

CEE 5614: Analysis of Air Transportation Systems
Quiz 1 : Open Notes

Spring 2023

Solution Set

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Instructions

Write your solutions in the spaces provided. Add any additional pages with calculations as needed. Make sure each additional page has your name.

Honor Code Pledge

The information provided in this exam is my own work. I have not received information from another person while doing this exam.

_____ (your signature/name)

Problem 1 (50 Points)

Task 1. Read the Aviation Week and Space Technology article (<https://aviationweek.com/air-transport/aircraft-propulsion/nasa-picks-boeings-transonic-truss-based-wing-sustainable-x-plane>) to understand the context of the problem.

Use the new generation Transonic Truss-Braced Wing (TTBA) aircraft provided in class (http://128.173.204.63/courses/cee5614/cee5614_pub/SUGAR_class.m) to answer the following questions.

Model a flight using the future TTBA aircraft traveling from **Punta Cana, Dominican Republic (PUJ)** to **New York (JFK)**. The aircraft has the following parameters: OEW of 40,000 kg., carries 16,000 kg. of fuel, and 14,000 kg of payload (passengers and belly cargo).

Task 2. Use the default climb speed profile provided in the aircraft data file to calculate the fuel burn in climb and the time to climb to an initial altitude that provides at least 150 m/minute climb rate. Use ISA atmospheric conditions in your calculations.

- a) Based on the data provided, estimate the fuel used to reach the Top of Climb Point (TOC) if the pilot wants to cruise at an altitude that provides a minimum climb rate of 152 m/minute. Airline dispatch suggest a cruise speed of Mach 0.75. For the selected altitude, estimate the mass of the aircraft at the Top of Climb (TOC) point. State the distance and travel time to reach the TOC point.

The parameters for the climb are estimated using the Unrestricted Climb Matlab code provided in class. The following initial conditions are used:

Takeoff mass = 70,000 kilograms

Temperature profile = ISA

Initial airport elevation = 0 (sea level)

After running the Unrestricted Climb Analysis code, we obtain the following:

Altitude with 150 m/min climb capability is 37,400 feet (see Figure 1)

Altitude with 150 m/min climb capability is 11,402 meters

Final Cruise Altitude is 37,000 feet

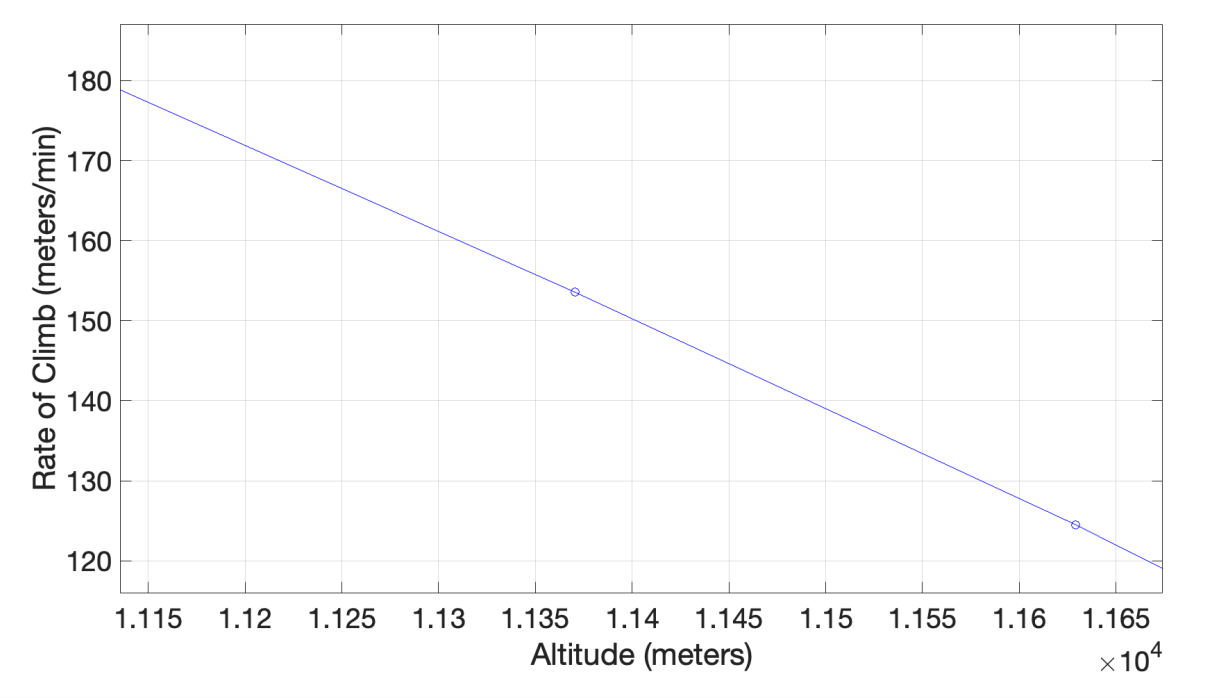
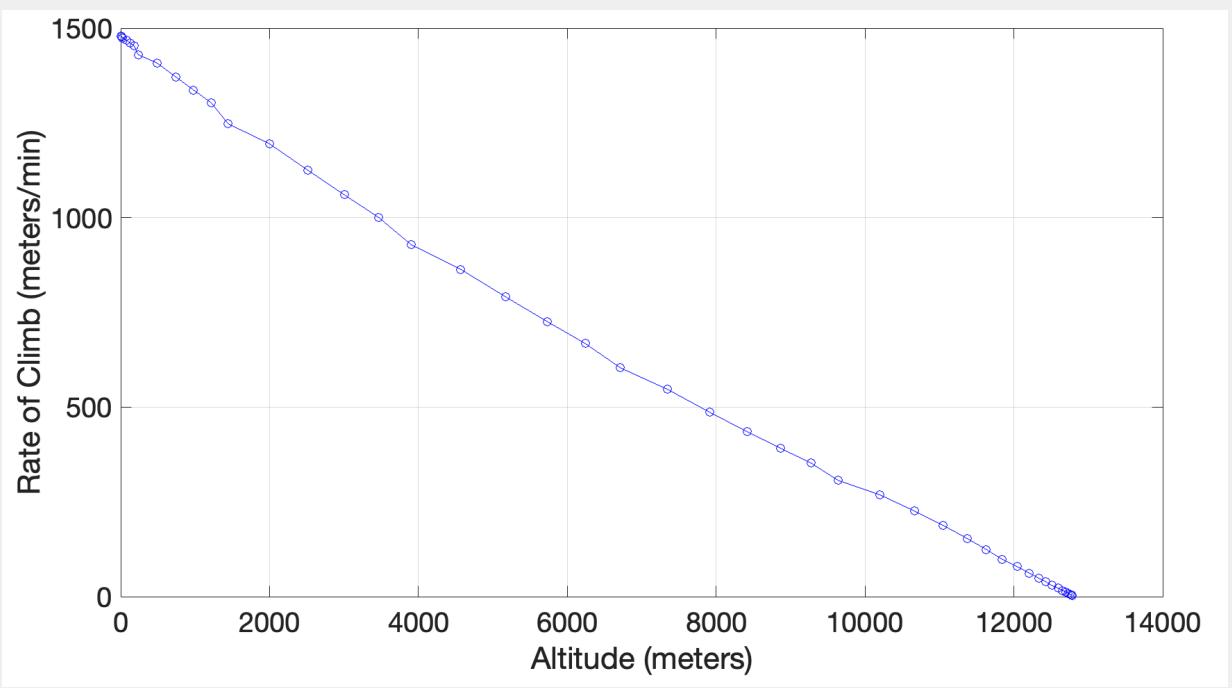
Final Cruise Altitude is 11,280.5 meters (37,000 feet)

