Under this condition the aircraft travels 8,600 nm but can only carry 20,900 kg of payload (includes cargo and passengers), and a fuel complement of fuel (171,100 liters or 137,460 kg.).

The total brake release gross weight is still 297.6 metric tons for condition (II).

Payload-Range Diagrams Explanations

Operating Point (III) represents the ferry range condition where the aircraft departs with maximum fuel on board and zero payload. This condition is typically used when the aircraft is delivered to its customer (i.e., the airline) or when a non-critical malfunction precludes the carrying of passengers.

This operating point would allow this aircraft to cover 9,600 nautical miles with 137,460 kg.of fuel on board and zero payload for a brake release gross weight of 275,560 kg. (137,460 + 138,100 kg.) or below MTOW.

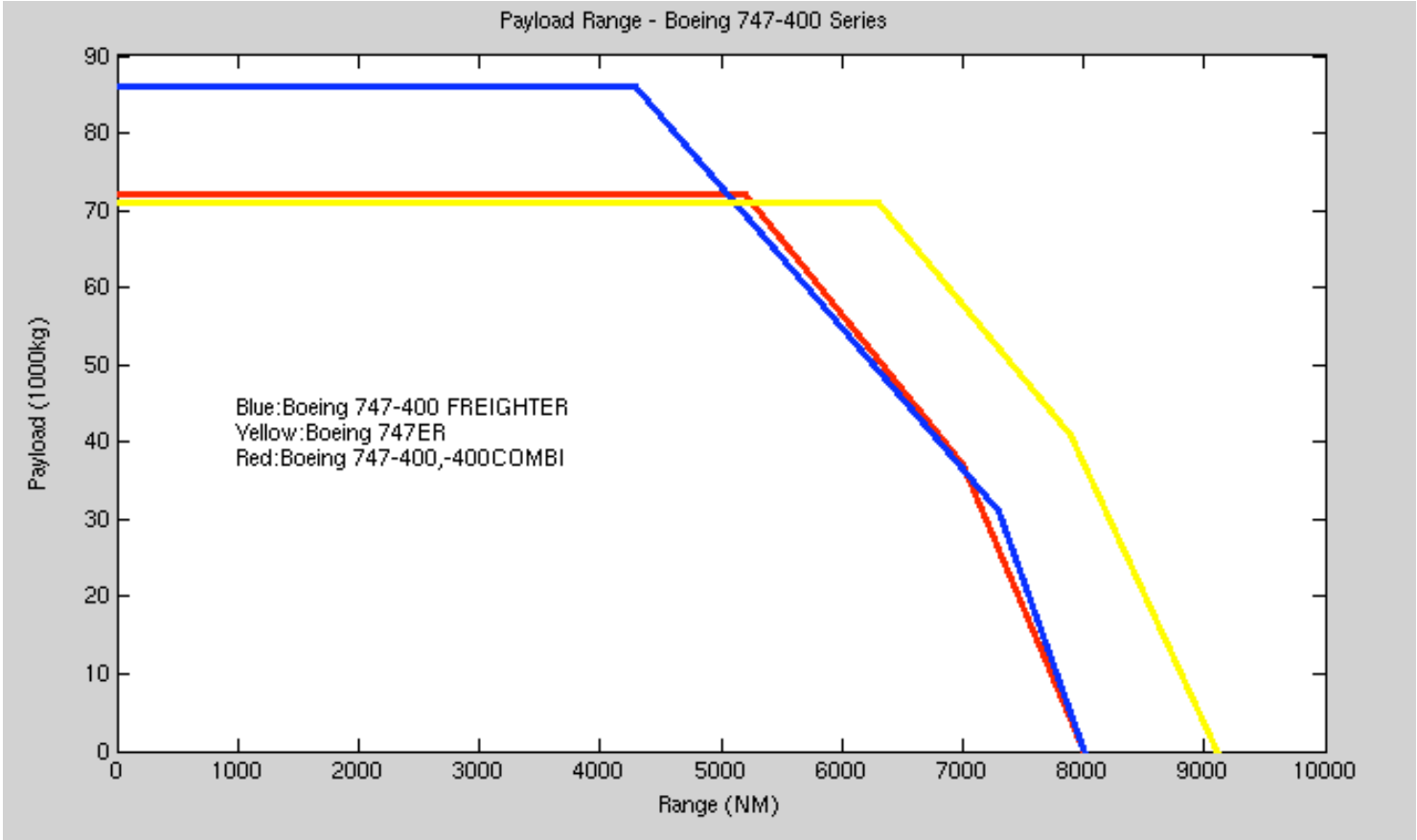
Limitations of P-R Diagram Information

A note of caution about payload range diagrams is that they only apply to a given set of flight conditions.

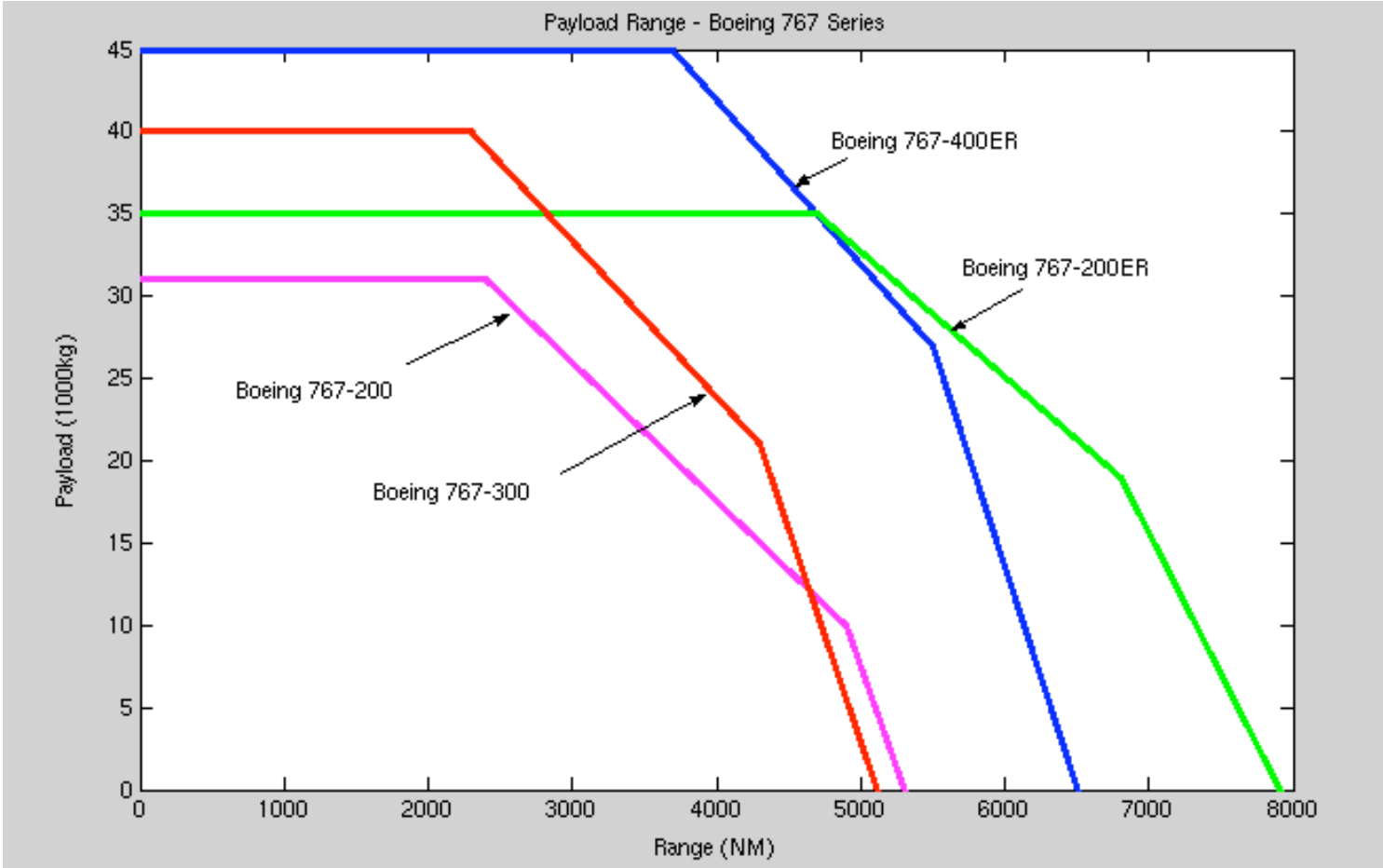
For example, the previous Payload-Range diagram is only applicable to zero wind conditions, 0.84 Mach, standard day conditions (e.g., standard atmosphere) and Air Transport Association (ATA) domestic fuel reserves (this implies enough fuel to fly 1.25 hours at economy speed at the destination point).

If any of these conditions changes so does the payload-range diagram. Later on we examine the sensitivity of Range to various other conditions.

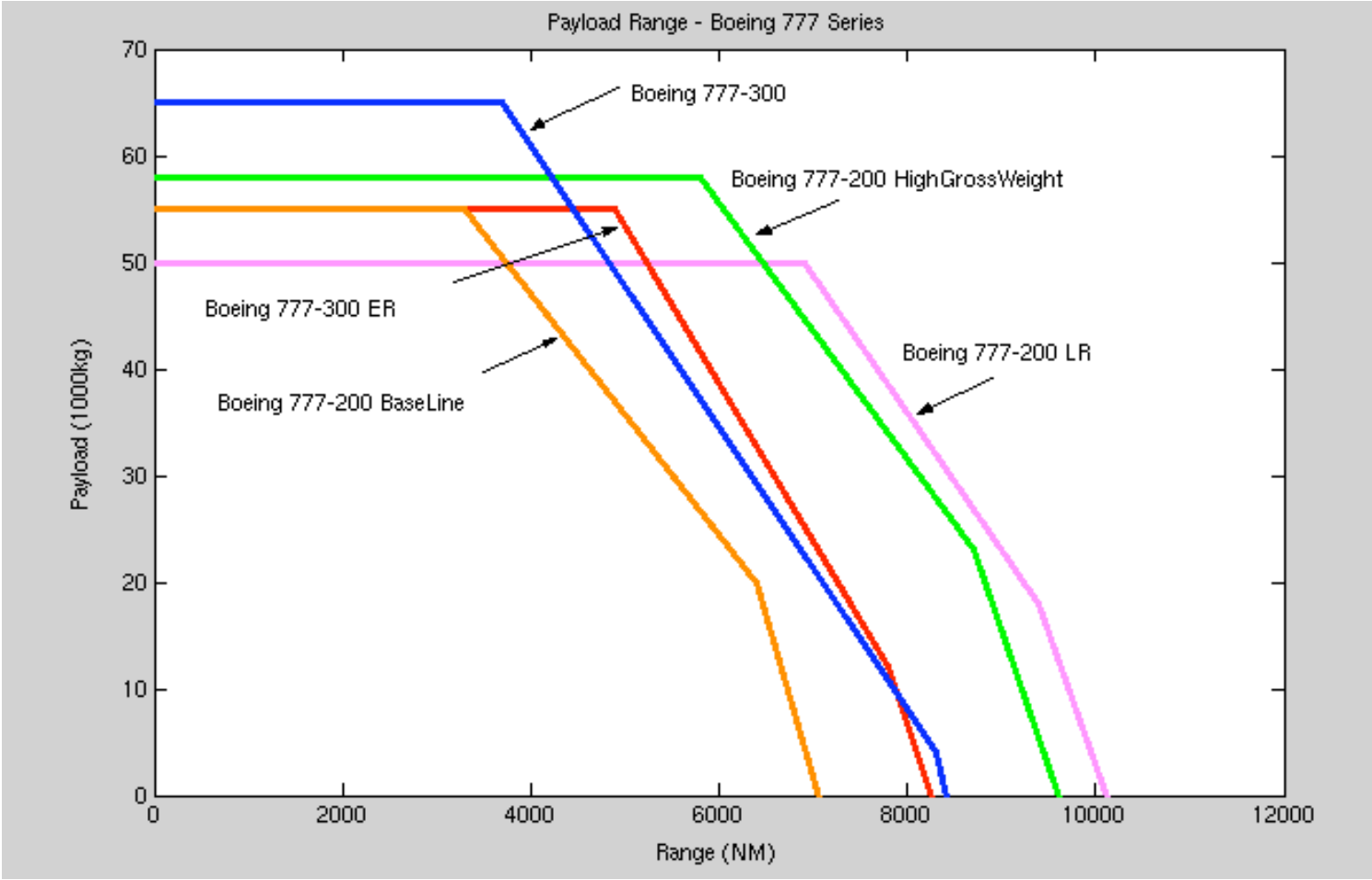
Sample Payload Range DiagramsPayload Range Diagrams (B747)



