

# Assignment # 7 Problem # 1 2014 Solution

## CEE 5614 Analysis of Air Transportation Systems

Dr. Antonio A. Trani  
Professor

# Problem Description

- Work the problem and estimate the IMC saturation capacity using both runways 36 and 09L for departures. Runways 09L and 09R handle arrivals as well. Consider both ATC time lag to clear a departure on runway 36 and also consider engine spool-up time and pilot time lags (another 10 seconds) after receiving a runway takeoff clearance.
- Plot the Pareto diagram with the new assumptions (part a) and compare the new solution with obtained in the handout.

# Problem Description

- The airport to be studied in this problem is shown in Figure 1
- The airport has two 9,000 foot runways with a configuration shown in the Figure 1 (see Page 5)
- The airport has an airport surveillance radar (ASR) which tracks aircraft up to 60 miles from the airport site
- Tables 1 and 2 show the typical ATC separations at the airport under IMC conditions
- Tables 3 and 4 show the separations under VMC conditions
- The airport has the following technical parameters: a) in-trail delivery error of 16 seconds, b) departure-arrival separation for both VMC and IMC conditions is 2 nautical miles, c) probability of violation is 5%
- Arriving aircraft are “vectored” by ATC to the final approach fix (see Figure) located 7 miles from the runway threshold

