Quiz 2 - Solution

Date Due: Wednesday November 20, 2013

Instructor: Trani

Problem #1 (50 points)

The North Atlantic Organized System (OTS) is typically made up of 6 Eastbound Track as shown in Figure 1. Each track is delineated by waypoints spaced every 10 degrees of longitude as shown in Figure 1. The OTS track configuration shown in Figure 1, has 6 tracks with an average track length of 1,700 nm. Canadian controllers “meter” (or space) traffic at the blue circles to enforce correct procedural separation.

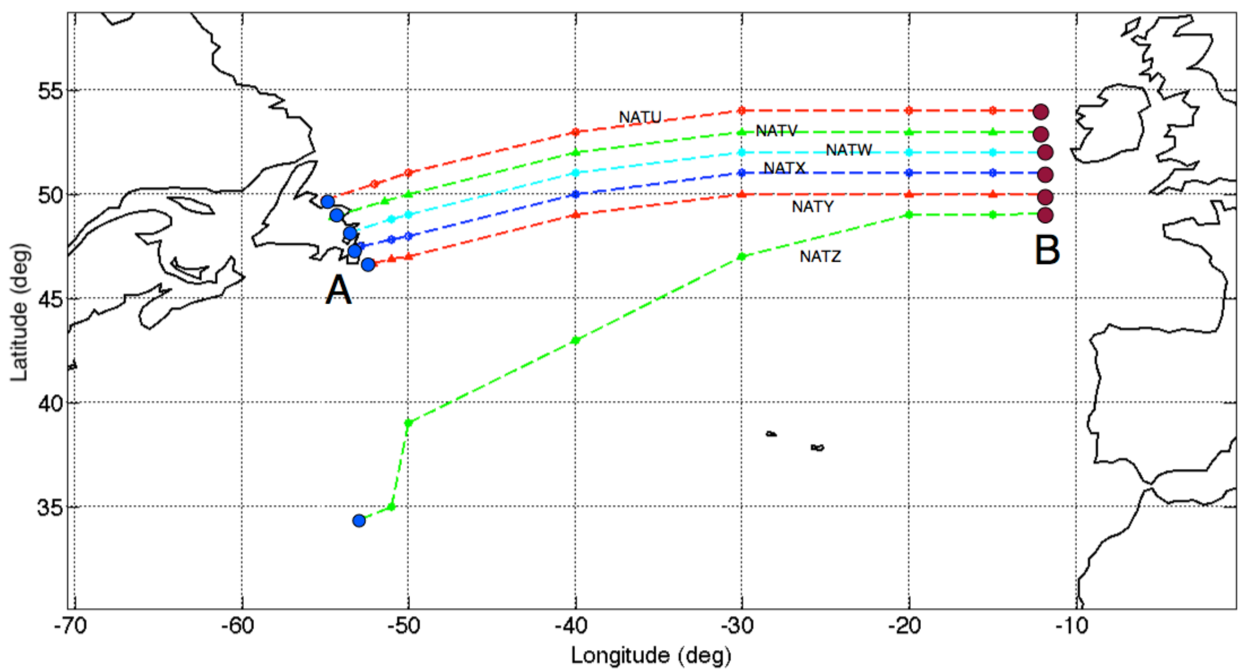

Figure 1. NATS OTS Track System (Eastbound).

a) Use the time-space technique described in class to derive saturation capacity two simple mathematical headway separation rules used by Oceanic controllers if the aircraft traveling at the same speed on the same track and cruise flight level are separated by a **nominal headway** of 10 minutes plus ATC buffers. The headway or time between successive arrivals to the NAT track system needs to be adjusted to account for “closing” or “opening” conditions considering pairwise flight operations (i.e., a lead and a following aircraft). For example, an air traffic controller in Canada (entry point for Eastbound flights) adjusts the entrance of a “following” aircraft into a track and cruise flight level if the “lead” aircraft is slower so that both aircraft will have a **nominal headway** condition (10 minutes) as the “leader” crosses the red dots (point B in Figure 1) near the European side. On the other hand,If the lead aircraft is faster than the following aircraft, the minimum separation between the two aircraft is enforced at the NAT OTS entry point (off Canadian Coast) is 5 minutes. ATC Oceanic controllers apply normally distributed buffers with a standard deviation of the delivery accuracy of 20 seconds. Oceanic controllers are more risk avert than their terminal area counterparts and tolerate 1% probability of violation between two successive operations. Inside the OTS track system pilots are required to fly at constant mach number in the .

The analysis is done for Flight levels 360 and above where most of the North Atlantic traffic flies. The selection of such flight levels allows us to estimate true airspeed conditions. The probability matrix assuming random arrivals is shown in Table 1a.

Table 1a. Probability Matrix for Aircraft Using the NATS OTS System.