Climb Fuel is 1209 kilograms (2,660 lbs.)

Distance to Reach TOC is 106 nm (196 kilometers)

I selected 37,000 feet as the initial cruise altitude because while the aircraft can reach 37,400 feet with 150 m/minute climb rate, Air Traffic Control assigns whole flight levels. Figure 1 shows the climb rate as a function of altitude. Note that it takes around 106 nautical miles to reach FL 370.

Figure 1. Rate of Climb Profile for TTBA Aircraft (Takeoff Mass 70,000 kgs), ISA Conditions.



b) Estimate the fuel used by the aircraft in cruise (**TOC to TOD only**) assuming the standard 6% detour factor to account for ATC restrictions and weather deviations. Perform the analysis using two cruise scenarios:

1. One climb at mid point between TOC and TOD
2. Schedule multiple climbs throughout the flight so that a climb is requested and granted when the aircraft mass allows the aircraft to climb at a minimum of 152 m/min at the start of each climb point (see Figure 1). Since the flight is mostly over the Atlantic ocean, assume climbs can be granted at 1,000 foot intervals (with Reduced Vertical Separation Minima).

Estimate the route length flown from Punta Cana to New York. Use the Great Circle Mapper application and add 6% to the route to account for ATC and weather diversions. The GCD distance is 1,350 nm or 1,432 nm adjusted distance.

We start by estimating the conditions for the descent profile. Use the unrestricted descent Matlab code provided to estimate the fuel and distance used in the descent profile. The TOD altitude is not precisely known. However, we can use 38,000 feet as a reasonable guess since the aircraft is at 37,000 feet at the TOC. The mass at the TOD point needs to be estimated as well. The route length is 1,432 nm or about half of the range of the aircraft (TTBA is expected to have similar range as the Boeing 737-800).

Takeoff weight = 70,000 kgs.

