# Example of Aircraft Climb and Maneuvering Performance

## CEE 5614
## Analysis of Air Transportation Systems

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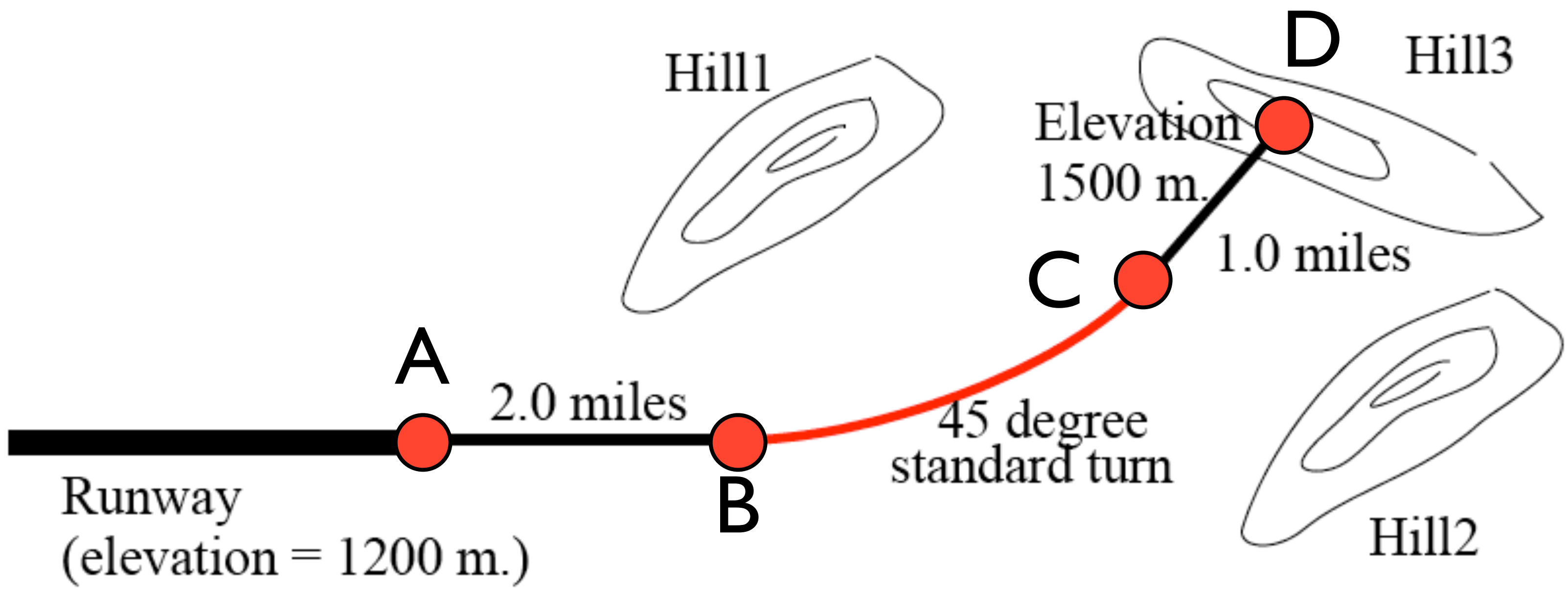
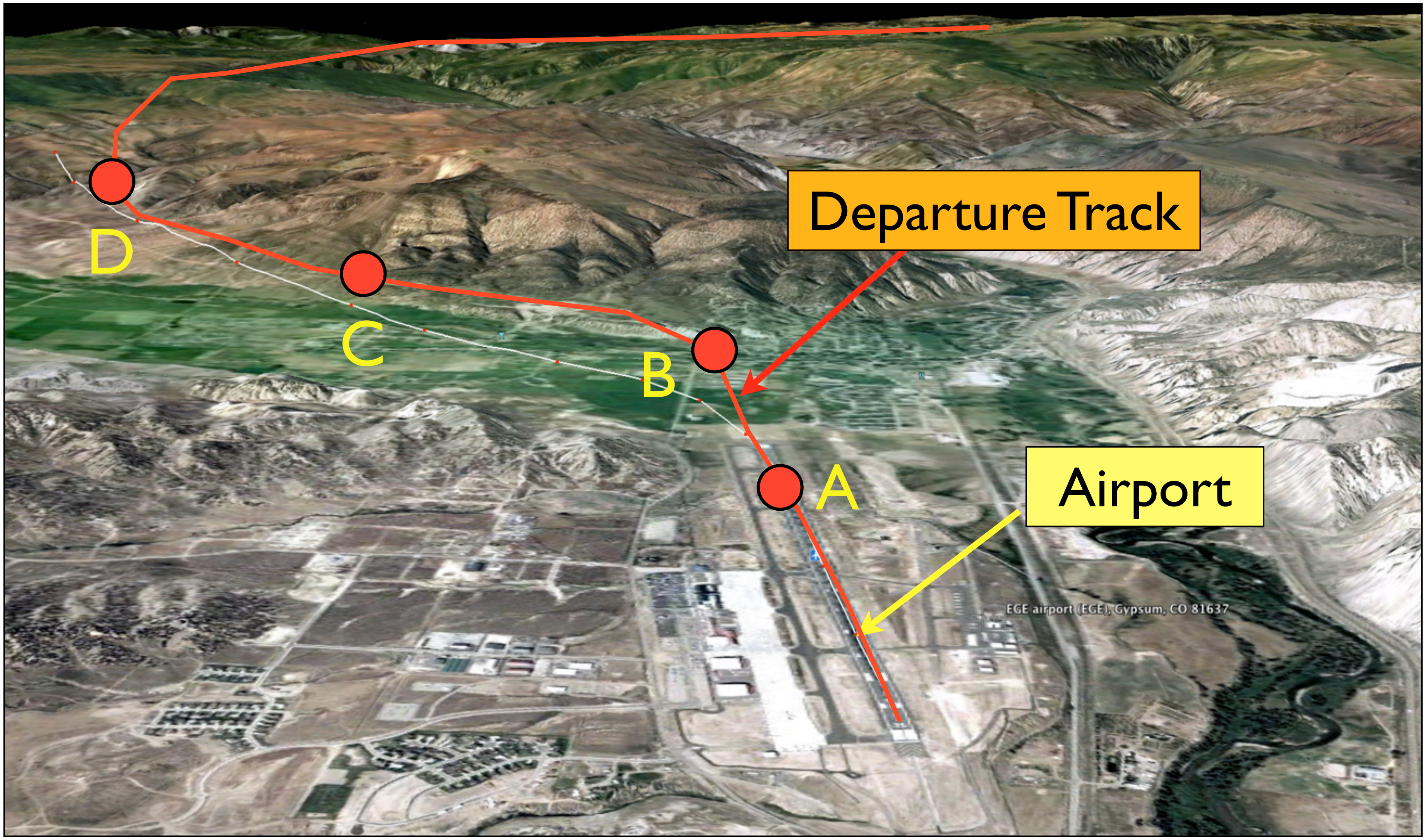
# Example - Aircraft Climb Performance