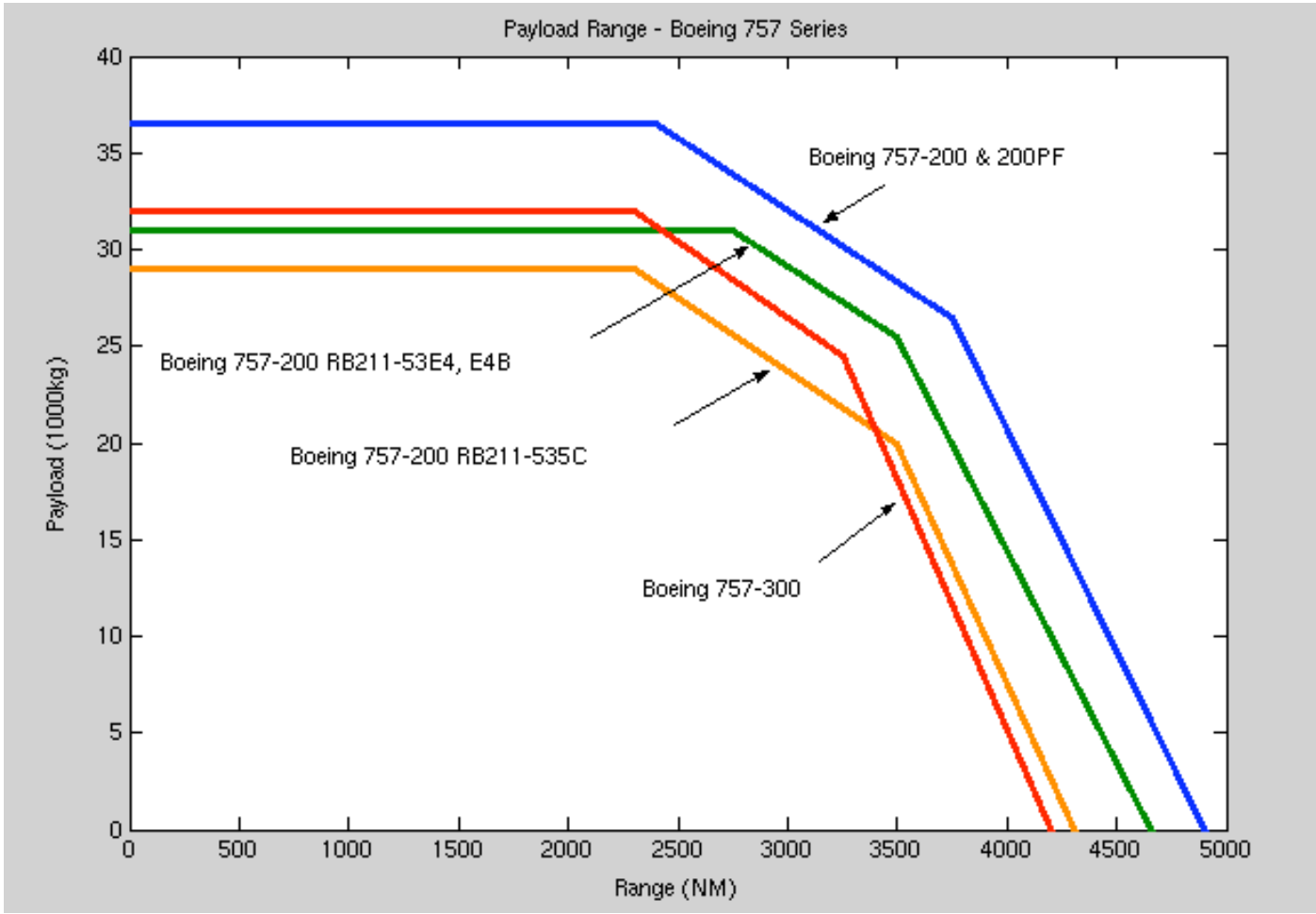
Payload Range Diagrams (B767)



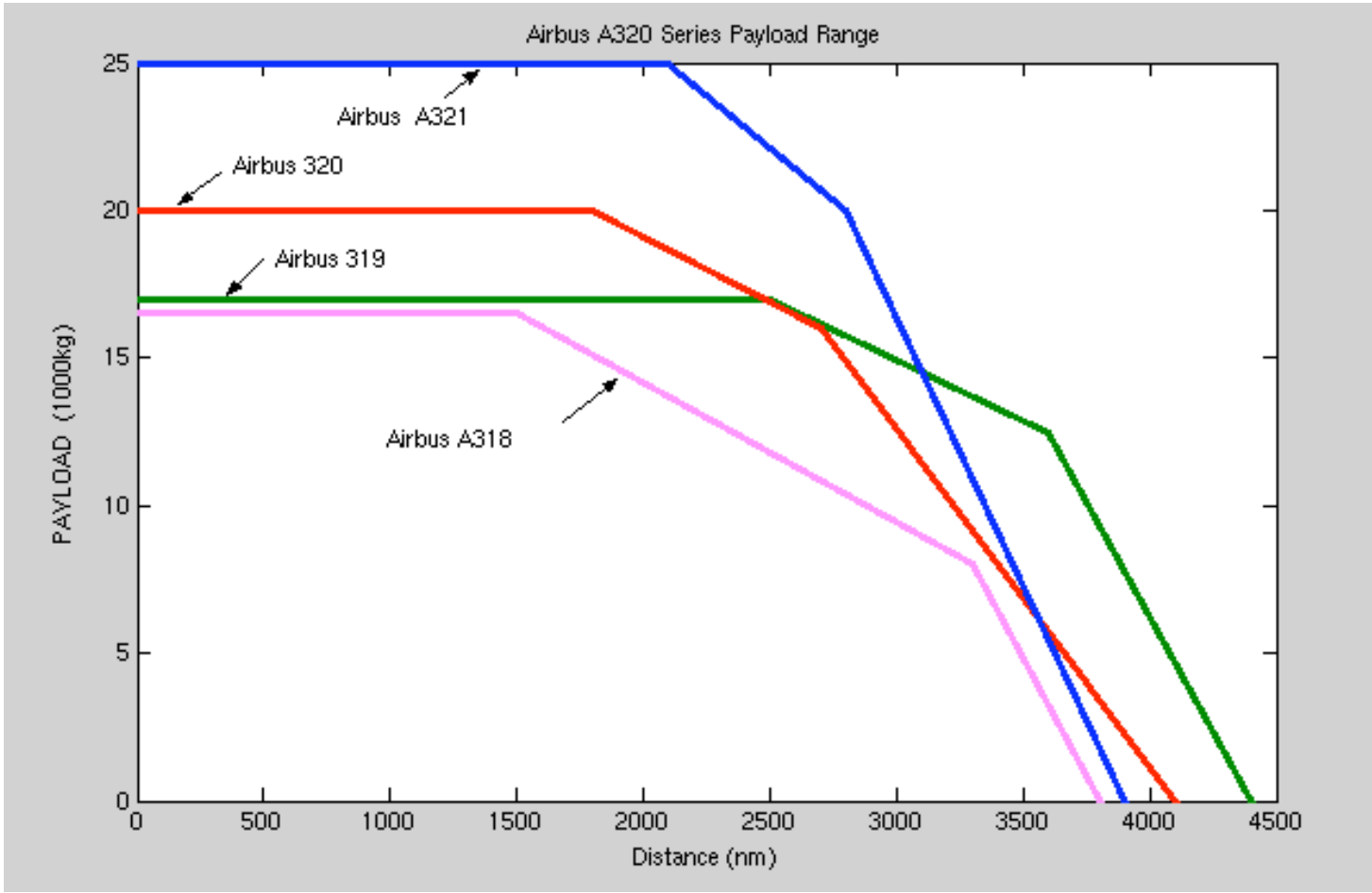
Payload Range Diagrams (B777)