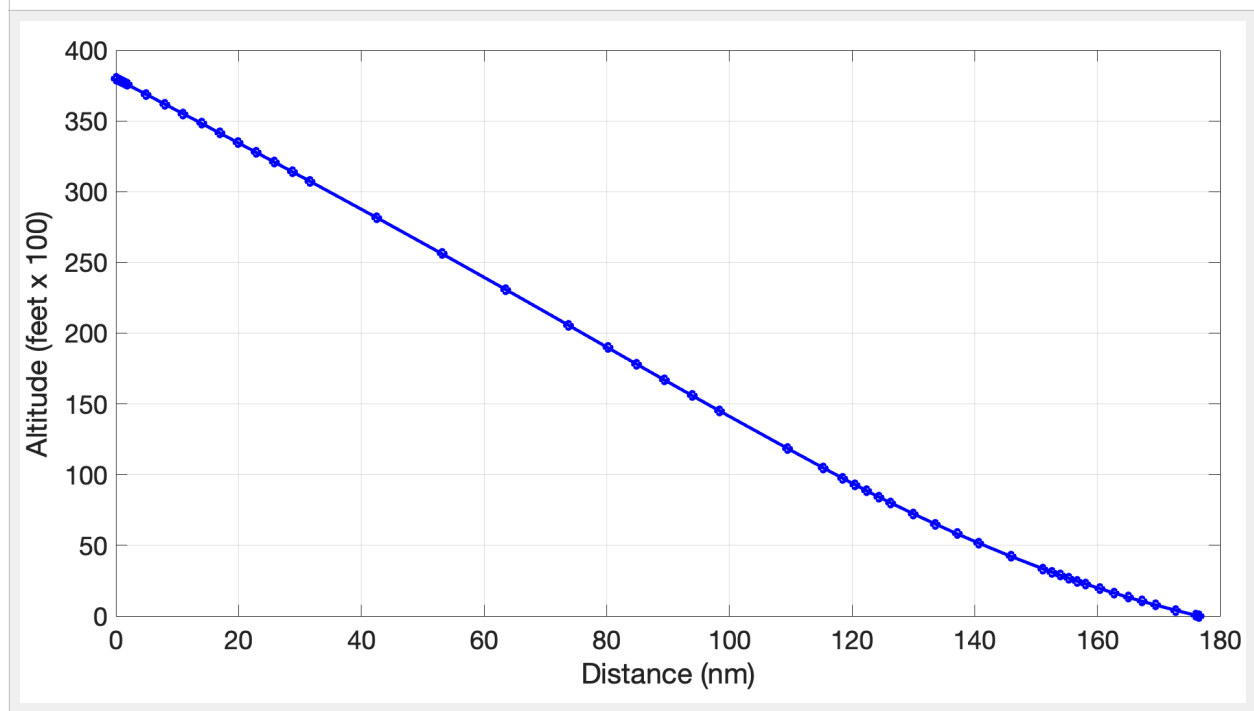
Fuel mass = 16,000 kgs.

Payload = 14,000 kgs.

If we assume that 2/3 of the fuel is used up to the TOD, the mass at the TOD is 59,440 kilograms. The descent fuel and distance flown in the descent profile are less sensitive to the mass at TOD. Figure 2 shows the unrestricted descent profiles. I used 1/10 of the maximum thrust as residual thrust in the descent profile.

$t_{Residual} = 1/10$; % residual thrust (dim) during the descent profile

Figure 2. Descent Profile for TTBA Aircraft (Mass at TOD is 60,000 kgs), ISA Conditions.



Descent distance is 176 nautical miles. The fuel used is 329 kilograms of fuel. **The estimated distance between TOC and TOD points is 1,150 nautical miles (rounded off).** Such distance is used to look at two scenarios.

Scenario 1. One climb at mid point between TOC and TOD.

Mid point is 575 nm from TOC. Use the numerical simulation technique explained in class to estimate the fuel burn from TOC to the mid point. The parameters of the simulation are:

Mass at TOD = 68,791 kilograms

Cruise altitude 37,000 feet

Distance flown = 575 nm

Proposed cruise speed = Mach 0.75

Running the Numerical Cruise Fuel Calculation Matlab code with 100 steps we obtain:

Computational Steps 100

Cruise Mach Number (dim) 0.75

Cruise Distance (nm) 575

Cruise Fuel (kg) 2368.0139

Cruise Time (minutes) 80.1913

Cruise Time (hrs) 1.3365

Initial Mass (kg) 68791

Final Mass at Mid Point (kg) 66423

Perform a 1,000 foot climb at mid point (750 nm from TOC). First we need to estimate the climb rate capability at the mid point (mass = 66,423 kilograms).