- Aircraft maneuvering performance analysis (initial climb profile)
- Use the the vehicle characteristics for the very large capacity transport aircraft in the Matlab files for CEE 5614 to solve this problem (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)



# Departure Procedure (Sample Airport)




source: Google Earth (2009)

# Problem Description

- The aircraft climbs at 180 knots (IAS) with 10 degrees of flaps down after departure as shown in the diagram
- As the aircraft climbs past the runway threshold at an altitude of 100 meters (and with the landing gear already retracted), the aircraft loses the inner starboard engine
- The pilot flies the departure profile shown in the figure to avoid terrain around the field.
- With the wing flaps deflected, the aircraft drag polar is modified with an extra term as follows:

# Modified Drag Function

- Drag polar with an added term to account for drag due to flaps:

$$C_d = C_{do} + C_{di} + C_{df}$$

$$C_d = C_{do}(M) + \frac{C_l^2}{\pi A \mathrm{Re}} + C_{df}$$


where:

$C_d$ = total drag coefficient (dim)

$C_{do}$ = zero lift drag coefficient (lift independent) (dim)

$C_{di}$ = induced drag coefficient (lift dependent) (dim)

$C_{df}$ = drag coefficient due to flaps (dim)

$AR$ = wing aspect ratio (dim)

$e$ = efficiency factor (dim)

$C_l$ = lift coefficient (dim)