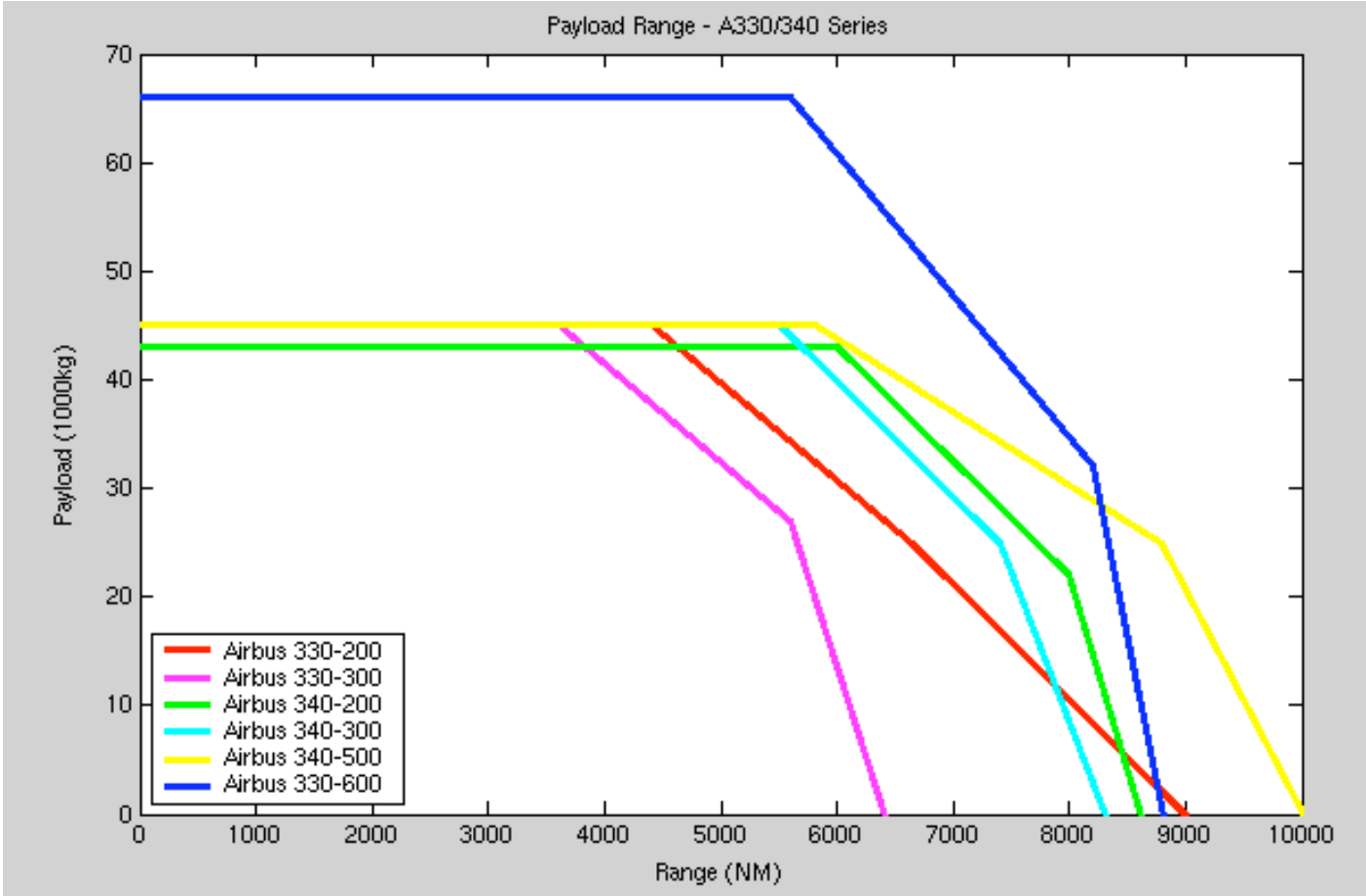
Payload Range Diagrams (B757)



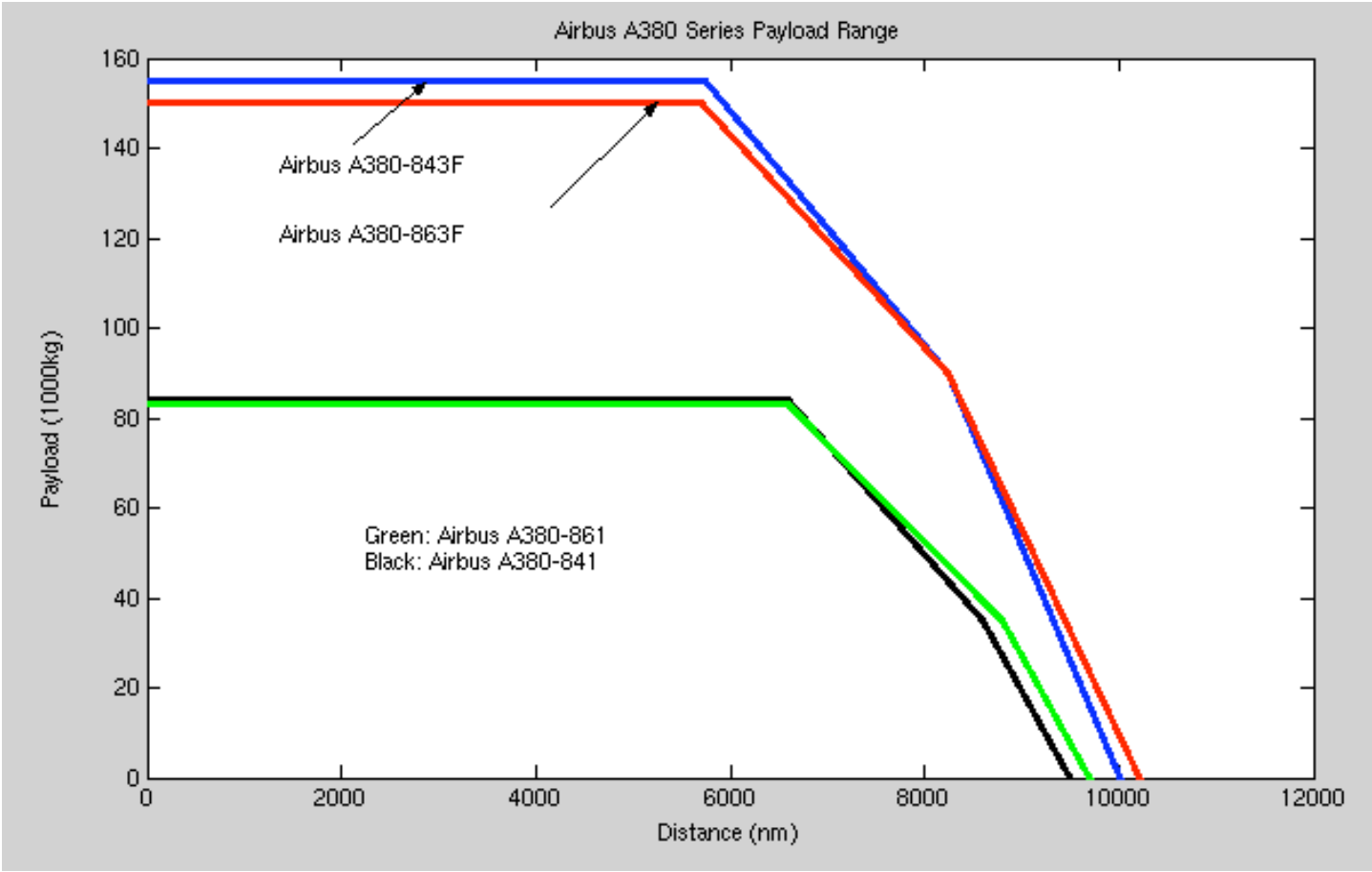
Payload Range Diagrams (A320)



Payload Range Dagrams (A330)



Payload Range Dagrams (A380)



Back to Our Problem!

Our critical aircraft flying (B777-200 HGW option) would fly 4,200 nm with full passengers.

- From the P-R diagram read off the DTW as ~230,000 kg.
- OEW + PYL = 165,827 kg.
- The amount of fuel carried for the trip would be:

FW = DTW - OEW - PYL

FW = 64,173 kg.

Since the P-R diagram tells us the DTW we could even skip the fuel computation and use DTW in our runway length analysis directly (see following pages).

Presentation of Runway Length Information

For the aircraft in question we have two sets of curves available to compute runway length:

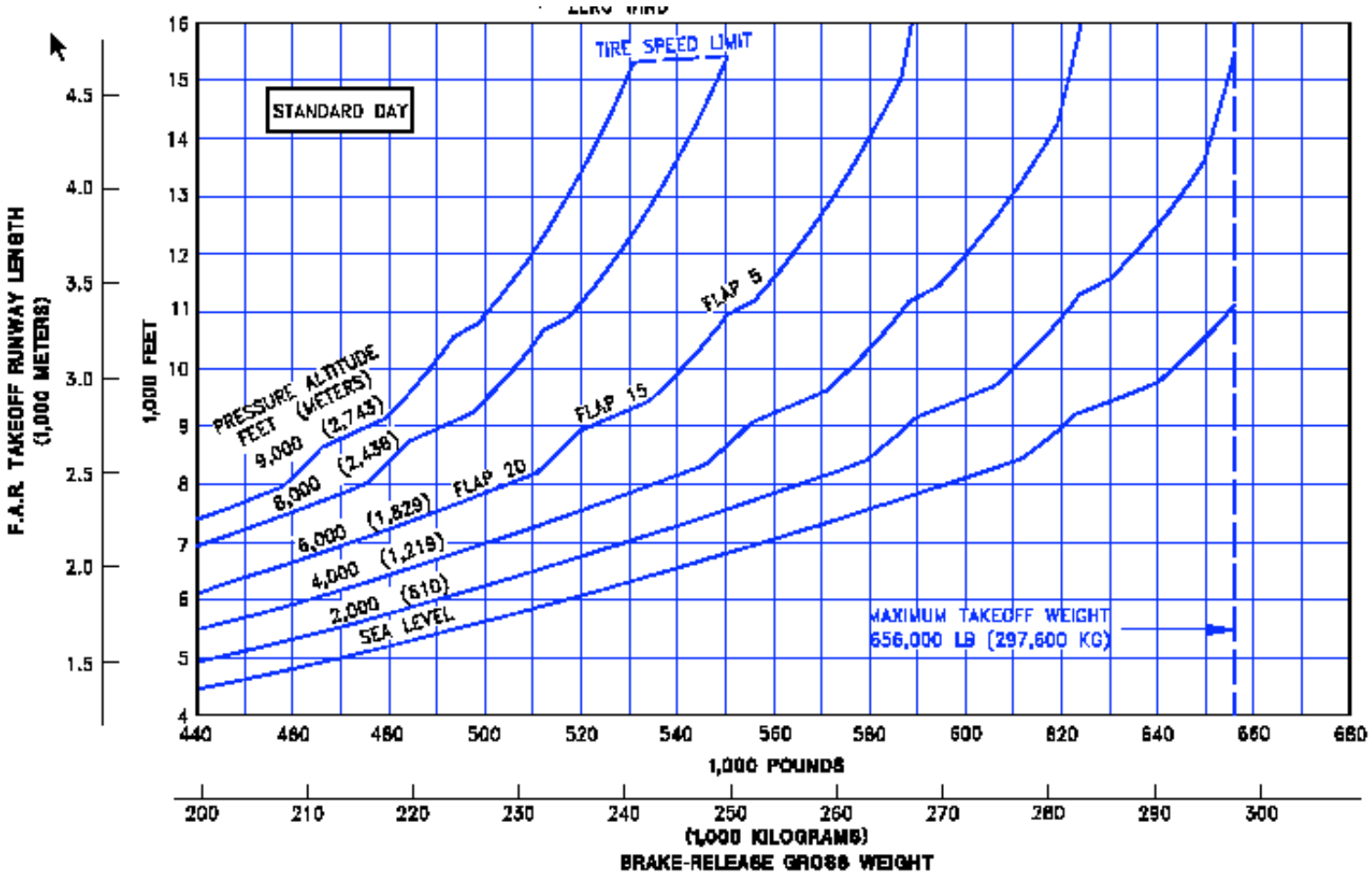
- Takeoff
- Landing

These curves apply to specific airfield conditions so you should always use good judgement in the analysis. Typically two sets of curves are presented by Boeing:

- Standard day conditions
- Standard day + ΔT conditions

where ΔT represents some increment from standard day conditions (typically 15°).