Cruise Mach Number (dim) : 0.750

True airspeed (knots) : 430.54

Altitude (m) : 11280.4878

Aircraft Mass (kg) : 66423

Lift Coefficient (dim) : 0.37212

Drag Coefficient (dim) : 0.021146

Drag (N) : 37027.5294

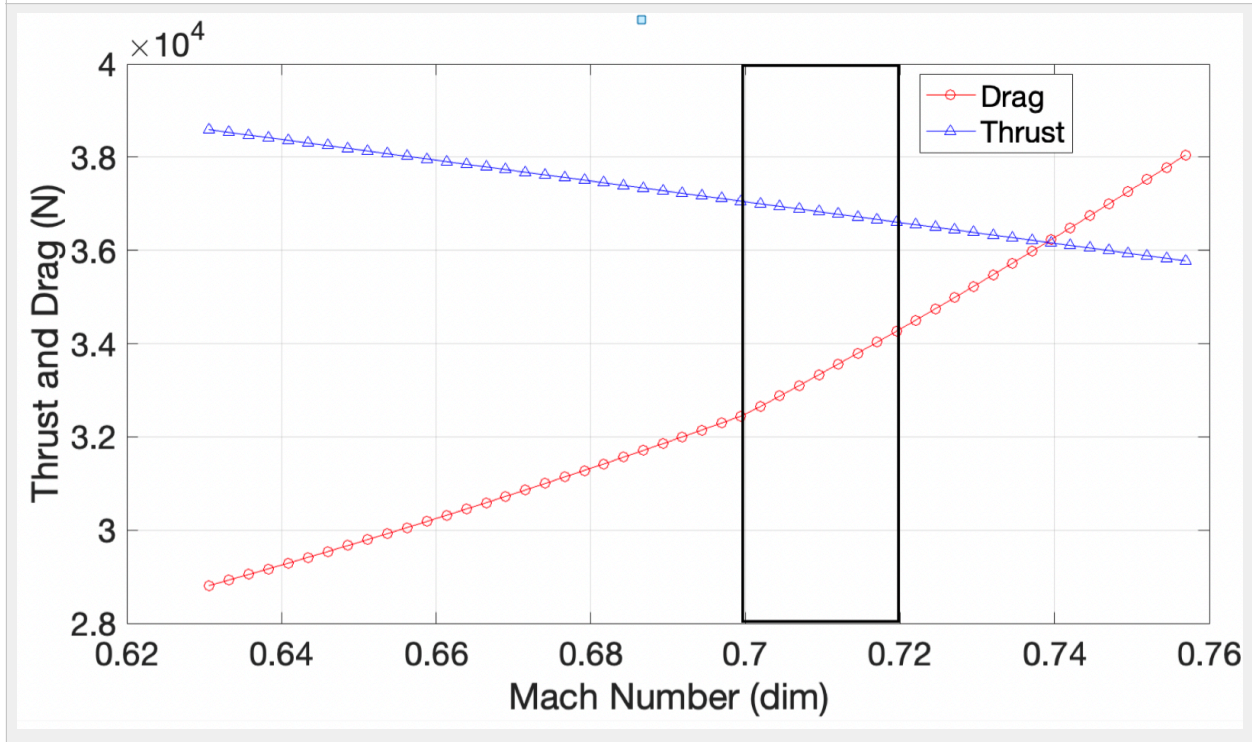
Thrust (N) : 35915.9579

Rate of climb (m/s) : -0.37784

Rate of climb (ft/min) : -74.3593

Note that the **aircraft cannot climb while flying at Mach 0.75 at the mid point.** This indicates that the initial cruise **Mach number of 0.75 is not achievable at FL370 at ISA conditions.** The climb profile specified in the aircraft file flies the aircraft at IAS 230 knots up to FL 370. 230 knots IAS is equivalent to Mach 0.63. Figure 3 shows the drag and thrust performance diagram at FL 370 and mass 68,791 kilograms. It is clear that the **cruise speed should be no more than Mach 0.72.**

Figure 3. TTBA Drag and Thrust at 37,000 feet and 68,791 kilograms (mass at the TOC). The Rectangular Region Is the Feasible Cruise Mach Number for this Example.



Since Mach 0.75 is not a feasible speed at 37,000 feet, we repeat the fuel burn calculations using a **cruise speed of Mach 0.72 instead (see Figure 3)**. From TOC to the mid point, flying at Mach 0.72 and 37,000 feet the following parameters are calculated using simulation analysis.

- Computational Steps 100
- Cruise Mach Number (dim) 0.72
- Cruise Distance (nm) 575
- Cruise Fuel (kg) 2266.8447
- Cruise Time (minutes) 83.5326
- Cruise Time (hrs) 1.3922
- Initial Mass (kg) 68791**
- Final Mass (kg) 66524.2**

Now verify the rate of climb at mid point to climb to FL 380 (next flight level).

- Cruise Mach Number (dim) : 0.72004
- True airspeed (knots) : 413.0382
- Altitude (m) : 11280.4878

Altitude (feet) : 37000
Aircraft Mass (kg) : 66524
Lift Coefficient (dim) : 0.40496
Drag Coefficient (dim) : 0.02107
Drag (N) : 33955.6734
Thrust (N) : 36596.0945

Rate of climb (m/s) : 0.85972

Rate of climb (ft/min) : 169.1919

Rate of climb (m/min) : 51.5829

The aircraft is capable of climbing to FL 380 at the mid point but only at 51.6 meters/minute and not at the desired 152 meters/minute. Check the climb rate at FL 380. Use the mass of 66524 kilograms .

Cruise Mach Number (dim) : 0.72
True airspeed (knots) : 413.00
Altitude (m) : 11585.3659
Altitude (feet) : 38000
Aircraft Mass (kg) : 66524
Lift Coefficient (dim) : 0.42501
Drag Coefficient (dim) : 0.021394
Drag (N) : 32849.707
Thrust (N) : 34708.6303

Rate of climb (m/s) : 0.60521

Rate of climb (ft/min) : 119.1058

Rate of climb (m/min) : 36.3127

You can estimate the average climb rate between FL 370 (51.6 m/min) and FL 380 (36.3 m/min). The solution using average values in the climb are shown below:

Average Rate of Climb (ROC) = 44 m/min

Climb speed = Mach 0.72

Average thrust (N) = 3.5652e+04

Average fuel burn in climb (N) = 0.4725 kilograms/second

Time to climb from FL 370 to FL 380 = 6.92 minutes

Fuel used in climb = 196.4 kilograms

Distance used in climb = 47.7 nautical miles.

An alternative method is to run the unrestricted climb analysis to determine the time and distance to climb from FL 370 to 380.

The fuel used in the climb from FL 370 to FL 380 is 196.4 kilograms. The mass of the aircraft reaching FL 380 is the 66,328 kilograms. Running the numerical integration code with the mass and other factors yields:

- Computational Steps 100
- Cruise Mach Number (dim) 0.72
- Cruise Distance (nm) 527
- Cruise Fuel (kg) 1988.1739
- Cruise Time (minutes) 76.5594
- Cruise Time (hrs) 1.276
- Initial Mass (kg) 66328
- Final Mass (kg) 64339.8

Figure 3. TTBA Drag and Thrust at 37,000 feet and 68,791 kilograms (mass at the TOC).