# Example - Aircraft Climb Performance Data File

- Very large capacity transport aircraft (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)

```
% Aircraft file to support other computational modules
% Aircraft = Similar to Airbus A380 (heavy transport)

global A e S neng tsfc macht Cdoct mass thrust_table mach_table lapse_rate_factor Vclimb altc g
```

```
S = 858;            % wing area (square m)
A = 9.0;            % Wings aspect ratio
e = 0.84;           % Oswald's efficiency factor
g = 9.81;
neng = 4;               % Number of engines
tsfc = 1.6e-4;          % thrust specific fuel consumption (N/N/s)
mass = 540000;          % mass at operating point (kg)
```

geometric, mass and specific fuel consumption

```
% Drag characterictics - CDO function (zero lift drag function)

Cdoct  = [0.020   0.020   0.0204    0.022   0.037   0.038   0.040];
macht  = [0.0     0.75    0.80      0.85    0.90    0.95    1.00];
```

drag data

```
% Thrust parameters for Eclipse aircraft (at sea level)

thrust_table = [338000 180000];    % Thrust limits (sea level in Newtons)
mach_table = [0.0  0.9];                  % mach number limits to bound thrust
lapse_rate_factor = 0.96;                  % thrust lapse rate factor
```

engine thrust data

```
% Computes the aircraft profile given altitude (Vcas given) - typical for
% four engine aircraft similar to the Airbus A380 aircraft

Vclimb = [180 200  220 250 270  280  290  300  300  300  300  300 300  300  300];    % knots IAS
Vdescent = [ 300 300  300 300 300  300  300  300  300  300  300  300 300  300  300];  % knots IAS
altc = [ 0 1000  2000 3000 4000  5000  6000  7000  8000  9000  10000  11000 12000  13000  14000]; % altitude (meters)
```

speed profile data

# Effects of Wing Flaps on $C_{df}$

- The following table provides values of the drag coefficient due to flaps for various flap angles
- Note that flap setting are discrete and unique to each aircraft

| Flap Angle (degrees) | $C_{df}$ |
|---|---|
| 0 | 0 |
| 2 | 0.006 |
| 5 | 0.010 |
| 10 | 0.015 |
| 20 | 0.023 |

# Example - Very Large Capacity Aircraft Data File

- An aircraft similar in size and performance as the Airbus A380
- Four turbofan engines each developing 34,400 kg (338,000 N) at sea level
- Takeoff mass is 450,000 kg. for this problem