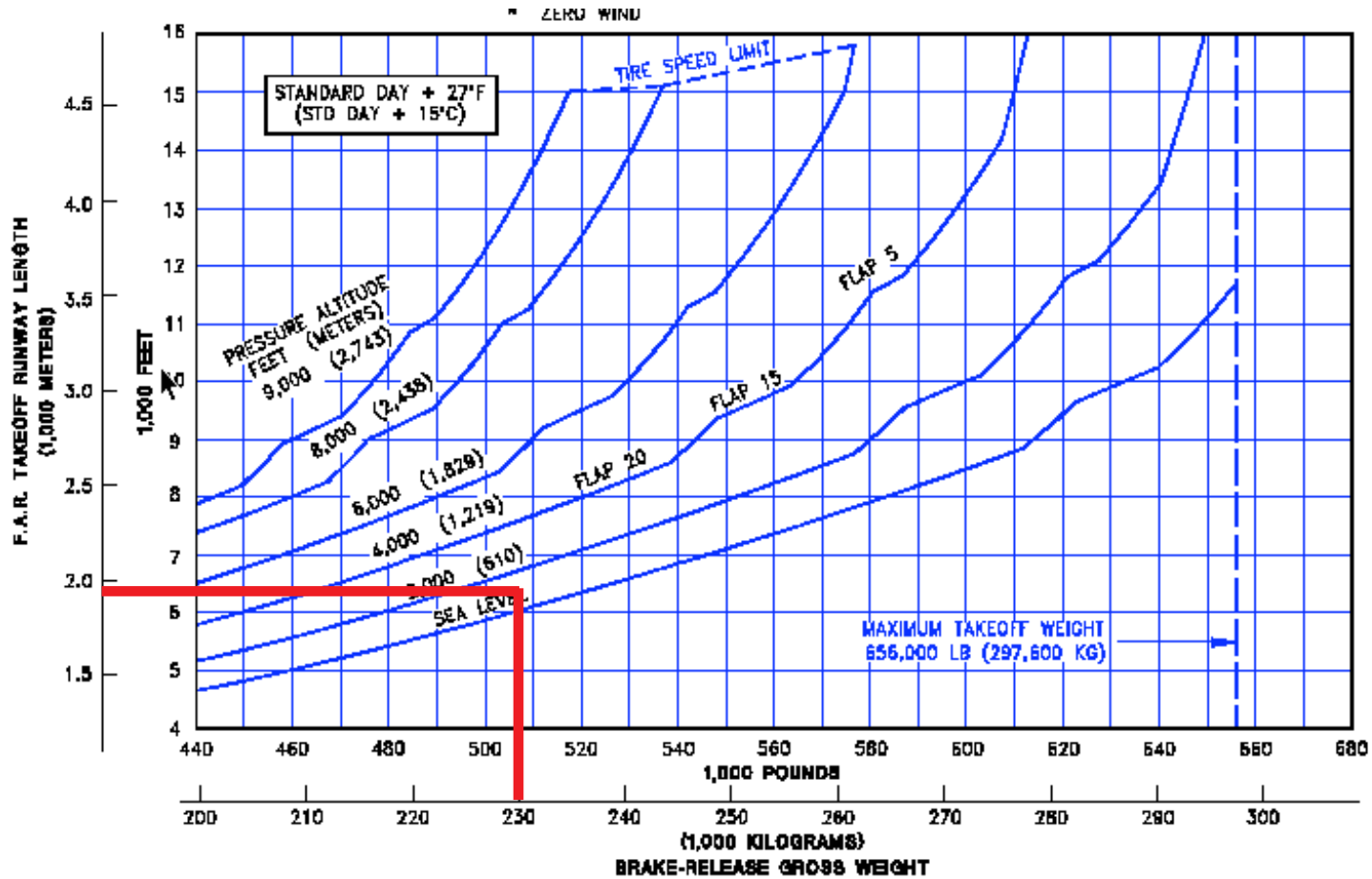
Boeing 777-200 HGW Takeoff Performance



3.3.3 FAA TAKEOFF RUNWAY LENGTH REQUIREMENTS - STANDARD DAY

MODEL 777-200 (HIGH GROSS WEIGHT AIRPLANE)

Boeing 777-200 HGW Takeoff Performance



3.3.4 FAA TAKEOFF RUNWAY LENGTH REQUIREMENTS

STANDARD DAY +27°F (STD + 15°C)

MODEL 777-200 (HIGH GROSS WEIGHT AIRPLANE)

Takeoff Runway Length Analysis

From the performance chart we conclude:

- **$RL_{takeoff}$ = 1,950 m.**
- Optimum flap setting = 20 degrees for takeoff (see flap setting lines in the diagram)
- DTW is way below the maximum capability for this aircraft.

Landing Analysis

This analysis is similar to that performed under FAA AC 150/5325-4. Consider an emergency situation and compute the landing weight at the departing airport.

DTW = 230,000 kg.

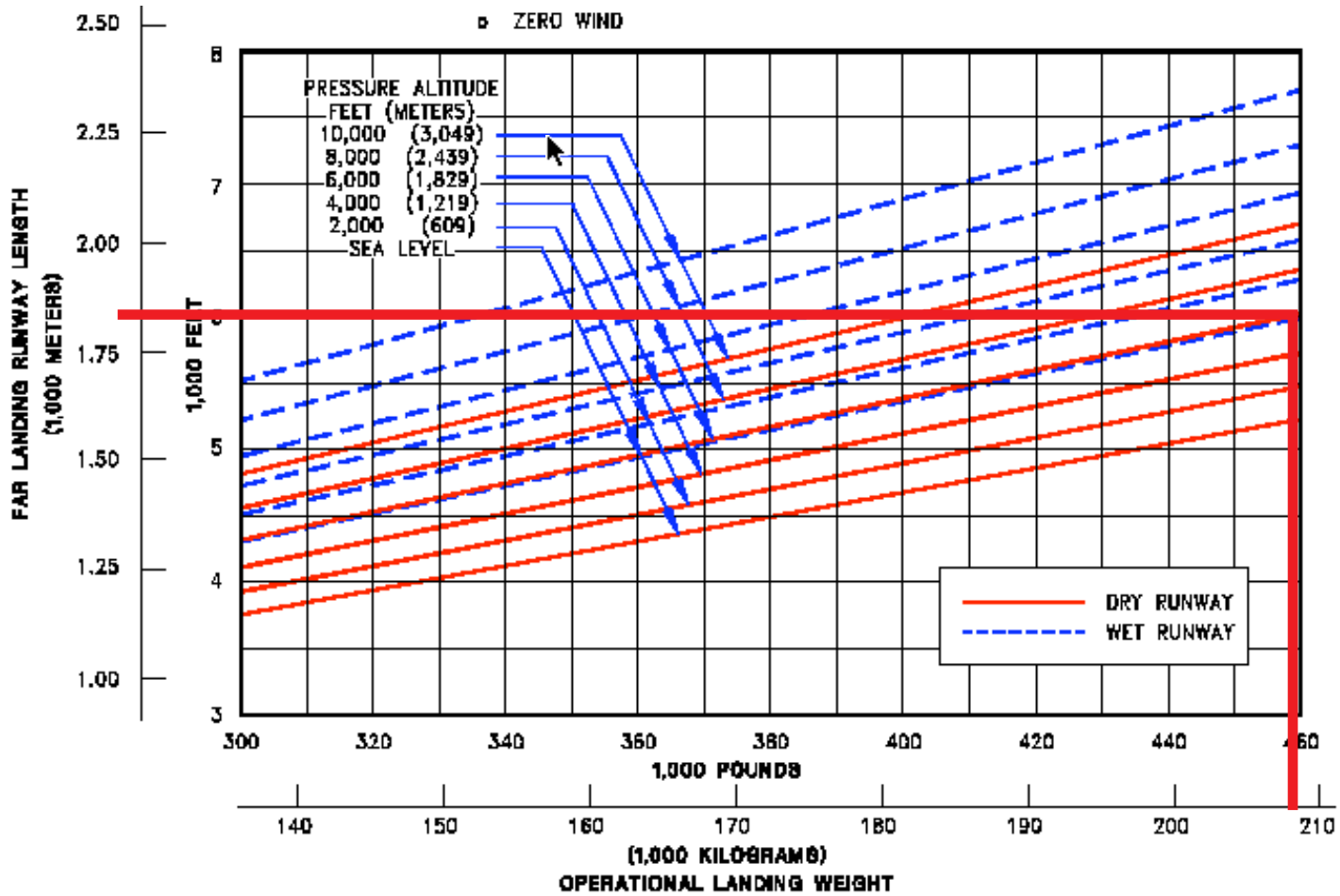
The maximum allowable landing weight for the aircraft is:

MALW = 208,700 kg.

Since DTW > MALW use MALW in the rest of the calculations.

RL_{land} = 1,850 m.

Boeing 777-200 HGW Landing Performance



3.4.1 FAA LANDING RUNWAY LENGTH REQUIREMENTS

MODEL 777-200

Reconcile Takeoff and Landing Cases

Select worst case scenario and use that as runway length requirement.

$RL_{takeoff}$ = 1,950 m.

RL_{land} = 1,850 m.

Takeoff dominates so use the $RL_{takeoff}$ as the design number.

Observe Some Trends from Takeoff Curves

- If DTW increases the RL values increase non-linearly (explain using the fundamental aircraft acceleration equation)
- As field elevation increases (pressure altitude) the RL values increase as well (temperature effect on air density)
- As DTW and field elevation increase the optimum flap setting for takeoff decreases
  - This is consistent with our knowledge of C_d and C_L. Hot and high airfield elevations require very low flap settings during takeoff to reduce the drag of the aircraft.
- High airfield elevations (and large to moderate DTWs) could hit a tire speed limit boundary. Aircraft tires are certified to this limit and thus an airline would never dare to depart beyond this physical boundary.

Other Considerations in Runway Length Analysis

- So far the runway length analysis assumed that we have plenty of land to build the runway.
- There are many practical situations when this is not true.
- Under land limited conditions use the Declared Distance Concept for runway length estimation described in Appendix 14 of FAA AC 150/5300-13.
- The application of declared distance is done on a case-by-case basis and should be part of the Airport Layout Plan (ALP)

Basic Concept

According to the FAA “by treating the airplane's runway performance distances independently, provides an alternative airport design methodology by declaring distances to satisfy the airplane's takeoff run, takeoff distance, accelerate-stop distance, and landing requirements”.

Declared distances are:

- *Takeoff Run Available (TORA)*
- *Takeoff Distance Available (TODA)*
- *Accelerate to Stop Distance Available (ASDA)*
- *Landing Distance Available (LDA).*

Some Runway Design Terms

The following are some definitions of terms employed in the declared distance concept analysis.