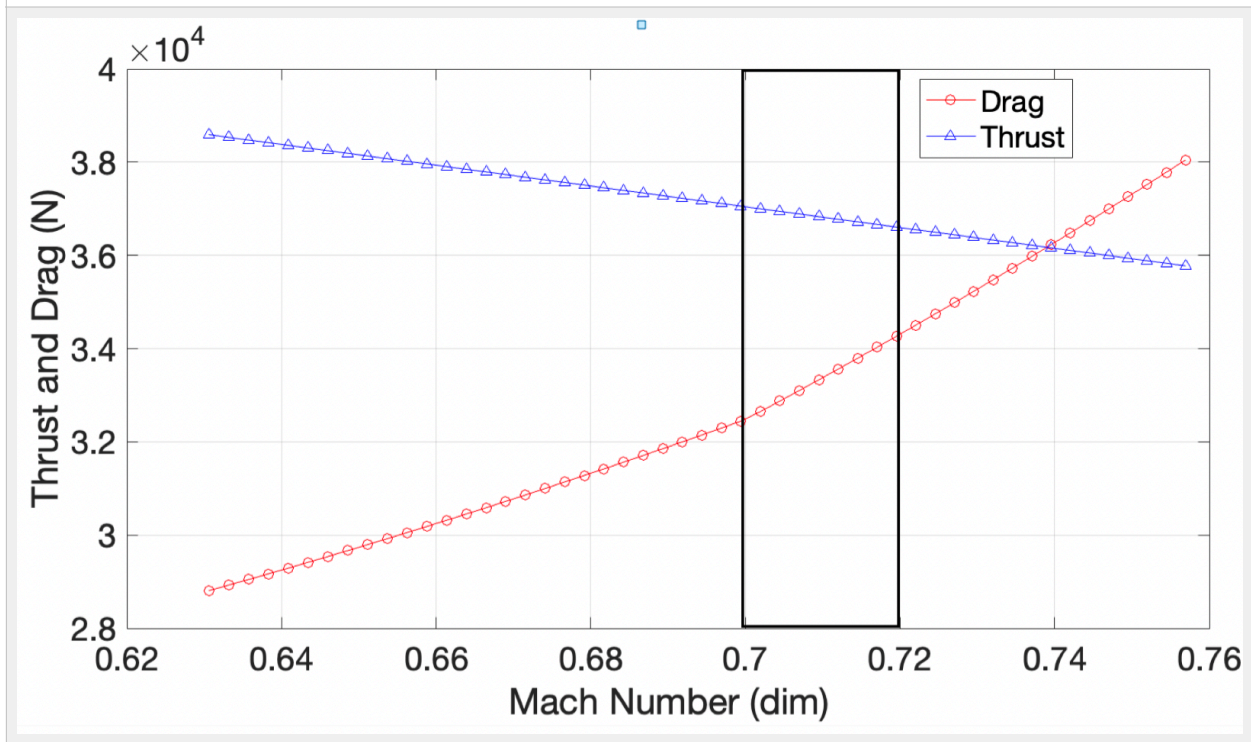


Figure 4. Flight Profile with a 1000-foot Climb at Mid Point.

Phase	Fuel Used (kgs)	Distance Flown (nm)	Initial Mass (kgs)	Final Mass (kgs)	Rate of Climb at Starting Point (m/min)
Climb	1,209	106	70,000	68,791	Variable
Cruise at FL 370	2,267	575	68,791	66,524	43.1
Climb (FL 370 to FL 380)	196	47.7	66,524	66,328	51.6
Cruise at FL 380	2,117	527	66,328	64,340	36.3
Descent (FL 380 to sea level)	329	176	64,340	64,011	N/A
Total	6,118	1431.7			131

Scenario 2. Schedule multiple climbs throughout the cruise.

If we are strict about a climb requirement of 150 m/minute, the multiple climb scenario is not possible because the aircraft is limited by thrust to be able to climb in such route. Longer routes can produce one or two climbs for this kind of aircraft.

Scenario 3. Flight with a constant cruise altitude

This scenario is possible if the aircraft stays at a constant cruise altitude of FL 370. For the constant cruise segment (1150 nm) we have:

- Computational Steps 200
- Cruise Mach Number (dim) 0.72
- Cruise Distance (nm) 1150
- Cruise Fuel (kg) 4510.4965
- Cruise Time (minutes) 167.0652
- Cruise Time (hrs) 2.7844
- Initial Mass (kg) 68791
- Final Mass (kg) 64280.5

Figure 5. Flight Profile Flown at Constant Altitude of 37,000 feet.

Phase	Fuel Used (kgs)	Distance Flown (nm)	Initial Mass (kgs)	Final Mass (kgs)	Rate of Climb at Starting Point (m/min)
Climb	1209	106	70000	68791	Variable
Cruise at FL 370	4510	1150	68791	64280	N/A
Descent (FL 370 to sea level)	329	176	64280	63951	N/A
Total	6048	1432			

c) What is the fuel savings using the multi-climb profile for the TTBA?

No savings. The climb at the mid point is so slow (very low rate of climb) that the additional fuel spent in the climb makes the climb counterproductive. The constant cruise altitude profile saves 70 kilograms compared to the profile with one climb.

d) Calculate the additional cost to the airline per flight (between the two profiles estimated in part (b)) if the fuel price today in large volumes is \$2.70 per gallon of Jet-A fuel (<http://www.iata.org/publications/economics/fuel-monitor/Pages/index.aspx>). Comment if the cost differential would be significant if the airline has three daily flights between PUJ and JFK.

The weight of jet-A fuel is 0.8 kgs per liter. One gallon is 3.785 liters. A gallon of Jet-A weights 3.028 kgs per gallon. The price of Jet-A fuel per kilogram is \$0.892. Cruising at constant altitude saves \$62.4 per flight.