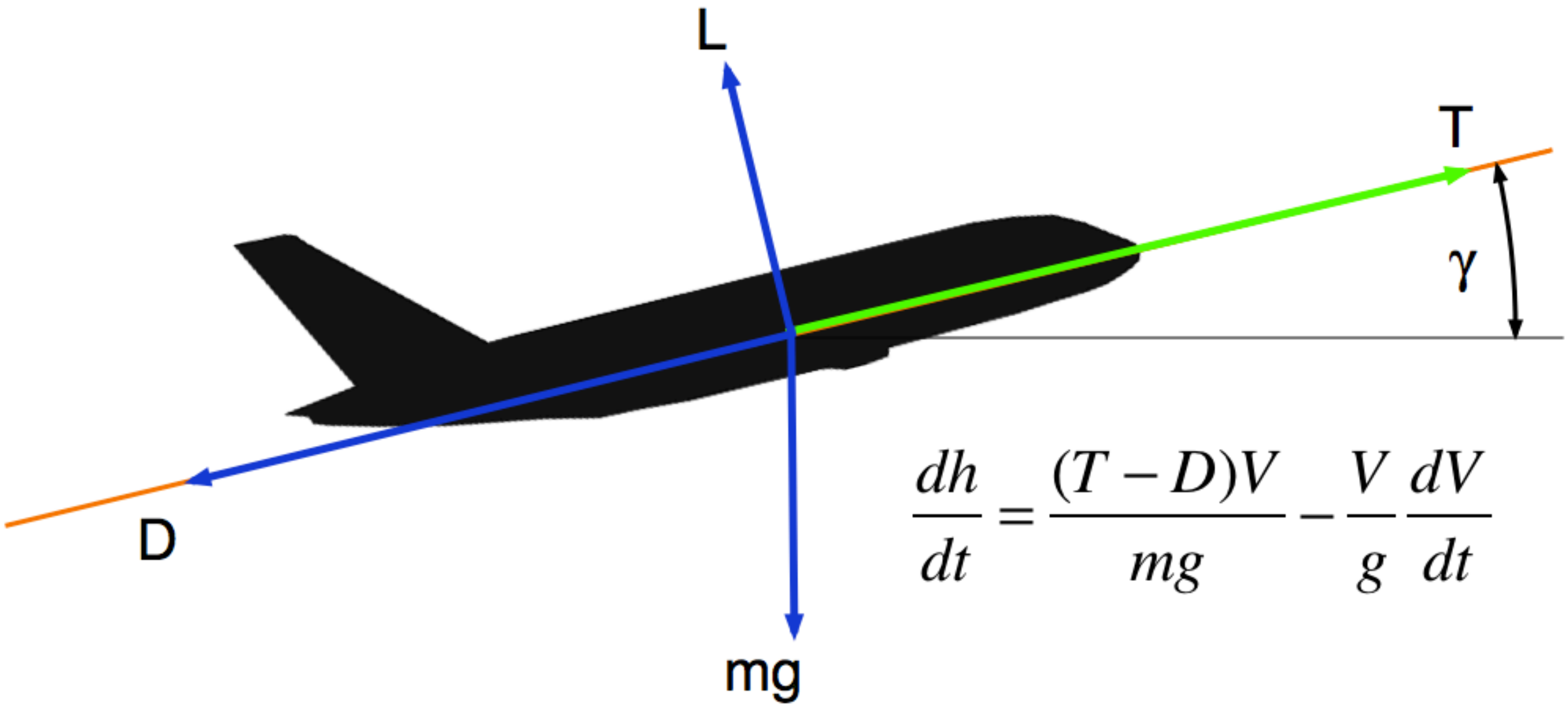

Airbus A380 taxies to the gate at LAX (A.A. Trani)

# More Problem Description

- Find if the aircraft clears the critical obstacle (Hill 3) with the required 305 meter (1000 ft.) margin of safety
- In your calculations, consider the degradation in the climb rate induced by the 45 degree turn maneuver
- During the climb maneuver the aircraft climbs at 180 knots (IAS) with 10 degrees of flaps

# Example - Aircraft Climb Performance
## Picture the Situation

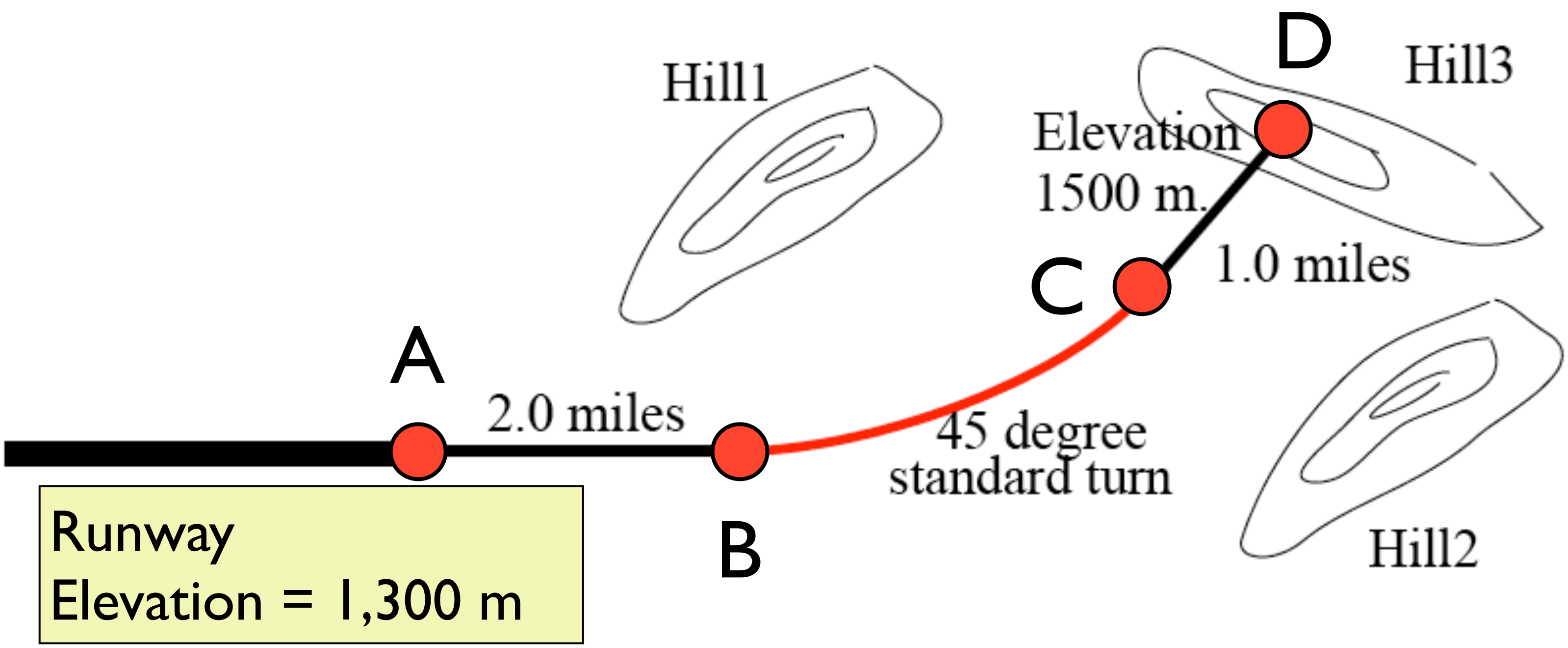
- For this analysis we will ignore the second term in the Right Hand Side (RHS) of the differential equation (acceleration term)
- This simulates that the pilot is interested in climbing as fast as possible and thus using all the engine thrust to climb