Runway Safety Area (RSA) -

Runway Protection Zone (RPZ)

Runway Object Free Area (ROFA)

These critical runway areas are defined in Chapters 2 and 3 of the FAA AC 150/5300-13

Runway Protection Zone (RPZ)

Trapezoidal shape area at the end of every runway and centered with the runway centerline

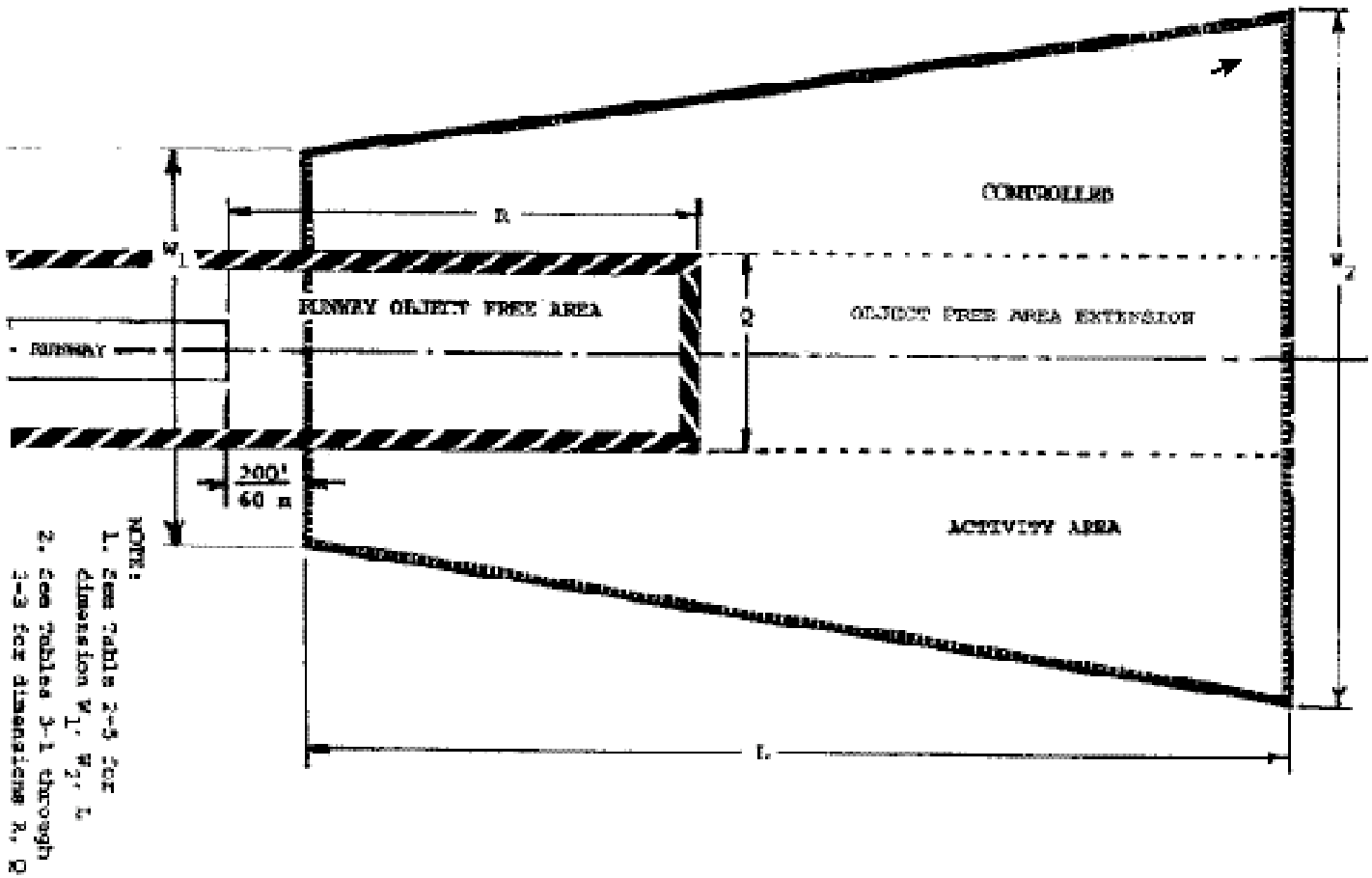
Two components make up the PRZ:

- Controlled activity area
- A portion of the Runway Object Free Area (ROFA)

According to the FAA AC 5300-13 the function of the RPZ is to "enhance the protection of people and property on the ground."

- The airport controls the RPZ
- RPZ are clear of incompatible objects
- Ideally the control is exercised by buying the land of the RPZ

Sketch of RPZ



RPZ Dimensions (Table 2.4 in AC 5300-13)

Approach Visibility Minimums 1/	Facilities Expected To Serve	Dimensions			
		Length L feet (meters)	Inner Width W_1 feet (meters)	Outer Width W_2 feet (meters)	RPZ acres
Visual and Not lower than 1-Mile (1 600 m)	Small Aircraft Exclusively	1,000 (300)	250 (75)	450 (135)	8.035
	Aircraft Approach Categories A & B	1,000 (300)	500 (150)	700 (210)	13.770
	Aircraft Approach Categories C & D	1,700 (510)	500 (150)	1,010 (303)	29.465
Not lower than 3/4-Mile (1 200 m)	All Aircraft	1,700 (510)	1,000 (300)	1,510 (453)	48.978
Lower Than 3/4-Mile (1200 m)	All Aircraft	2,500 (750)	1,000 (300)	1,750 (525)	78.914

Runway Object Free Area (ROFA or OFA)

The runway object free area (OFA) is centered on the runway centerline and extends beyond the runway thresholds.

Clearing standards:

- no ground objects protruding above the runway safety area edge elevation
- Navigation equipment can be located inside OFA
- Maneuvering aircraft OK
- No parked aircraft or agricultural operations are allowed inside OFA

Check out **Tables 3.1 through 3.3** for OFA dimensional standards.

OFA Dimensions (Approach Cat. A/B and 3/4 mile)

ITEM	DIM 1/	AIRPLANE DESIGN GROUP I 2/	I	II	III	IV
Runway Length	A	- Refer to paragraph 301 -				
Runway Width	B	60 ft 18 m	60 ft 18 m	75 ft 23 m	100 ft 30 m	150 ft 45 m
Runway Shoulder Width		10 ft 3 m	10 ft 3 m	10 ft 3 m	20 ft 6 m	25 ft 7.5 m
Runway Blast Pad Width		80 ft 24 m	80 ft 24 m	95 ft 29 m	140 ft 42 m	200 ft 60 m
Runway Blast Pad Length		60 ft 18 m	100 ft 30 m	150 ft 45 m	200 ft 60 m	200 ft 60 m
Runway Safety Area Width	C	120 ft 36 m	120 ft 36 m	150 ft 45 m	300 ft 90 m	500 ft 150 m
Runway Safety Area Length Beyond RW End 3/	P	240 ft 72 m	240 ft 72 m	300 ft 90 m	600 ft 180 m	1,000 ft 300 m
Obstacle Free Zone Width and Length		- Refer to paragraph 306 -				
Runway Object Free Area Width	Q	250 ft 75 m	400 ft 120 m	500 ft 150 m	800 ft 240 m	800 ft 240 m
Runway Object Free Area Length Beyond RW End 3/	R	240 ft 72 m	240 ft 72 m	300 ft 90 m	600 ft 180 m	1,000 ft 300 m

OFA Dimensions (Approach Cat. C and D)

ITEM	DIM 1/	AIRPLANE DESIGN GROUP I	II	III	IV	V	VI
Runway Length	A	- Refer to paragraph 301 -					
Runway Width	B	100 ft 30 m	100 ft 30 m	100 ft2/ 30 m2/	150 ft 45 m	150 ft 45 m	200 ft 60 m
Runway Shoulder Width3/		10 ft 3 m	10 ft 3 m	20 ft2/ 6 m2/	25 ft 7.5 m	35 ft 10.5 m	40 ft 12 m
Runway Blast Pad Width		120 ft 36 m	120 ft 36 m	140 ft2/ 42 m2/	200 ft 60 m	220 ft 66 m	280 ft 84 m
Runway Blast Pad Length		100 ft 30 m	150 ft 45 m	200 ft 60 m	200 ft 60 m	400 ft 120 m	400 ft 120 m
Runway Safety Area Width 4/	C	500 ft 150 m	500 ft 150 m	500 ft 150 m	500 ft 150 m	500 ft 150 m	500 ft 150 m
Runway Safety Area Length Beyond RW End 5/	P	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m
Obstacle Free Zone Width and Length		- Refer to paragraph 306 -					
Runway Object Free Area Width	Q	800 ft 240 m	800 ft 240 m	800 ft 240 m	800 ft 240 m	800 ft 240 m	800 ft 240 m
Runway Object Free Area Length Beyond RW End 5/	R	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m	1,000 ft 300 m

Runway Safety Area (RSA)

Another area surrounding the runway that should be clear of objects, except for objects that need to be located in the runway or taxiway safety area because of their function (i.e., navigation equipment)

- Objects higher than 3 inches (7.6 cm) should be mounted on frangible structures
- Manholes should be constructed at grade (or 7.6 cm. in height at most)
- No underground fuel storage facilities are allowed inside RSA (or taxiway safety areas)

Check out **Tables 3.1 through 3.3** for RSA dimensional standards.

Example Runway Design for Boeing 777-200

Assume a precision approach is needed for IFR conditions

RPZ =>

- W_1 = 1,000 ft.
- W_2 = 1,750 ft.
- L = 2,500 ft.