Problem 2 (30 Points)

A low-cost airline started operations of Boeing 737-9 Max (with 194,700 lbs. max. design takeoff weight and 193 seats) between Providence (PVD) and Dublin (DUB). The engine used is the CFM Leap 1B27 engine.

- a) Evaluate the suitability of the aircraft for the route in question.

Characteristics of PVD (Providence) airport.

Airport elevation = 54 feet

Longest runway = 8,700 feet

Design temperature = 81.8 deg.F.

ISA Temperature at 54 feet = 58.8 deg. F.

Temperature difference = 23 deg. F. Use the charts ISA + 27 deg. F. If available.

Distance between PVD-DUB = 2,639 nm (2,797 nm)

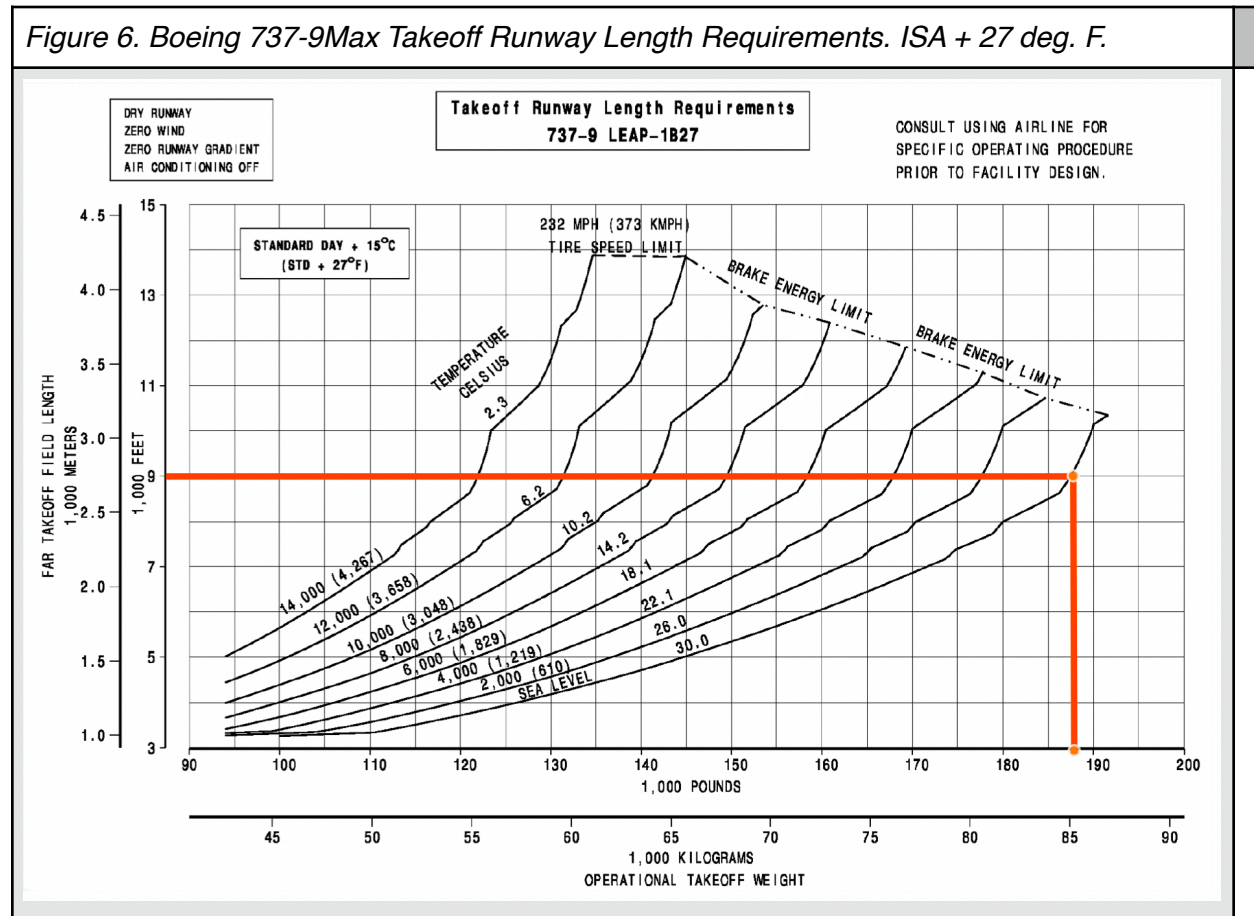
<i>Figure 6. Boeing 737-9Max Analysis for PVD-DUB.</i>	
Parameter	Boeing 737-9 Max
OEW (lbs)	104000
MTOGW (lbs)	194700
Passenger Seats	193
Passenger Load (lbs)	42460
OEW + Payload (lbs)	146460
Fuel (lbs)	41540
DTW (lbs)	188000
Stage length (nm)	2797
Fuel per passenger (lbs)	215
Runway Length needed (feet). No adjustment	9,000
Runway Length needed (feet). With adjustment	9,100
Runway length Available	8,700
SAR (nm/lb)	0.067

The fuel required for the trip is less than the maximum usable fuel (45,694 kilograms).

The payload required is less than the maximum structural payload.

b) Find the runway length needed at Providence (PVD) that satisfies FAA/EASA requirements.

Figure 6 shows the takeoff runway length requirements (without grade adjustment). The existing runway 5/23 at PVD has threshold elevation difference of 8.6 feet. Adding 86 feet to the solution yields a runway length requirement of 9,100 feet (rounded from 9086 feet). A 400 foot extension will be needed.



c) If the total operating cost of the aircraft is 3.3 times the cost of the fuel used in the route, estimate the average fare required to breakeven with a load factor of 85%. The cost of fuel is \$2.7 per gallon at today's prices.