$$\frac{dh}{dt} = \frac{(T-D)V}{mg} - \frac{V}{g}\frac{dV}{dt}$$

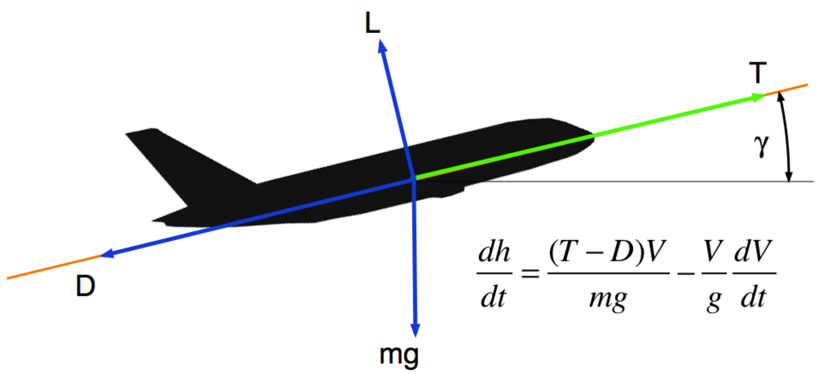
# Calculation of Performance at Key points in the Climb Profile

- The analysis assumes the aircraft is studied as a point mass system. We evaluate the performance at discrete points
- The analysis will estimate the climb performance at four key points in the initial climb maneuver. This will allow us to check if the aircraft clears the obstacle with a 1,000 foot clearance needed by Federal Aviation Regulations (FAR Part 25)



# Calculation Procedure

- Step 1: Estimate true airspeed using atmospheric model
- Step 2: Estimate the lift coefficient needed to sustain flight using the basic lift equation
- Step 3: Estimate drag coefficient
- Step 4: Estimate total drag (D)
- Step 5: Estimate the thrust produced by the engines at altitude (T)
- Step 6: Find the rate of climb (dh/dt)




$$\frac{dh}{dt} = \frac{(T-D)V}{mg} - \frac{V}{g}\frac{dV}{dt}$$

# Aircraft Climb Performance
# 1300 meters (above sea level) and 180 knots IAS
# (Engine Failure Point - Point A)

- Using the standard expression to estimate the true mach number of the aircraft at altitude,
- The true mach number is 0.2896
- The true airspeed (TAS) is 97.1 m/s or 188.6 knots

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2*(450{,}000)(9.81)}{(1.0797)(97.1)^2(858)} \frac{(kg)(m/s^2)}{(kg/m^3)(m/s)(m^2)} = 1.011$$

$$C_d = C_{do} + C_{di} + C_{df} = 0.020 + \frac{1.011^2}{\pi(9.0)(0.84)} + .015 = 0.078$$

# Aircraft Climb Performance
# 1300 meters (above sea level) and 180 knots IAS
# (Engine Failure Point - Point A)

- The total drag is,

$$D = \frac{1}{2}\rho V^2 S C_d = \frac{1}{2}(1.0797)(97.1)^2(858)(0.078) = 340{,}743N$$

- The calculated drag has units of Newtons (verify by yourself)
- The thrust produced by **3 engines** in the very large capacity transport is estimated using the simple linear model

$$T_{0,M} = T_{0,M=0} - \lambda M_{true}$$

$$T_{h,M} = T_{0,M}\left(\frac{\rho_h}{\rho_0}\right)^m$$

# Aircraft Climb Performance
# 1300 meters (above sea level) and 180 knots IAS
# (Engine Failure Point - Point A)

- The thrust developed by 3 remaining engines is calculated to be:
- $T_{total}$ = 762,801 N
- The rate of climb is then,

$$\frac{dh}{dt} = \frac{(T_{total} - D)V}{mg} =$$

$$\frac{dh}{dt} = \frac{(762,801 - 340,743)(97.1)}{450,000(9.81)} \frac{(N - N)}{kg(m / s^2)}$$