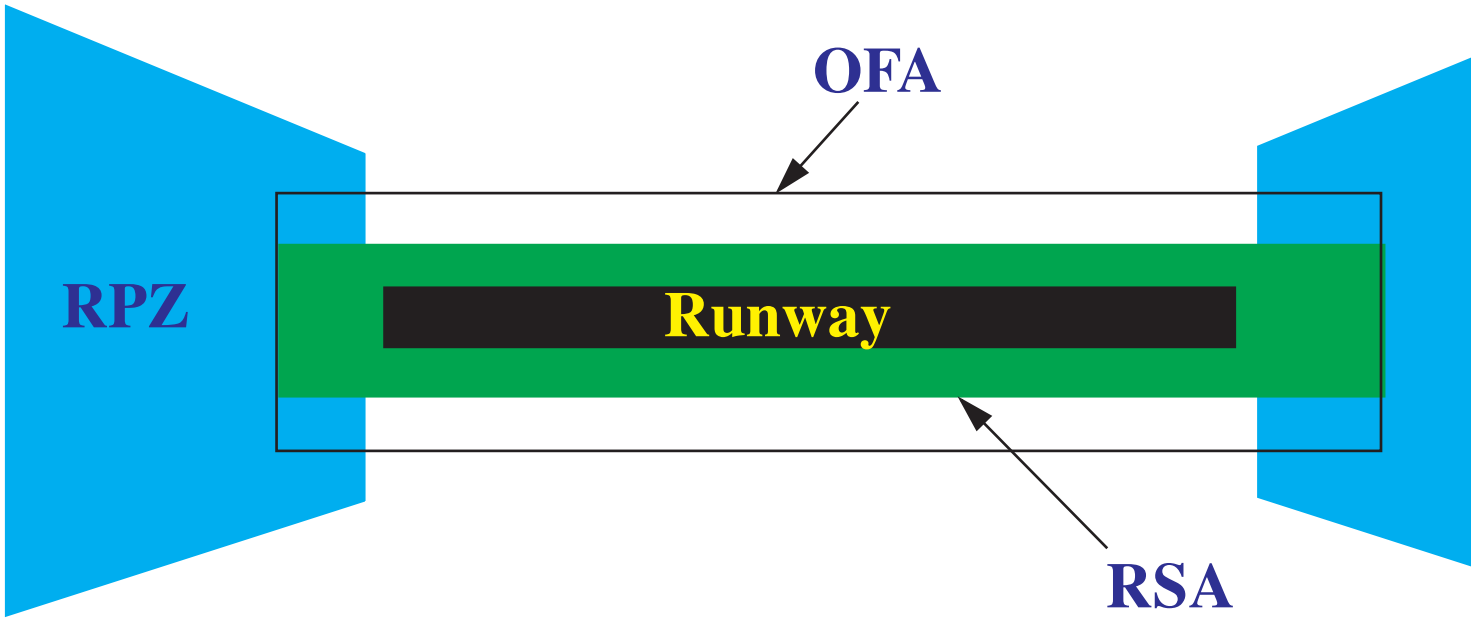
OFA

- 800 ft. width and 1,000 ft. beyond runway end

RSA

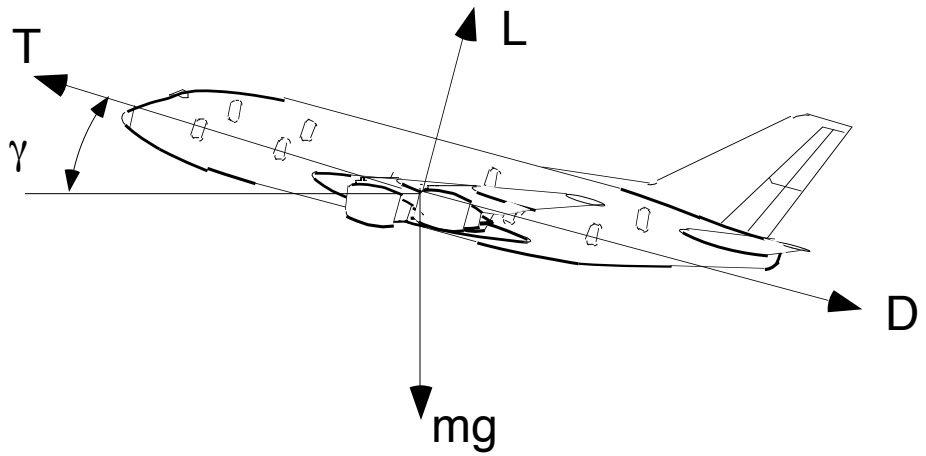
- 500 ft. width and 1,000 ft. beyond runway end

Example Runway Design for Boeing 777-200



Climb Performance

Many airport and airspace simulation models employ simplified algorithms to estimate aircraft climb performance in the terminal area.



Basic Climb Performance Analysis

The basic equations of motion along the climbing flight path and normal to the flight path of an air vehicle are:

$$m\frac{dV}{dt} = T - D - mg\sin\gamma \tag{25}$$

$$m\frac{d\gamma}{dt}V = L - mg\cos\gamma \tag{26}$$

where: m is the vehicle mass, V is the airspeed, T and D are the tractive and drag forces, respectively; γ is the flight path angle. L is the lift force and $mg\cos\gamma$ is the gravitational component normal to the flight path.

Climb Performance Model Simplifications

For small γ (flight path angle):

$$\sin\gamma = \frac{T-D}{mg} - \frac{1}{g}\frac{dV}{dt} \tag{27}$$

where: the first term in the RHS accounts for possible changes in the potential state of the vehicle (i.e., climb ability) and the second terms is the acceleration capability of the aircraft while climbing. Further algebraic manipulation yields,

$$V\sin\gamma = \frac{dh}{dt} = \frac{V[T-D]}{mg} - \frac{V}{g}\frac{dV}{dt}$$

where: dh/dt is the rate of climb and V is the true airspeed. Note that if one neglects the second term (acceleration factor) assuming small changes in V as the vehicle climbs one can easily estimate the rate of the climb of the vehicle for a prescribed climb schedule.

Incorporation of a Parabolic Drag Polar Model

Let lift and drag be expressed in the simple parabolic form,

$$L = \frac{1}{2}\rho S C_L V^2 \tag{28}$$

$$D = \frac{1}{2}\rho S C_D V^2 \tag{29}$$

where: C_L and C_D are the lift and drag coefficients (nondimensional), V is the airspeed, S is the wing area (reference area) and ρ is the density of the air surrounding the vehicle.

Final Derivation of Climb Rate Expression

The functional form of the lift and drag coefficients (C_L, C_D) in its simplest form is,

$$C_D = C_{D0} + C_{Di} = C_{D0} + \frac{C_L^2}{\pi A R e} \tag{30}$$

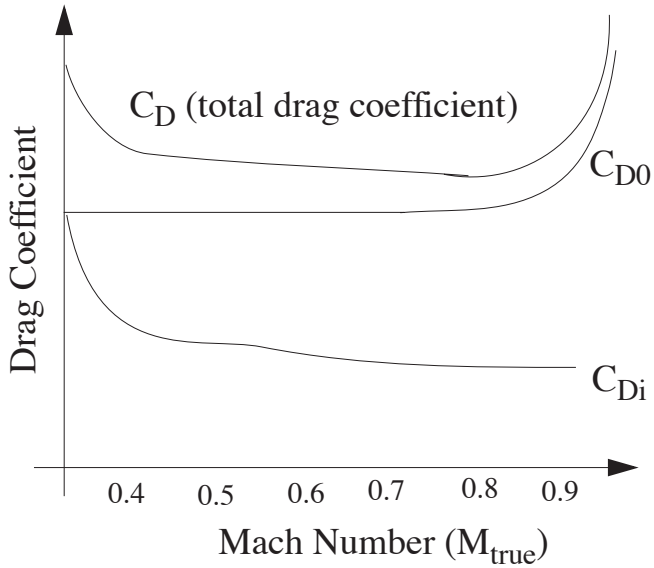
$$C_L = \frac{2mg}{\rho S V^2} \tag{31}$$

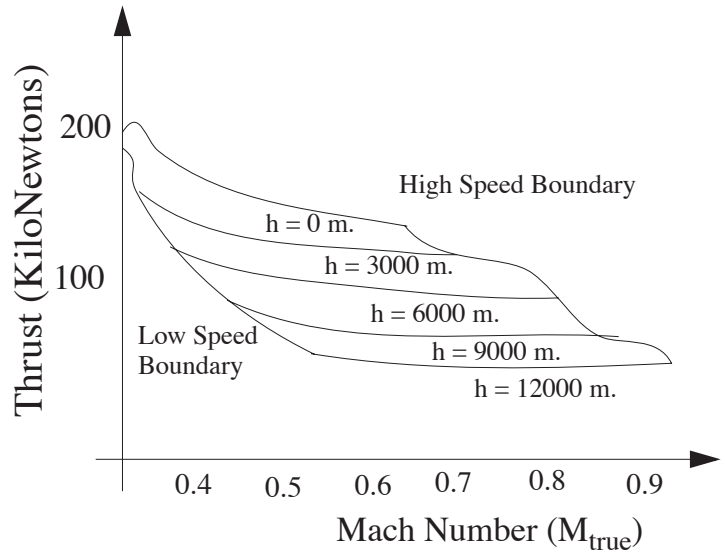
where: C_{D0} is the zero lift drag coefficient, and the second drag term accounts for drag due to lift generation (i.e., induced drag).Then the rate of climb function becomes,

$$\frac{dh}{dt} = \frac{V\left[T(\rho, V) - \frac{1}{2}\rho V^2 S\left\{C_{D0}(M) + \frac{C_L^2(M, V)}{\pi A R e}\right\}\right]}{mg} \tag{32}$$

Mathematical Approximation for Aircraft Thrust and Drag

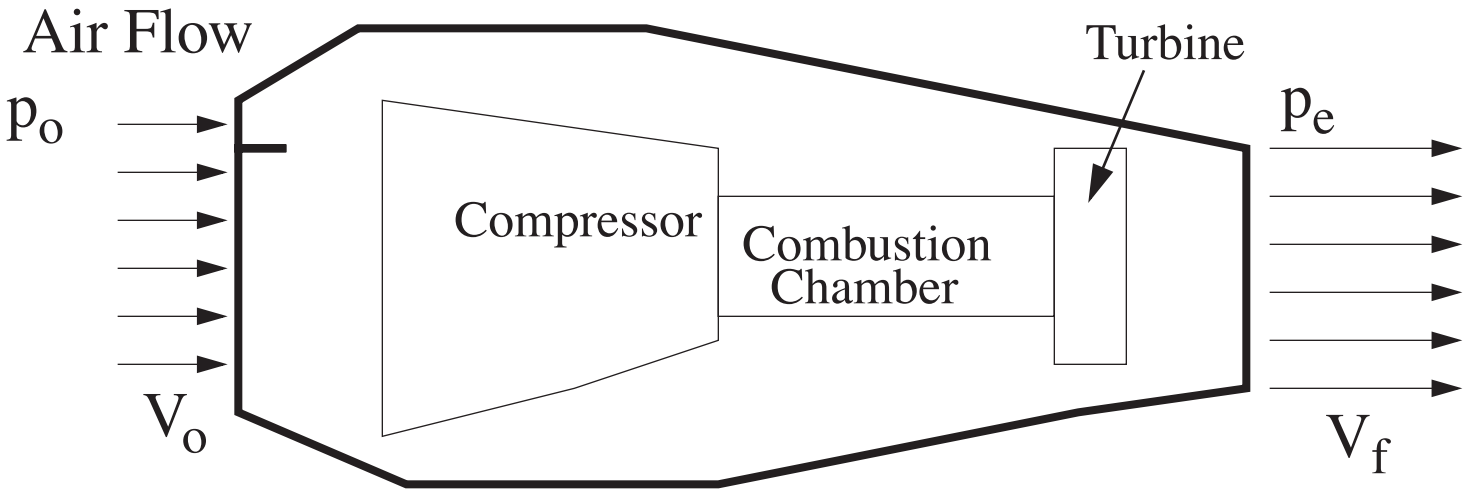
Thrust and drag are two fundamental variables extracted from wind-tunnel and flight tests.