Cost of fuel is \$0.892 per kilogram (\$0.405 per pound). The total fuel carried in the trip is 41,540 lbs. (18,881 kilograms or 6200 gallons). The cost of fuel is \$16,740. The total cost of the flight is then \$55,242.

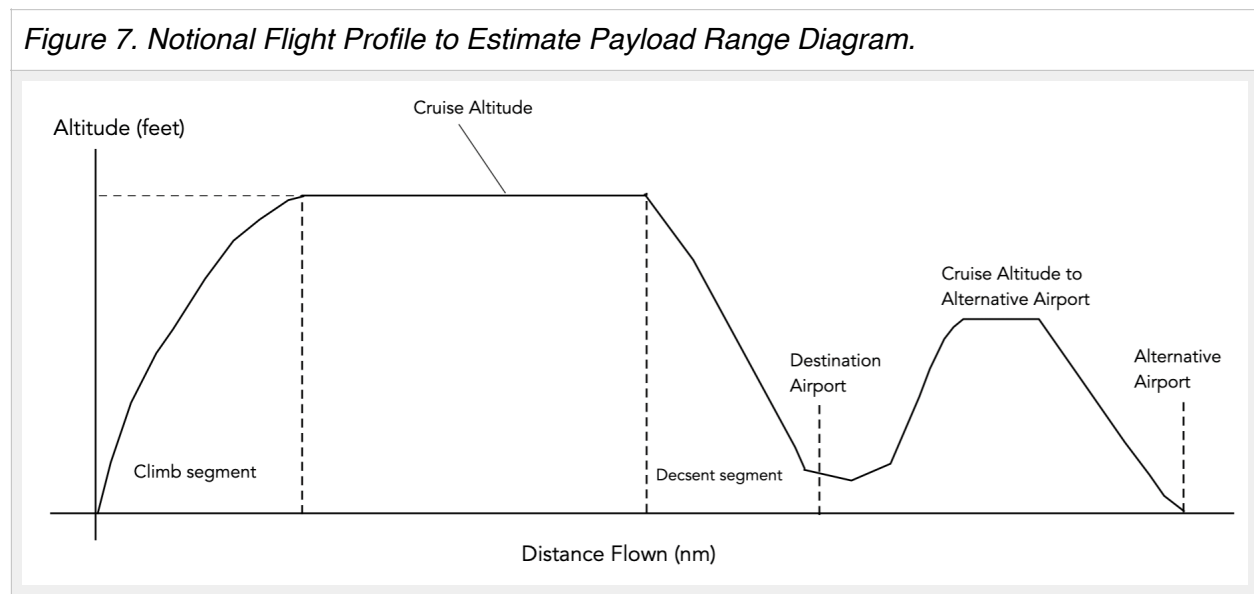
The airfare with load factor of 85% is \$336.8 per ticket.

Problem 3 (30 Points)

The Payload-Range diagram of an aircraft offers useful information about the efficiency of operations across a wide spectrum of routes (stage length) and payload conditions. An important parameter to evaluate the efficiency of aviation operations is the Specific Air Range (SAR) which measures the distance traveled for every pound (or kilogram) of fuel used.

- a) Using the TTBA aircraft explain the steps that you would follow to construct a basic payload-range diagram for the aircraft with three boundaries defined by the maximum zero fuel, the maximum takeoff, and the maximum fuel boundaries.

The process starts defining some key parameters for the calculations. Figure 7 illustrates the typical flight planning profile to estimate the aircraft range. The profile includes a diversion to an alternate airport. The diversion to the alternate airport may require specify information such as a cruise altitude and Mach number to the alternate. An alternative method is to account for the diversion to the alternate as a fraction of the total fuel used to fly from the origin to the destination airport or to set a time-based diversion (i.e., 1.5 hours of additional flight to account for the diversion).



The computational process of estimating a payload-range diagram is outlined in the Matlab code provided (`pay_range_2004.m` available at http://128.173.204.63/cee5614/matlab_files_cee5614.html).

Step 1

Define operational parameters:

Airport departure elevation (feet or meters)

Cruise altitude (feet or meters)

Cruise speed (typically Mach number)

Cruise altitude to alternate (feet or meters)

Cruise speed to alternate (Mach number)

Fuel reserve time (hours) % time for fuel reserve computation (hours)

Define the aircraft characteristics (for example use the `SUGAR_class.m` file)

Step 2

Estimate the climb fuel requirement using the unrestricted climb profile scripts. The analysis can include an algorithm to select the cruise altitude with checks for Mach number (see Problem 1) and cruise altitude compatible with user input.

This step produces distance flown to TOC, travel time to TOC, and fuel used to TOC.

Step 3

Estimate the fuel reserve to carry on any flight.

The calculation of fuel reserve is done in lines 93-98 in the **pay_range_2004** script. The calculation estimates the drag in the cruise condition to the alternate and then estimate the fuel burn. Fuel burn is translated to a total fuel quantity using the fuel reserve time.

Step 4

Estimate range using the maximum payload boundary in the payload-range diagram.

The calculation of the range-payload diagram (max payload boundary) is done in lines 100-120 in the **pay_range_2004** script. The calculation uses a loop to estimate range for a given fuel quantity (defined as **fuel_mass1** in line 111). The Breguet range equations is used to estimate range. Alternatively, you could use the numerical range calculation procedure demonstrated in class.