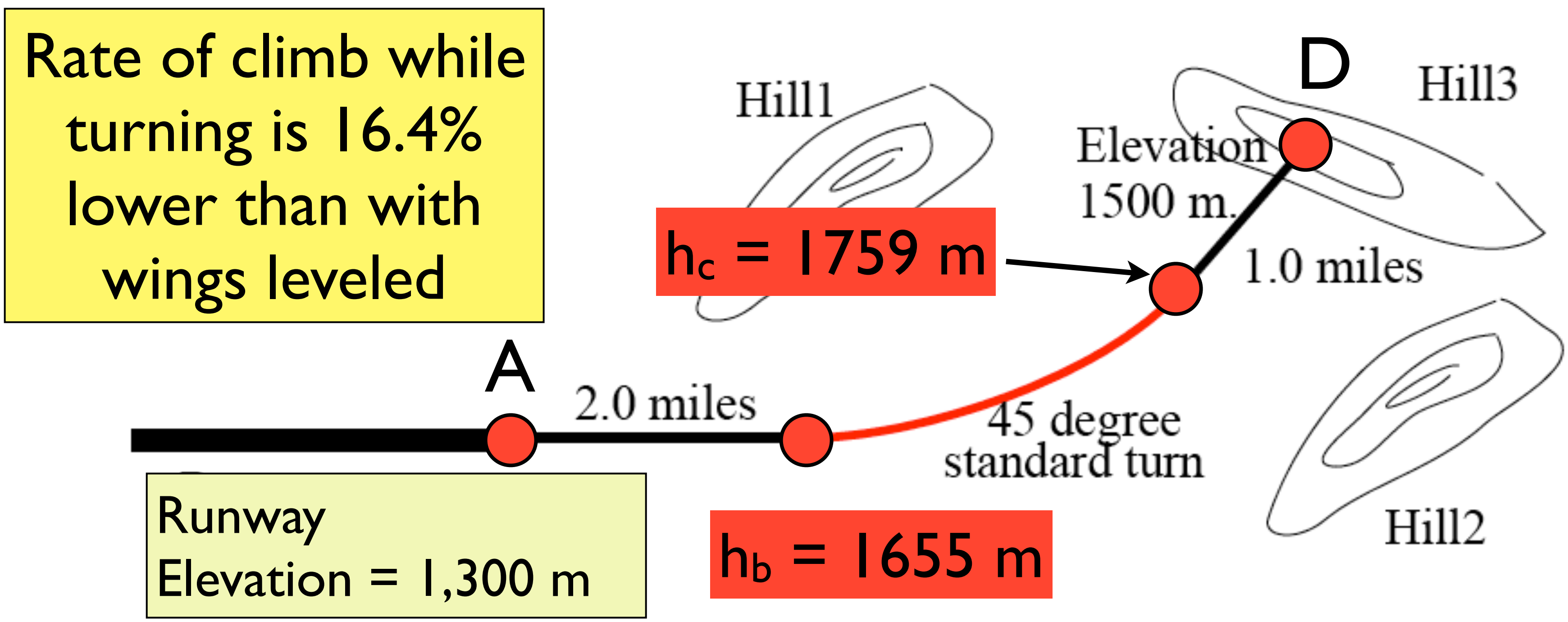
$$\frac{dh}{dt} = 9.28 \ m / s$$

# Aircraft Climb Performance
# 1300 meters (above sea level) and 180 knots IAS
# (Engine Failure Point - Point A)

- The first segment is 2 nm long
- 2 nm is 3,704 meters
- The aircraft takes 38.2 seconds to cover the 2 nm.
- The aircraft gains 355 meters in 2nm
- The altitude reached at point B is 1,655 meters

# Aircraft Climb Performance
# 1655 meters (above sea level) and 180 knots IAS
# (Turning - Point B)

- The 45-degree turn is performed in 15 seconds at 3 degrees per second
- Therefore, the aircraft gains another 114 meters while turning 45 degrees
- At the end of the 45- degree turn, the final altitude is estimated to be 1,769 meters