Modeling Aircraft Thrust

- Thrust is a function of aircraft speed and altitude
- Basic thermodynamics dictates that thrust is the net result of the speed differential between inlet and outlet of the engine



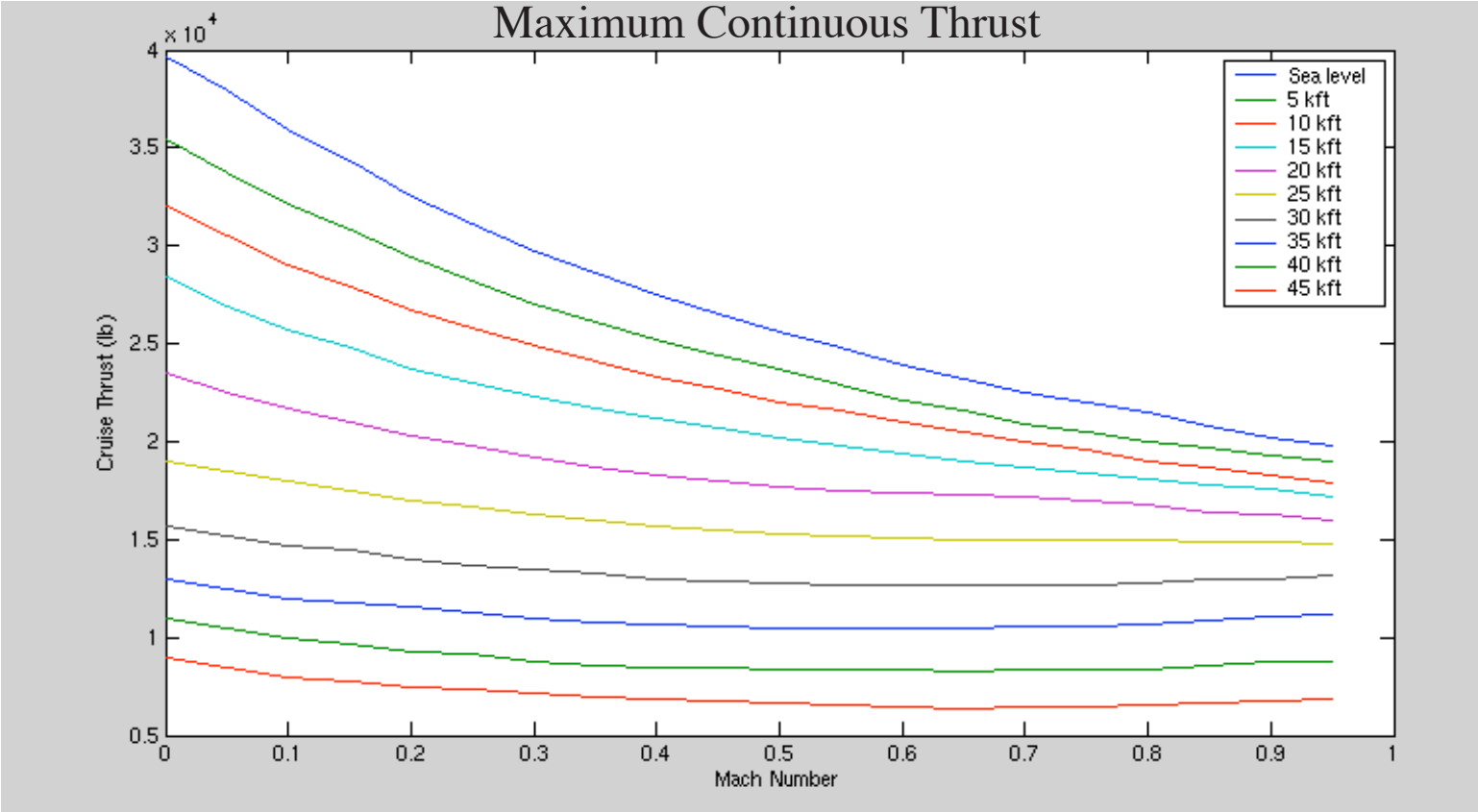
$$T = f(V_f - V_o) \text{ and } f(p_o - p_e)$$

Basic Propulsion Forces Modeling Ideas

- Thrust is a function of altitude (or density)
  - + A general thrust lapse function can be obtained using real engine data (empirical data)
- Thrust is a function of aircraft speed
  - + A more complex function can be obtained using real engine data (empirical data)
  - + The thrust losses during takeoff troll are significant as illustrated in the figures below
- Thrust functions are provided by the engine manufacturer in terms of tables (thrust vs altitude and mach number and thrust specific fuel consumption vs altitude and mach number)

Sample Thrust Variations (PW JT9D Engine)

Observe the large variations of thrust with respect to aircraft altitude



Modeling Thrust Using a Thrust Lapse Gradient

A simple way to model thrust as a function of altitude is presented below:

$$T_h = T_0 \left(\frac{\rho_h}{\rho_0} \right)^m \tag{33}$$

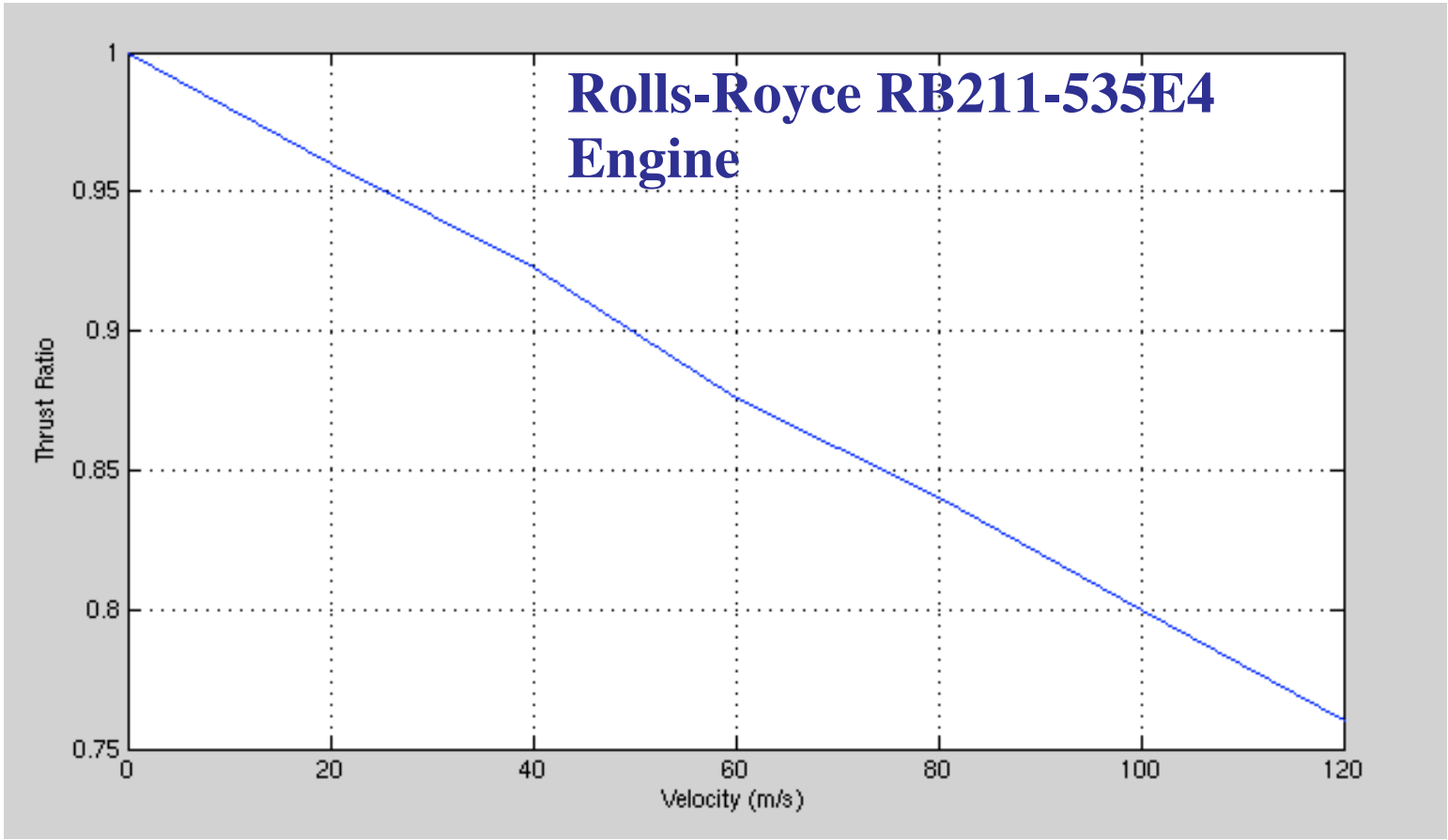
where:

