# Quiz 1 (Open Book/Notes) 

## Solution

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## Honor Code Pledge

The information provided in this exam is my own work. I have not received information from another person while doing this exam.

## Signature Space

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## Problem \#1 (50 points)

Providence International Airport (PVD) is studying a runway extension program to attract a new cargo operator to the airport. Review Airnav and the satellite images at Google Earth to answer the following questions. The objective is to estimate the runway extension needed for the longest runway at PVD if the following aircraft are considered by the cargo operator (see Table 1).

More detailed information about the airport can be found at the Airnav database available on the web at: http://www.airnav.com/ airports/. In your analysis use the latest version of the Boeing and Airbus documents for airport design (http://128.173.204.63/ courses/cee5614/sites_ce_5614.html\#Aircraft_Data).

Table 1. Proposed Services from PVD to Other Airports.

| Origin-Destination Airport Pair | Aircraft Expected to Fly the Route |
| :--- | :--- |
| PVD-TLV | Boeing 747-8F with General Electric GEnx-2B67. Aircraft <br> maximum design takeoff weight is $975,000 \mathrm{lb}$. |
| PVD-DOH | Boeing 747-8F with General Electric GEnx-2B67. Aircraft <br> maximum design takeoff weight is 975,000 Ib. |

a) Find the runway length needed for each one of the routes. Determine which one of the trips constitutes the critical stage length and state the new runway length extension needed (if any).

PVD is located at 55 feet above sea level (practically at sea level conditions). Runway $5 / 23$ is 7166 feet long. Runway $16 / 34$ is 6081 feet long. Both runways are 150 feet wide. The mean high temperature of the hottest month of the year is 83 deg. F. (28.3 deg. C.). This is illustrated in Figure 1.

## Monthly Average/Record Temperatures



Figure 1. Temperature Profile for PVD Airport (source: weather.com).

| CHARACTERISTICS | UNITS | 747-8F |
| :---: | :---: | :---: |
| MAX DESIGN TAXI WEIGHT | POUNDS | 978,000 |
|  | KILOGRAMS | 443,613 |
| MAX DESIGN TAKEOFF WEIGHT | POUNDS | 975,000 |
|  | KILOGRAMS | 442,253 |
| MAX DESIGN LANDING WEIGHT | POUNDS | 761,000 |
|  | KILOGRAMS | 345,184 |
| MAX DESIGN ZERO FUEL WEIGHT | POUNDS | 725,000 |
|  | KILOGRAMS | 328,854 |
| OPERATING EMPTY WEIGHT <br> (1) | POUNDS | 434,600 |
|  | KILOGRAMS | 197,131 |
| MAX STRUCTURAL PAYLOAD (1) | POUNDS | 290,400 |
|  | KILOGRAMS | 131,723 |
| TYPICAL CARGO - MAIN DECK CONTAINERS | CUBIC FEET | 24,462 |
|  | CUBIC METERS | 693 |
| MAX CARGO - LOWER DECK CONTAINERS (LD-1) | CUBIC FEET | 5,850 |
|  | CUBIC METERS | 166 |
| MAX CARGO - LOWER DECK BULK CARGO | CUBIC FEET | 520 |
|  | CUBIC METERS | 14.7 |
| USABLE FUEL CAPACITY | U.S. GALLONS | 59,734 (2) |
|  | LITERS | 226,118 |
|  | POUNDS | 400,218 |
|  | KILOGRAMS | 181,536 |

Figure 2. Characteristics of Boeing 747-8F Aircraft (source: boeing.com).

Table 2 shows the critical route distances. Using values published in Figure 2. OEW = 197,131 kilograms. Maximum Structural Payload is 131,723 kilograms. For maximum flexibility assume the aircraft carries the maximum structural payload. Hence:
OEW + PAY $=197131+131723=328854$ kilograms.

Note that this value is the Maximum Zero Fuel Weight of published by Boeing in Figure 2. For maximum flexibility, we would expect the operator to use Maximum Takeoff Weight as shown in Figure 3. The figure illustrates that under MTOW and to cover the Doha segment, the aircraft OEW + PAY has to be be 300 metric tons. This implies that 28.854 metric tons of cargo estimated above cannot be carried. The maximum payload for the Doha route is then estimated to be 98.8 metric tons.

DTW = MTOW $=442.2$ metric tons. Fuel weight is estimated to be 146.2 metric tons. Check that maximum fuel is not above the maximum capacity for the aircraft ( 181.5 metric tons).

## Table 2. Distances from PVD Airport to Critical Destinations.

| Origin-Destination <br> Airport Pair | Great Circle Distance (nm) | Route Distance (nm) |
| :---: | :---: | :---: |
| PVD-TLV | 4809 | 5049 |
| PVD-DOH | 5703 | 5988 |

The critical stage length is Providence-Doha.


Figure 3. Payload-Range Diagram of 747-8F Aircraft (source: $\underline{\text { boeing.com) }}$ ).
The runway length requirement operating from PVD at ISA + 15 deg. C is shown in Figure 4. Note that 30 deg. Celsius is close to the design temperature of 28.3 deg. C. At sea level conditions the aircraft requires 10,600 feet with zero gradient. Runway $5 / 23$ has $\mathrm{a} 0.1 \%$ gradient. Correcting for gradient this yields:

Rc $=10600$ * $(7.17$ * 10) $=10,672 \sim 10,700$ feet for design
The correction is adding 10 feet for each 1 foot in maximum elevation difference in centerline elevations. A $0.1 \%$ gradient difference for a 7166 for runway is 7.17 feet.
The runway extension required at PVD is estimated to be 3,534 feet.