	Following Aircraft			
Lead Aircraft	**B757 Class**	**B767 Class**	**B777 Class**	**B747 Class**
B757 Class	2.25	4.50	4.50	3.75
B767 Class	4.50	9.00	9.00	7.50
B777 Class	3.75	7.50	9.00	7.50
B747 Class	4.50	9.00	7.50	6.25

Opening case:

Fixed rule at nominal 5 minutes plus buffers. Since the aircraft are under radar control on both sides of the Atlantic, we assume 20 second standard deviation of the in-trail delivery error and 1% probability of violation (i.e., 2.33 σ_o from mean - or 46.6 seconds or 0.78 minutes of additional headway above the minimum required). Tables 1b and 1c show the headway and distance based calculations for the opening cases. The vector of true airspeeds used is: Vtrue = [233.05 238.95 244.85 247.80] all values in meters/second.

The situation on how controllers act to separate traffic in the North Atlantic is shown graphically in Figure 1b.

Table 1b. Headway Separation (Minutes) Matrix (Opening Cases) for Aircraft Using the NATS OTS System.

	Following Aircraft			
Lead Aircraft	**B757 Class**	**B767 Class**	**B777 Class**	**B747 Class**
B757 Class				
B767 Class	5.78			
B777 Class	5.78	5.78		
B747 Class	5.78	5.78	5.78	

Table 1c. Equivalent Distance Separation (nautical miles) Matrix (Opening Cases) for Aircraft Using the NATS OTS System.

	Following Aircraft			
Lead Aircraft	**B757 Class**	**B767 Class**	**B777 Class**	**B747 Class**
B757 Class				
B767 Class	43.64			
B777 Class	43.64	44.75		
B747 Class	43.64	44.75	45.85	

Closing case:

The rule is 10 minutes plus buffers as the lead aircraft completes the crossing of the NAT OTS system. Figure 1a shows graphically the situation. Since the aircraft are under radar control on both sides of the Atlantic, we assume 20 second standard deviation of the in-trail delivery error and 1% probability of violation (i.e., 2.33 σ_o from the mean - or 46.6 seconds or 0.78 minutes of additional headway above the minimum required).

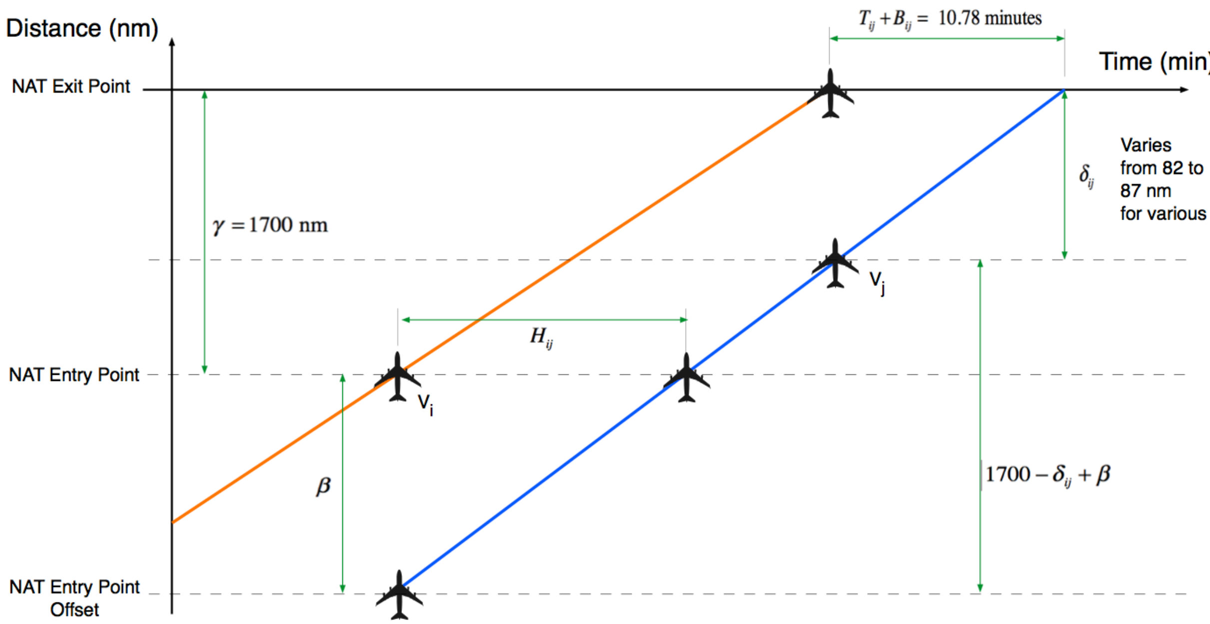

Figure 1a. Time-Space Diagram for Closing Case in the NAT OTS System.