T_h is the thrust at altitude, T_0 is the sea level static thrust,

ρ_h and ρ_0 are the density values at altitude and at sea level, respectively

m is an empirical coefficient derived from real data

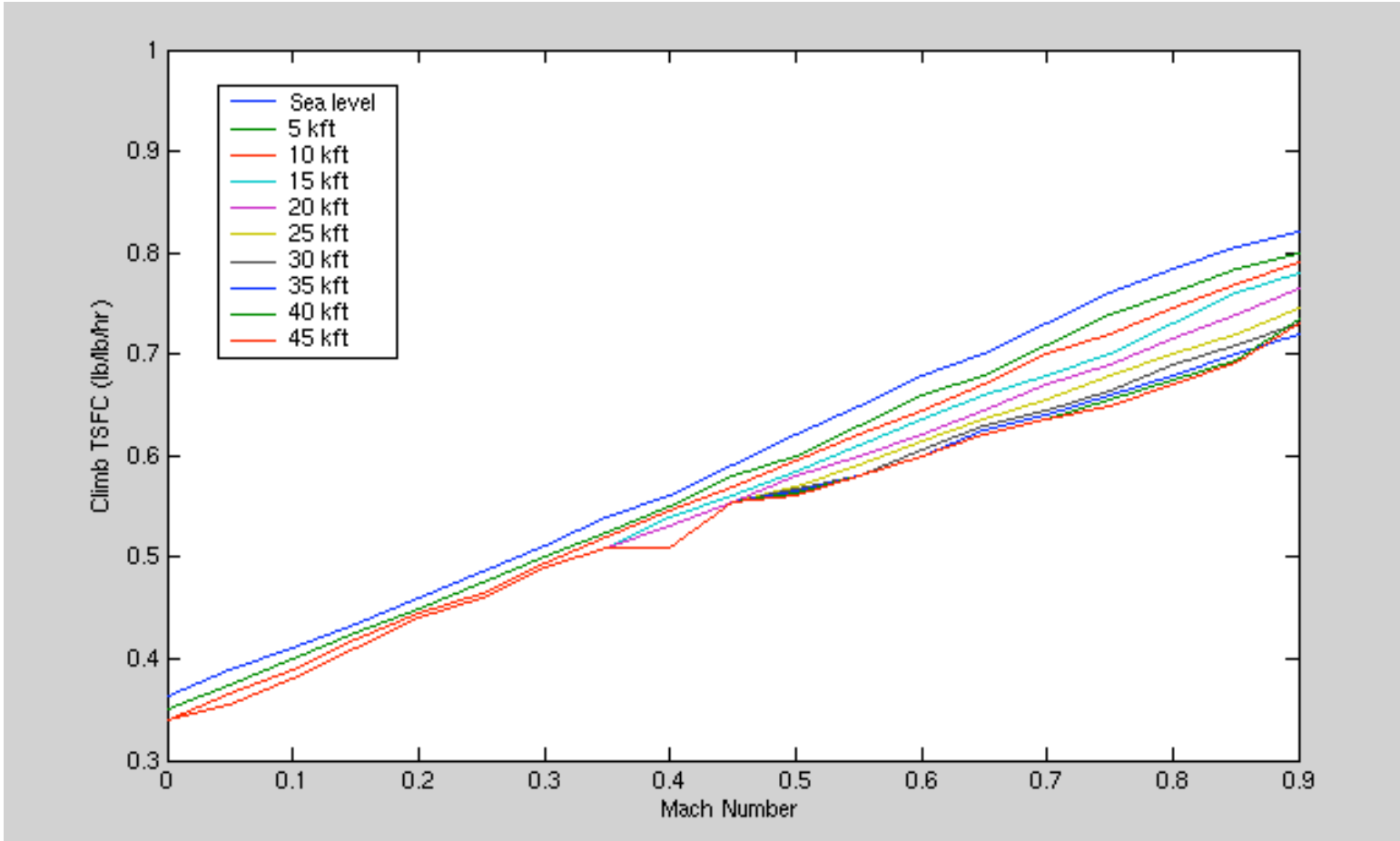
Takeoff Thrust Variations with Speed



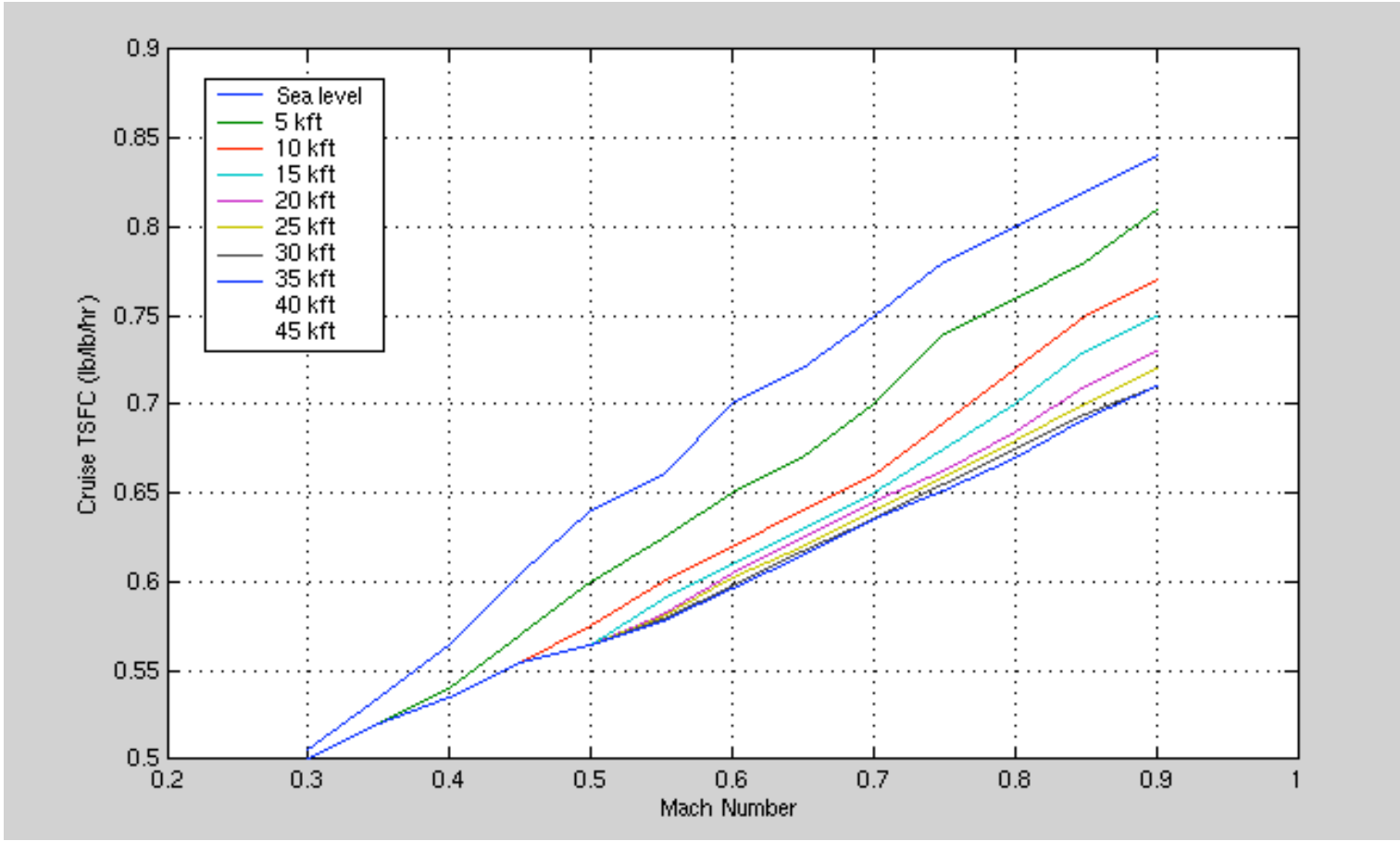
Thrust ratio = T_v / T_{static}

Source: Mair and Birdsall (1992)

Variations of Climb TSFC (PW JT9D Engine)



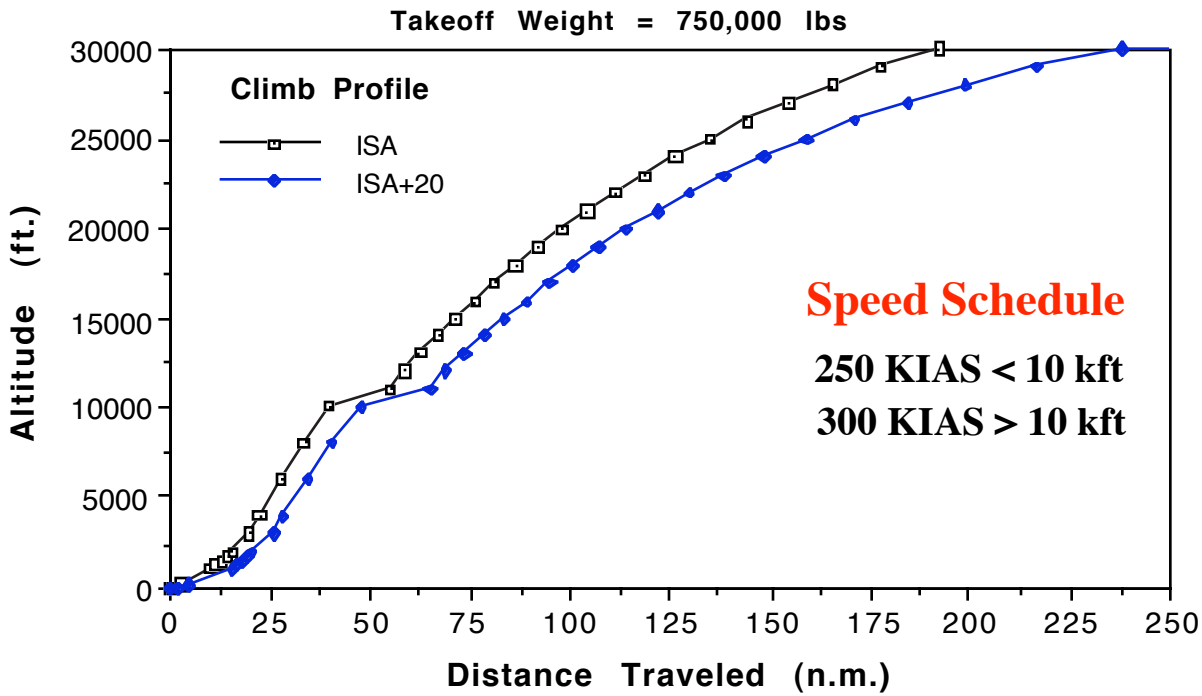
Variations of Cruise TSFC (PW JT9D Engine)

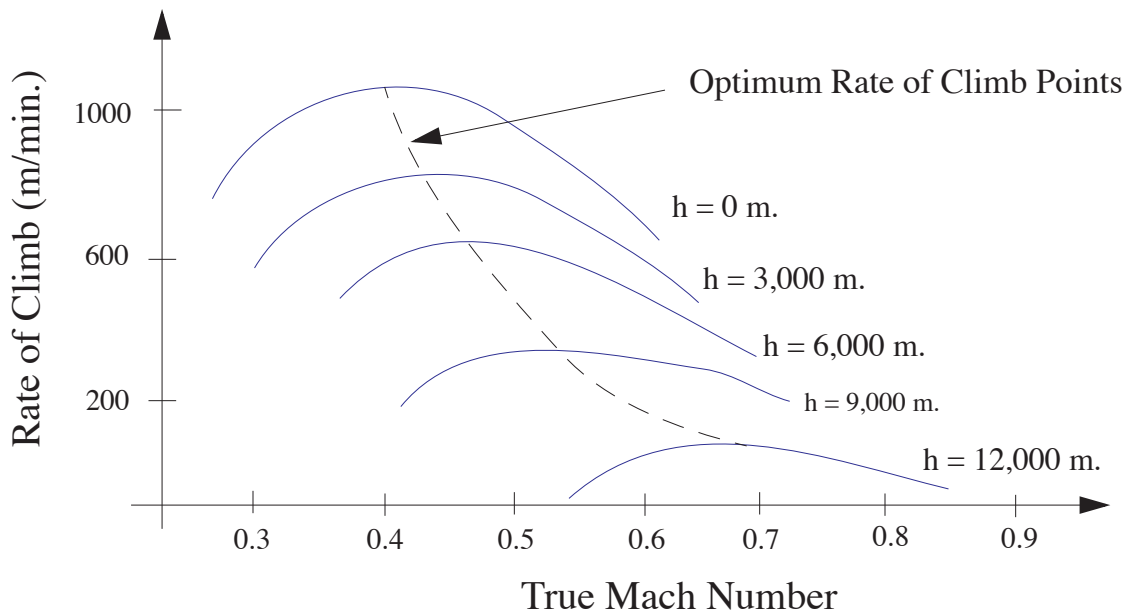


Sample Climb Trajectory Results

Numerical integration of equation (30) for a given flight speed schedule (speed time history) yields the following climb profiles.

Four engine, turbofan-powered aircraft



Typical Rate of Climb Envelope

Iterative analysis of the rate of climb equation yields the following results across the complete flight envelope.