Figure 4. Runway Length Requirements for 747-8F Aircraft (source: boeing.com). Performance at ISA +15 deg. $C$.
b) Find the maximum payload to be carried in each route with the extension you suggested in part (a).Comment on your solution.
Maximum payload was estimated to be 98.8 metric tons.
c) What is the ferry range for the Boeing 787-8F?

The ferry range is $8,600 \mathrm{~nm}$ directly from the payload-range diagram.

## Problem \# 2 (50 points)

The differential equation of motion that explains the behavior of a climbing aircraft is given as:

$$
V \sin \gamma=\frac{d h}{d t}=\frac{V[T-D]}{m g}-\frac{V}{g} \frac{d V}{d t}
$$

The terminology has been explained in the class notes.


In class so far we ignored the second term (the so-called acceleration term) in the equation.
a) Develop a procedure using a flowchart or steps (as needed) to account for the second term in the analysis. In other words, how would you solve the problem if the second term is no longer ignored.

The suggested method includes a variation of the calculations done in class.
Step 1: Estimate true airspeed using atmospheric model
Step 2: Estimate the lift coefficient needed to sustain flight using the basic lift equation
Step 3: Estimate drag coefficient
Step 4: Estimate total drag (D)
Step 5: Estimate the thrust produced by the engines at altitude ( T )
Step 6: Find the rate of climb (dh/dt) without the acceleration term
Step 7: Estimate the rate of change of speed as a function of time using the solution obtained in Step 6. Save the values of dV/dt obtained in this step for calculations in Step 7.

Step 8: Solve the rate of climb (dh/dt) problem using the new values of dV/dt obtained in Step 7.
Step 9: Compare two solutions
b) Use the regional jet aircraft performance file provided in the Matlab files for CEE 5614 (http://128.173.204.63/courses/ cee5614/cee5614_pub/regionalJet_class.m) to demonstrate (with actual calculations) the procedure developed in part (a). Assume:
i) The regional jet climbs at 220 knots (indicated) at 4,000 feet above sea level (state 1).
ii) Suppose the aircraft accelerates to a new state (called state 2), and reaches 240 knots (indicated) at 8,000 feet.
c) Compare the answer obtained in part (b) with the baseline conditions where the acceleration term is ignored. Comment on the two answers obtained.

A numerical simulation of the two profiles is shown in Figure 5. The blue profile is the uncorrected climb profile without the $\mathrm{dV} / \mathrm{dt}$ term added. The red profile is the corrected climb profile with the terms $\mathrm{dV} / \mathrm{dt}$ added. Note that for the aircraft climbing from 4000 feet to 8000 feet takes an extra 10 seconds to reach the new altitude after the rate of climb equation has been corrected.


Figure 5. Numerical Solution for Regional Jet Climb Performance. ISA Conditions.

Table 3. Parameters for Climb Profile at 4,000 feet. Regional Jet with Mass of 24,000 kg.

| Parameter | Uncorrected Profile | Corrected Profile |
| :---: | :---: | :---: |
| Altitude | 4000.0000 | 4000.0000 |
| IAS (knots) | 220.0000 | 220.0000 |
| True Mach | 0.3524 | 0.3524 |
| Speed of Sound (m/s) | 335.5400 | 335.5400 |
| TAS (m/s) | 118.2443 | 118.2443 |
| Mass $(\mathrm{kg})$ | 24000.0000 | 24000.0000 |
| Cl (dim) | 0.5298 | 0.5300 |
| Cd (dim) | 0.0304 | 0.0304 |
| Drag (N) | 13495.0000 | 13495.0000 |
| Thrust (N) | 61536.0000 | 61536.0000 |
| dV/dt (m/s-s) | 0.0000 | 0.1089 |
| Rate of Climb (m/s) | 24.1300 | 22.8100 |

Table 4. Parameters for Climb Profile at 8,000 feet. Regional Jet with Mass of 24,000 kg.

| Parameter | Uncorrected Profile | Corrected Profile |
| :---: | :---: | :---: |
| Altitude | 8000.0000 | 8000.0000 |
| IAS (knots) | 240.0000 | 240.0000 |
| True Mach | 0.3739 | 0.3739 |
| Speed of Sound (m/s) | 330.7900 | 330.7900 |
| TAS (m/s) | 123.7000 | 123.6824 |
| Mass $(\mathrm{kg})$ | 24000.0000 | 24000.0000 |
| Cl (dim) | 0.5472 | 0.5472 |
| Cd (dim) | 0.0312 | 0.0312 |
| Drag (N) | 13421.0000 | 13421.0000 |
| Thrust (N) | 53160.0000 | 53160.0000 |
| dV/dt (m/s-s) | 0.0000 | 0.1089 |
| Rate of Climb (m/s) | 20.8800 | 19.5000 |

At a rate of climb of $24.13 \mathrm{~m} / \mathrm{s}$ estimated in Table 3, it takes the aircraft 50.5 seconds to reach the new altitude. If the new speed is 240 knots IAS ( $123.7 \mathrm{~m} / \mathrm{s}$ ) then an initial estimate of $\mathrm{dV} / \mathrm{dt}$ is:
$\mathrm{dV} / \mathrm{dt}=(123.7-118.2) / 50.5=0.1089 \mathrm{~m} / \mathrm{s}^{2}$.
The estimated ROC with the acceleration term added is $19.5 \mathrm{~m} / \mathrm{s}$ versus $20.88 \mathrm{~m} / \mathrm{s}$. The difference is $1.37 \mathrm{~m} / \mathrm{s}$ or $6.6 \%$. If the same value of $\mathrm{dV} / \mathrm{dt}$ is used in Table 3, the difference in ROC is $5.5 \%$.