Step 5

Estimate range using the maximum takeoff weight boundary in the payload-range diagram.

The calculation of the range-payload diagram (maximum takeoff weight boundary) is done in lines 122-141 in the **pay_range_2004** script. The calculation uses a loop to estimate range for a given fuel quantity (defined as **fuel_mass2** in line 133). The Breguet range equations is used to estimate range. Alternatively, you could use the numerical range calculation procedure demonstrated in class.

Step 6

Estimate range using the maximum fuel boundary in the payload-range diagram.

The calculation of the range-payload diagram (maximum fuel boundary) is done in lines 143-162 in the **pay_range_2004** script. The calculation uses a loop to estimate range for maximum fuel and reducing the payload (defined as **fuel_mass3** in line 153). The Breguet range equations is used to estimate range. Alternatively, you could use the numerical range calculation procedure demonstrated in class.

Step 7

Collect all results of Steps 1-6 and plot.

The results of the calculations explained in Steps 1-6 are summarized into three vectors (lines 166-168) of the **pay_range_2004** script.

b) Provide a sample calculation to show me how would you construct the payload-range diagram for such aircraft.

I used the following parameters to generate sample payload-range plots of the TTBA aircraft. The `pay_range_2004` script generates the three boundaries of the payload-range diagram.

```

h_airport = 0;           % airport altitude (m)
rhos = 1.225;           % sea level density (kg/m-m-m)
hcruise = 36000/3.28;   % cruise altitude (m)
time_fuel_reserve = 1.0; % time for fuel reserve computation (hours)
SUGAR_class           % Aircraft file
mach_cruise = 0.73;    % cruising mach number (dim)
hcruise_alternate = 6000; % cruising altitude to alternate airport (meters)
mach_cruise_alternate = 0.70; % mach number in cruise to alternate airport

```

Figures 8 and 9 show two solutions of the payload-range diagram for the TTBA. Flying at Mach 0.73 we notice a reduction in the ferry range of 300 nautical miles. A similar reduction in range occurs at intersection of the maximum takeoff weight and maximum fuel boundaries.

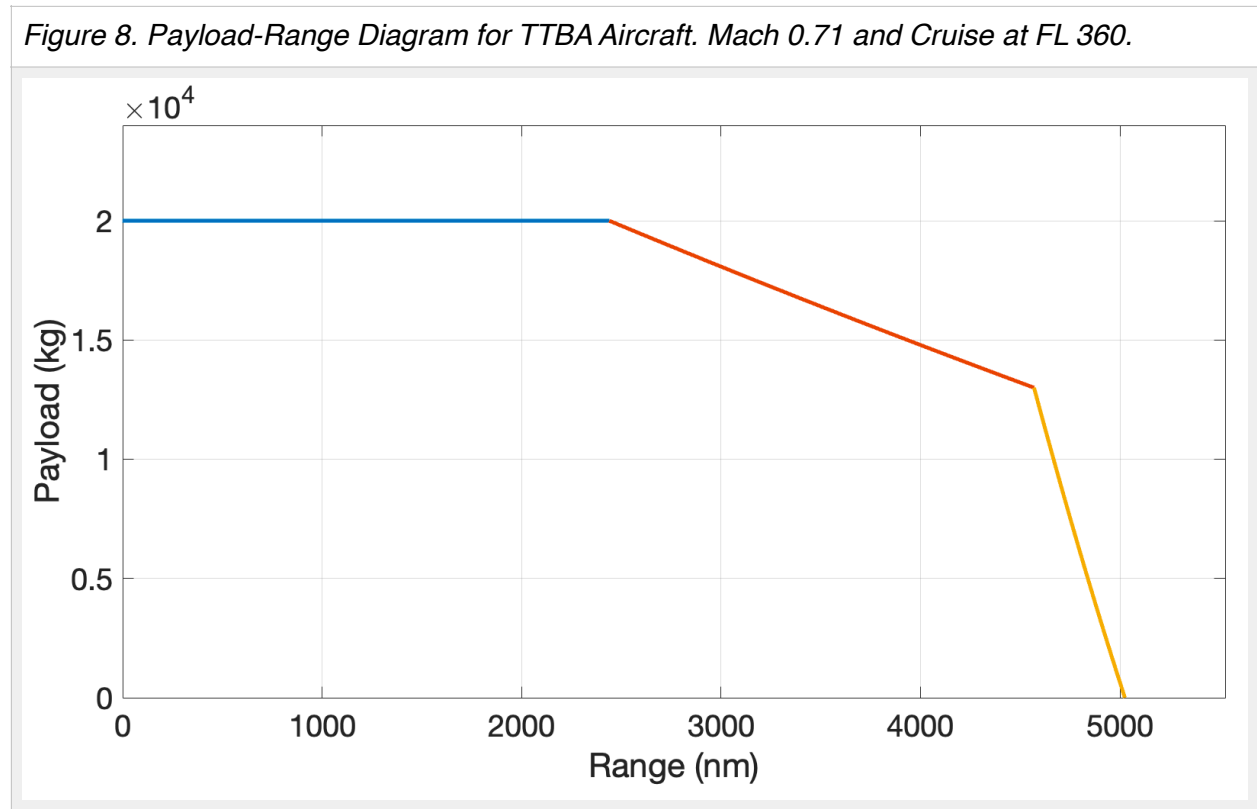


Figure 9. Payload-Range Diagram for TTBA Aircraft. Mach 0.73 and Cruise at FL 360.