# Aircraft Climb Performance
# 1655 meters (above sea level) and 180 knots IAS
# (Turning Maneuver at Point B)

- The lift coefficient to maintain flight while turning and banking at 27.7 degrees is,

$$C_l = \frac{2mg}{\rho V^2 S \cos(\phi)} = \frac{2*(450{,}000)(9.81)}{(1.024)(98.4)^2(858)\cos(27.7)} = 1.152 \text{ dim}$$

- The drag coefficient while turning and banking at 27.7 degrees is,

$$C_d = C_{do} + C_{di} + C_{df} = 0.020 + \frac{1.152^2}{\pi(9.0)(0.84)} + .015 = 0.091$$

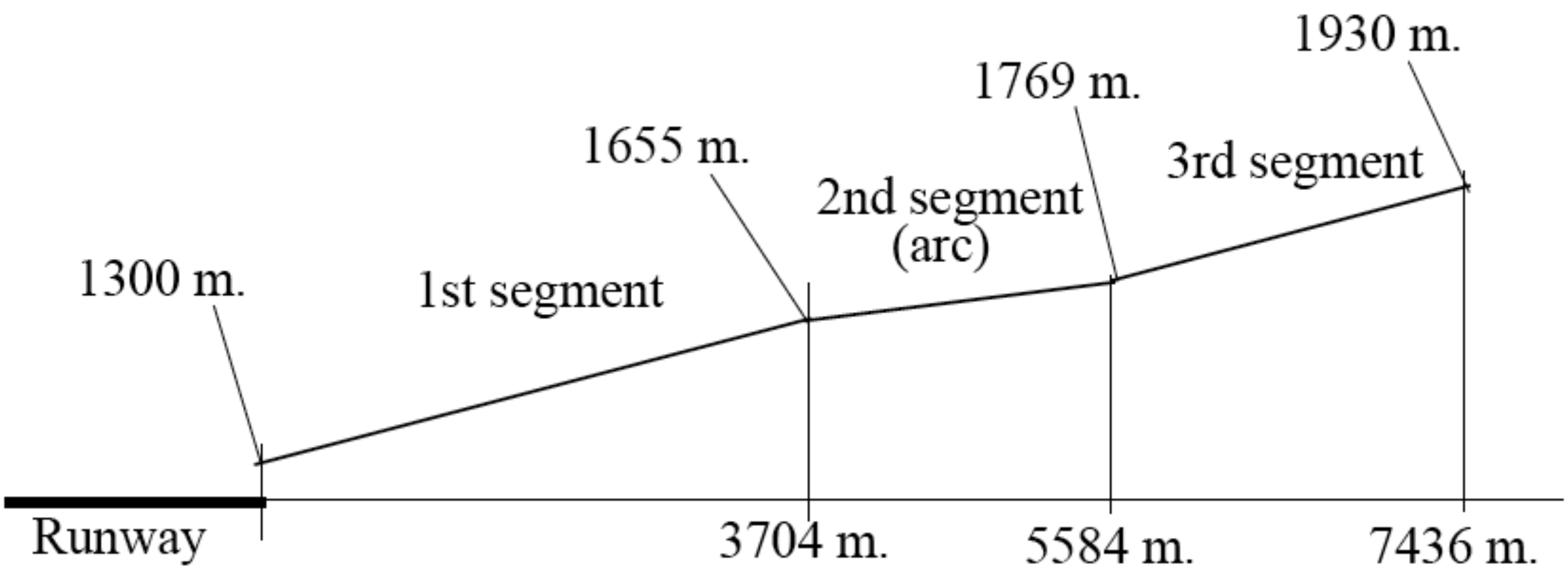
- Note that this value is substantially higher than before (i.e., no turn) ,

# Aircraft Climb Performance
# 1769 meters (above sea level) and 180 knots IAS
# (End of Turning Maneuver at Point C)

- The aircraft is still configured with 10 degrees of flaps and traveling at 180 knots indicated airspeed
- This translates into 98.8 m/s (true airspeed)
- The aircraft returns to wings level flight with a climb rate of 8.62 m/s
- The final altitude at Point D is estimated to be 1,930 meters
- The hill has an elevation of 1,500 meters above sea level
- The aircraft clears the hill by 430 meters (1930-1500 m)
- Therefore the obstacle clearance accountability area is satisfied