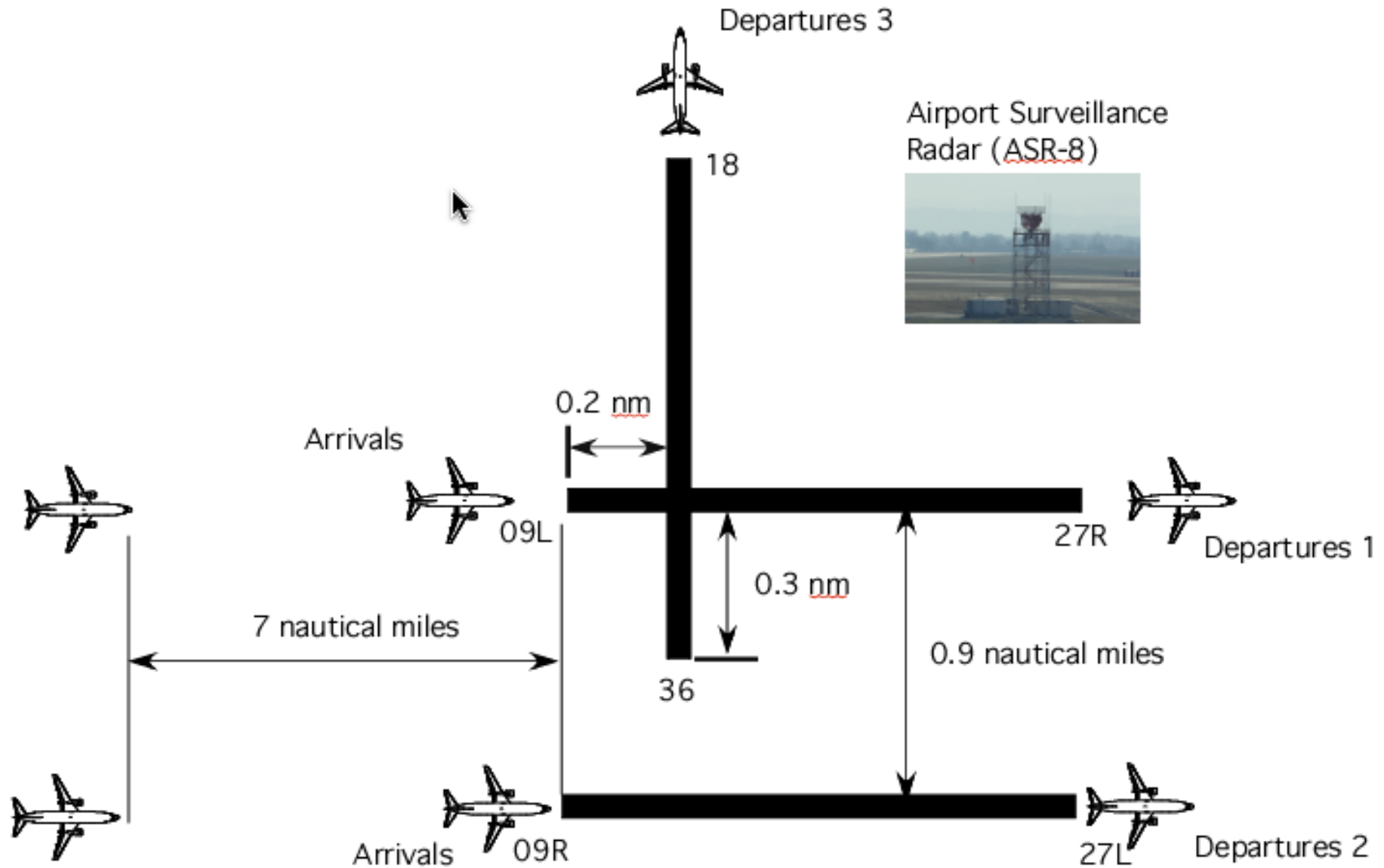
# Problem Description

- The airport has an aircraft fleet mix made up of 10% small, 65% large and 25% heavy wake class aircraft
- The characteristics of the aircraft are given in Table 5
- Observed runway occupancy times in the field are: 48, 55, and 62 seconds for small, large and heavy aircraft, respectively
- Assume the 3-point runway deceleration calculation method applies to this problem to estimate the time to cross the intersection

# Problem Description

- In your analysis assume departing aircraft accelerate on the runway at a constant rate of  $2.2 \text{ m/s}^2$
- Assume that ATC controllers release departures on runway 18-36, around 10 seconds after an arriving aircraft crosses the intersection between runways 09L-27R and 18-36
- Arrivals and departures are not airborne at the intersection
- For departures on runway 18-36 to occur, it is desired that when the departing aircraft is released for takeoff, the next arrival to runway 09L be no less than 2.5 nm from the arrival threshold
- This rule is used by ATC controllers to schedule departures on runway 36

# Problem Description



# Problem Description (IFR Separations)

Table 1. Minimum arrival-arrival separations under IMC conditions. Values in are nautical miles.

Minimum Separation Matrix (nm)				Arrivals-Arrivals	
Lead	Trailing				
	Small	Large	Heavy		
Small	3	3	3		
Large	5	3	3		
Heavy	6	5	3		

Table 2. Minimum departure-departure separations under IMC conditions. Values in are in seconds.

Departure-Departure Separation Matrix (seconds)				
Lead	Trailing			
	Small	Large	Heavy	
Small	60	60	60	
Large	60	60	90	
Heavy	120	120	120	

# Problem Description (VFR Separations)

Table 3. Minimum arrival-arrival separations under VMC conditions. Values in are nautical miles.

Minimum Separation Matrix (nm)			Arrivals-Arrivals	
Lead		Trailing		
	Small	Large	Heavy	
Small	2.4	2.4	2.4	
Large	5	2.4	2.4	
Heavy	6	4	2.7	

Table 4. Minimum departure-departure separations under IMC conditions. Values in are in seconds.

Departure-Departure Separation Matrix (seconds)				
Lead		Trailing		
	Small	Large	Heavy	
Small	50	50	50	
Large	50	50	75	
Heavy	90	90	90	



# Problem Description (Runway Performance)

Table 5. Runway Performance Data.

Aircraft Group	Parameters	Representative Aircraft
Small aircraft	Approach speed = 125 knots Touchdown location = 1,200 feet Average deceleration = $-4.2 \text{ ft/s}^2$ Free roll time = 2.0 seconds (after touchdown and before braking)	Cessna Citation 560, Citation 500, Beechcraft Jet 400
Large aircraft	Approach speed = 145 knots Touchdown location = 1,300 feet Average deceleration = $-4.2 \text{ ft/s}^2$ Free roll time = 2.0 seconds	Boeing 737-400 (B-737-400), Airbus A320 (A-320-200)
Heavy aircraft	Approach speed = 155 knots Touchdown location = 1,400 feet Average deceleration = $-4.2 \text{ ft/s}^2$ Free roll time = 2.0 seconds	Boeing 747-400, Airbus A340-600

# Questions

1. Calculate the arrival-departure saturation capacity diagram (Pareto diagram) under IMC conditions for this airport
  2. Calculate the arrival-departure saturation capacity diagram (Pareto diagram) under VMC conditions for this airport
- State all your assumptions in your calculations

# Solution Steps to the Problem

- Start with a single runway analysis for IMC conditions
- Identify interactions between runways
- Use the principle of superposition whenever possible (i.e., study independent runways and then add their capacity)
- Set-up a manual simulation scheme to look at various operational strategies for the airport

# Single Runway Analysis (Arrival Operations)

- Use the spreadsheet program provided in class or your own manual calculations

Pij Matrix				
		Trailing		
	Small	Large	Heavy	
Small	0.010	0.065	0.025	
Large	0.065	0.423	0.163	
Heavy	0.025	0.163	0.063	