Tables 1d and 1e show the headway and distance based calculations for the opening cases. The vector of true airspeeds used is: Vtrue = [453.01 464.48 475.95 481.68] all values in knots at 36,000 feet or above. An equation to estimate the required headway between two aircraft (closing case) as the lead aircraft passes the NAT OTS entry point is given below:

$$H_{ij} = H_{nom} + (\gamma - \delta_{ij})\left[\frac{1}{V_i} - \frac{1}{V_j}\right] = (T_{ij} + B_{ij}) + (\gamma - \delta_{ij})\left[\frac{1}{V_i} - \frac{1}{V_j}\right]$$

$$\gamma = 1700 \text{ nm}$$

$$T_{ij} = 10 \text{ minutes}$$

$$B_{ij} = \sigma_0 q_v$$

$$\sigma_0 = 1/3 \text{ minutes}$$

$$q_v = 2.33 \text{ (dimensionless)}$$

$$\sigma_0 q_v = 0.78 \text{ minutes}$$

An example calculation to separate a Boeing 757 (lead) and Boeing 747 (follower) class aircraft are shown below.

$$H_{ij} = H_{nom} + (\gamma - \delta_{ij})\left[\frac{1}{V_i} - \frac{1}{V_j}\right] = (T_{ij} + B_{ij}) + (\gamma - \delta_{ij})\left[\frac{1}{V_i} - \frac{1}{V_j}\right]$$

$$H_{B757-B747} = 10.78 + (1700 - 87)\left[\frac{1}{453} - \frac{1}{482}\right] = 23.6 \text{ minutes}$$

$$\beta_{B757-B747} = H_{B757-B747}(V_{B747}) = \frac{23.6}{60}\text{hr} \bullet \left[482 \ \frac{nm}{hr}\right] = 189.6 \text{ nm}$$

Note that the differential speeds of the aircraft are 29 knots. Flying a distance of 1613 nm inside the NAT OTS (1700-87 nm), the Boeing 747 class aircraft has to be delivered an additional 12.8 minutes (above the nominal headway) behind the Boeing 757 vehicle in order to satisfy the minimum required separation at the European side of the Atlantic. The headway behind the Boeing 757 class aircraft as the lead enters the NAT OTS boundary will be 23.6 minutes.

The situation on how controllers act to separate traffic in the North Atlantic is shown graphically in Figure 1b.

Table 1d. Headway Separation (Minutes) Matrix (Closing Cases) for Aircraft Using the NATS OTS System.

	Following Aircraft			
Lead Aircraft	**B757 Class**	**B767 Class**	**B777 Class**	**B747 Class**
B757 Class	10.78	15.9	21.1	23.6
B767 Class		10.78	18.6	16.0
B777 Class			10.78	13.3
B747 Class				10.78

Table 1e. Equivalent Distance Separation (nautical miles) Matrix (Closing Cases) for Aircraft Using the NATS OTS System.

	Following Aircraft			
Lead Aircraft	**B757 Class**	**B767 Class**	**B777 Class**	**B747 Class**
B757 Class	81.4	123.0	167.4	189.6
B767 Class		83.5	147.6	128.5
B747 Class			85.5	106.8
B777 Class				86.5

Quick Rule of Thumb for ATC Controllers:

Add 3 minutes to the nominal headway (10 minutes) for each 0.01 Mach Number difference between lead and trailing aircraft. For example, for an Airbus A380 class vehicle following a Boeing 757 ATC will apply 15 minutes (0.05 delta Mach number) above the nominal rule. The total headway will then be ~25 minutes.

Table 1f. Headway Separations for Various Combinations of Aircraft of all Cases (Flying inside the NATS OTS System).

	Following Aircraft			
Lead Aircraft	**B757 Class**	**B767 Class**	**B777 Class**	**B747 Class**
B757 Class	10.78	15.9	21.1	23.6
B767 Class	5.78	10.78	18.6	16.0
B777 Class	5.78	5.78	10.78	13.3
B747 Class	5.78	5.78	5.78	10.78

The expected value of headways is 684 seconds. **This produces a track/flight level capacity of 5.3 per hour.** Note that in theory since the NATS OTS has 6 tracks and 8 flight levels, this could result in a total OTS system hourly capacity of 253 aircraft per hour.

b) Using the rules proposed in part(a), **estimate the saturation capacity of track NATX** (see Figure 1) if the arrival distribution of flight arrivals is random to the NAT entry point and if the fleet mix using the track is shown in Table 1. In your analysis consider that each NAT OTS tracks includes all cruise flight levels between 320 to 390 (inclusive).

Table 1. Aircraft Population Operating in the North Atlantic Organized System.