# Aircraft Climb Performance
# Final Note

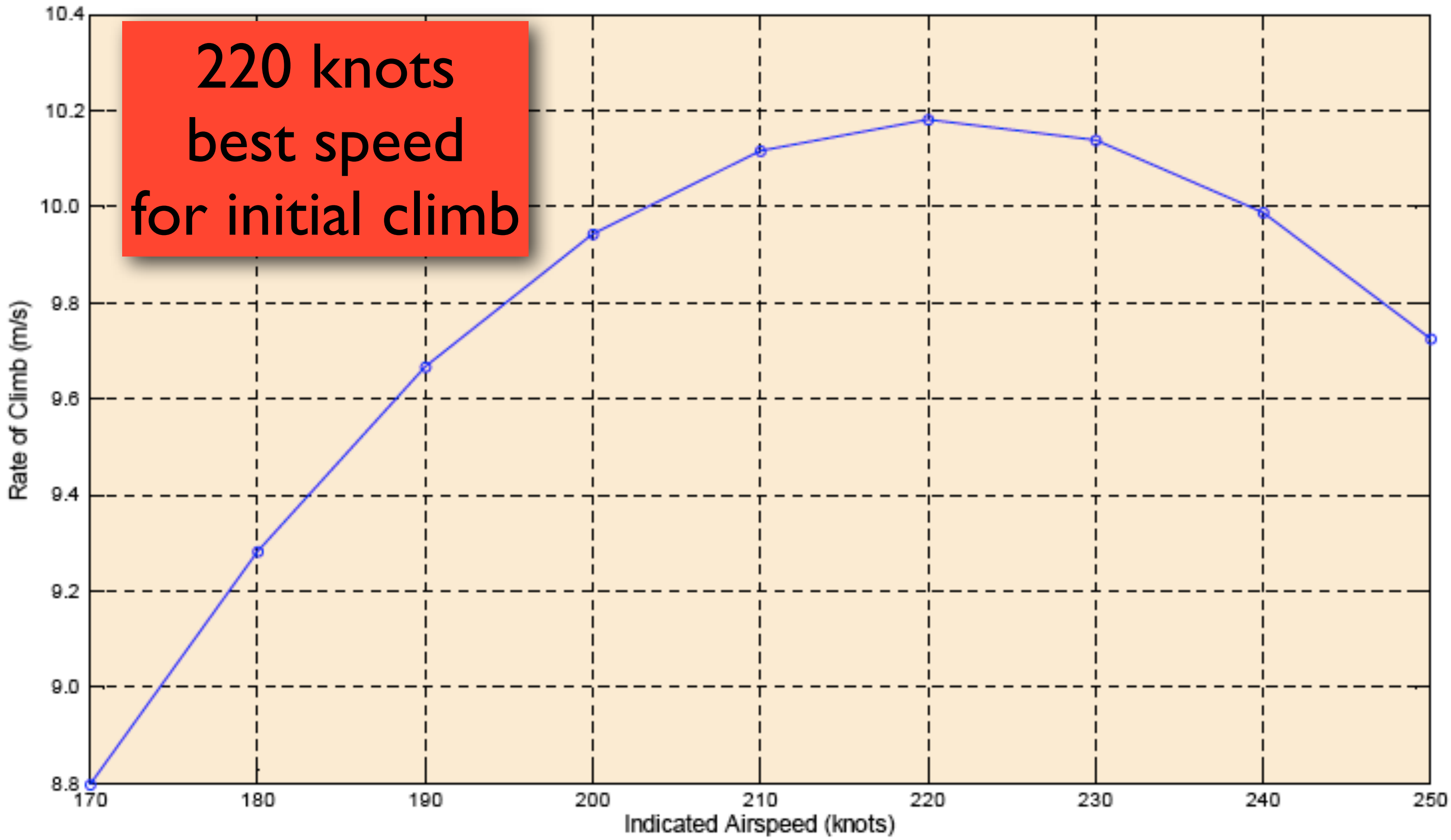
- The following diagram summarizes the calculations done for the engine failure condition




Path distance from runway

# Rate Of Climb Analysis

- Repeating the steps shown the previous pages we can estimate the speed in the initial climb that will maximize the rate of climb



# Observations

- Rate of climb is sensitive to bank angle
- The analysis of obstacle accountability areas is critical in real airline operations
- The aircraft takeoff weight could be limited by the engine out condition in climbing above obstacles around the airport and not the runway length (WAT curves in aircraft performance)
- This will be illustrated in the homework
- A clear example of initial climb limitations is illustrated in the incident on United Flight 863 departing SFO. Read the article at:
- http://www.post-gazette.com/headlines/19990320jumbojet4.asp