Aircraft Group	Typical Cruise Mach Number	Percent Mix (%)
B757, B737, Airbus 320	0.79	15
B767, A330, A340	0.81	30
B747,A380	0.84	25
B777, B772ER	0.83	30

c) The introduction of DataLink Communications (called CPDLC) and Future Area Navigation Systems (FANS 1/A1) onboard aircraft is expected to change the North Atlantic procedures described in part (a). For example, controllers handling aircraft equipped with both CPDLC and FANS 1/A1 will be able to use Reduced Longitudinal Separation Minima Procedures (RLongSM) so that nominal headways will be reduced to 5 minutes (from 10 minutes today). Using the new technology, **estimate the saturation capacity of track NATX** (see Figure 1) if the arrival distribution of flight arrivals is random and if the fleet mix is still as shown in Table 1. Contrast the saturation capacities obtained in parts (b) and (c).

$$H_{ij} = H_{nom} + (\gamma - \delta_{ij})\left[\frac{1}{V_i} - \frac{1}{V_j}\right] = (T_{ij} + B_{ij}) + (\gamma - \delta_{ij})\left[\frac{1}{V_i} - \frac{1}{V_j}\right]$$

$$H_{B757-B747} = 5.78 + (1700 - 87)\left[\frac{1}{453} - \frac{1}{482}\right] = 18.6 \text{ minutes}$$

$$H_{B757-B777} = 5.78 + (1700 - 86)\left[\frac{1}{453} - \frac{1}{476}\right] = 16.1 \text{ minutes}$$

Table 1g. Future Headway Separations for Various Combinations of Aircraft of all Cases (Flying inside the NATS OTS System) with CPDLC Capability.

	Following Aircraft			
Lead Aircraft	**B757 Class**	**B767 Class**	**B777 Class**	**B747 Class**
B757 Class	5.78	10.9	16.1	18.6
B767 Class	5.78	5.78	13.6	11.0
B777 Class	5.78	5.78	5.78	8.3
B747 Class	5.78	5.78	5.78	5.78

The expected value of headways is 494 seconds. **This produces a track/flight level capacity of 7.3 per hour.** Note that in theory since the NATS OTS has 6 tracks and 8 flight levels, this could result in a total OTS system hourly capacity of 350 aircraft per hour.

Problem 2 (50 Points)

For the aircraft climb and maneuvering example solved in class (http://128.173.204.63/courses/cee5614/cee5614_pub/aircraft_maneuvering_performance.pdf), find the maximum takeoff weight for the very large capacity aircraft to legally depart the airport and clear the 1,500 meter hill by 300 meters. Assume the flap setting is at 10 degrees. Also assume the aircraft fully retracts the landing gear at 50 meters of altitude and the climb speed is 190 knots (Indicated) in the initial climb for the first 6 miles after engine failure. The aircraft has 3 remaining engines after point A.

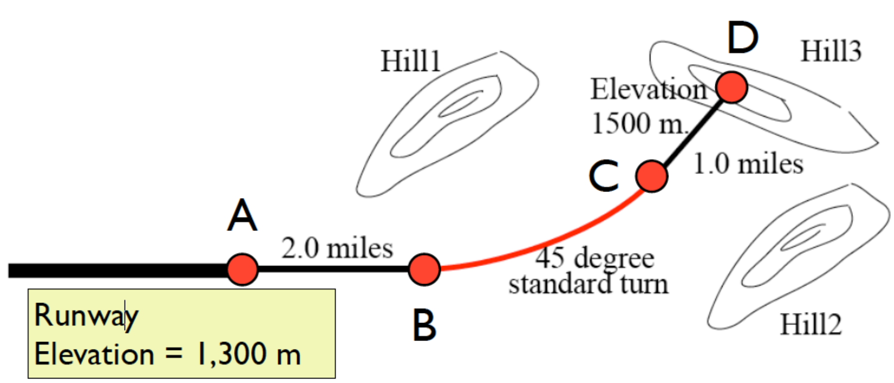

Figure 2. Engine Out Procedure for Problem 2.

Phase 1 - 2 nm (3.70 km) straight and level flight

Phase 2 - 0.8 nm (1.47 km) turning flight (at 190 knots) at 27.6 degrees of bank angle

Phase 3 - 1 nm (1.85 km) straight and level flight

Using a trial and error method we derive the maximum takeoff to overfly the 1500 meter obstacle. Table 2 shows the results. Note that the aircraft can takeoff with 525 metric tons of mass and still clear the critical obstacle by 300 meters or more.

Table 2. Airbus A380 Class Vehicle for Problem 2.

	M = 450 tons	M = 510 tons	M = 520 tons	M = 525 tons
Phase 1 Final Altitude (m) (B)	1696	1613	1602	1596
Phase 2 Final Altitude (m) (C)	1829	1713	1698	1690
Phase 3 Final Altitude (m) (D)	1982	1836	1815	1805
Altitude Above Obstacle	482	336	315	305