**IFR  
Conditions**

Augmented Matrix ( $T_{ij} + B_{ij}$ )				
		Trailing		
	Small	Large	Heavy	
Small	112.80	100.88	96.08	
Large	178.34	100.88	96.08	
Heavy	211.82	153.74	96.08	

**Arrivals-Only  
Capacity  
30.98 per  
hour**

# Single Runway Analysis (departure operations)

Pij Matrix				
		Trailing		
	Small	Large	Heavy	
Small	0.010	0.065	0.025	
Large	0.065	0.423	0.163	
Heavy	0.025	0.163	0.063	

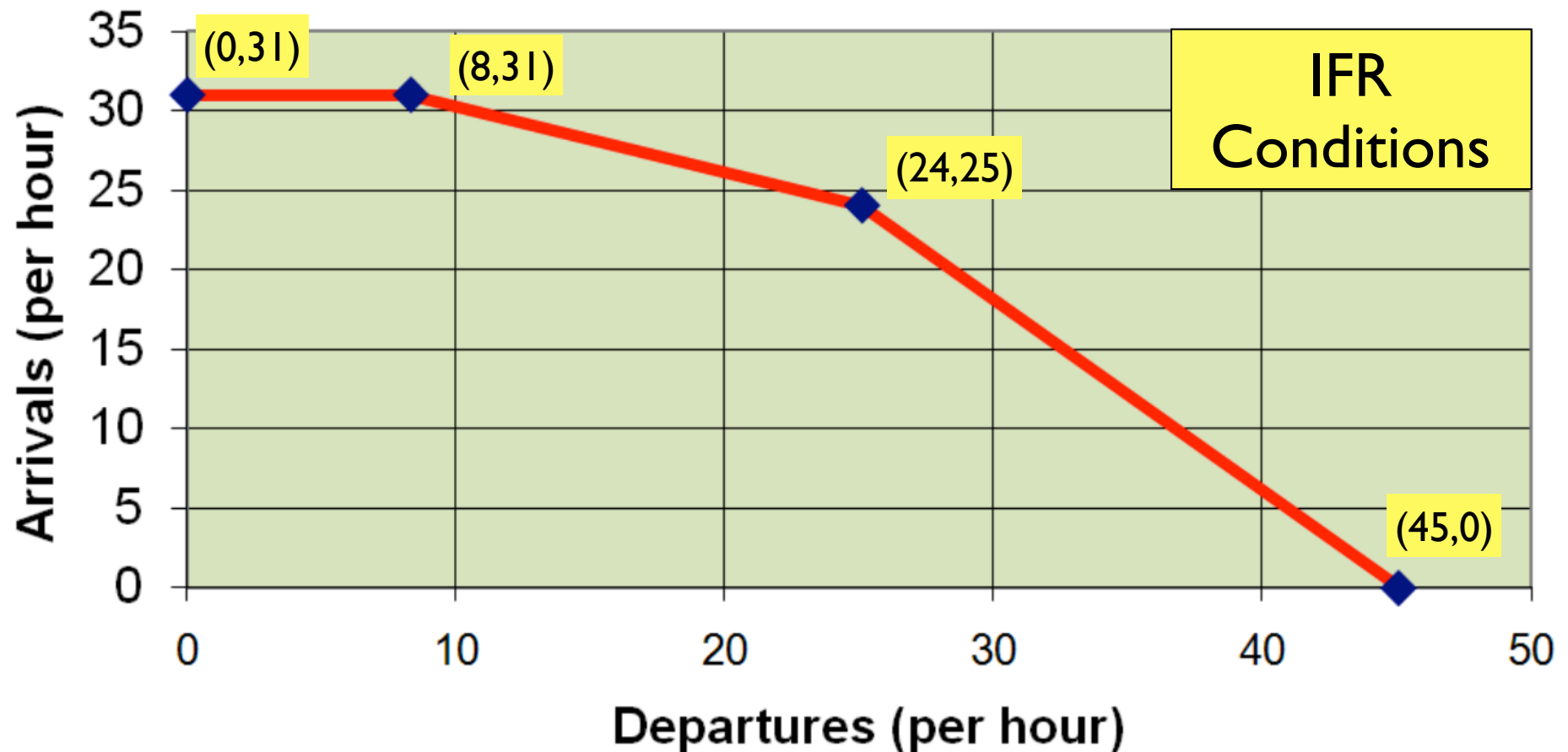
**IFR  
Conditions**

Departure-Departure Separation Matrix (seconds)				
		Trailing		
	Small	Large	Heavy	
Small	60	60	60	
Large	60	60	90	
Heavy	120	120	120	

**Departures-  
Only  
Capacity  
45.07 per  
hour**

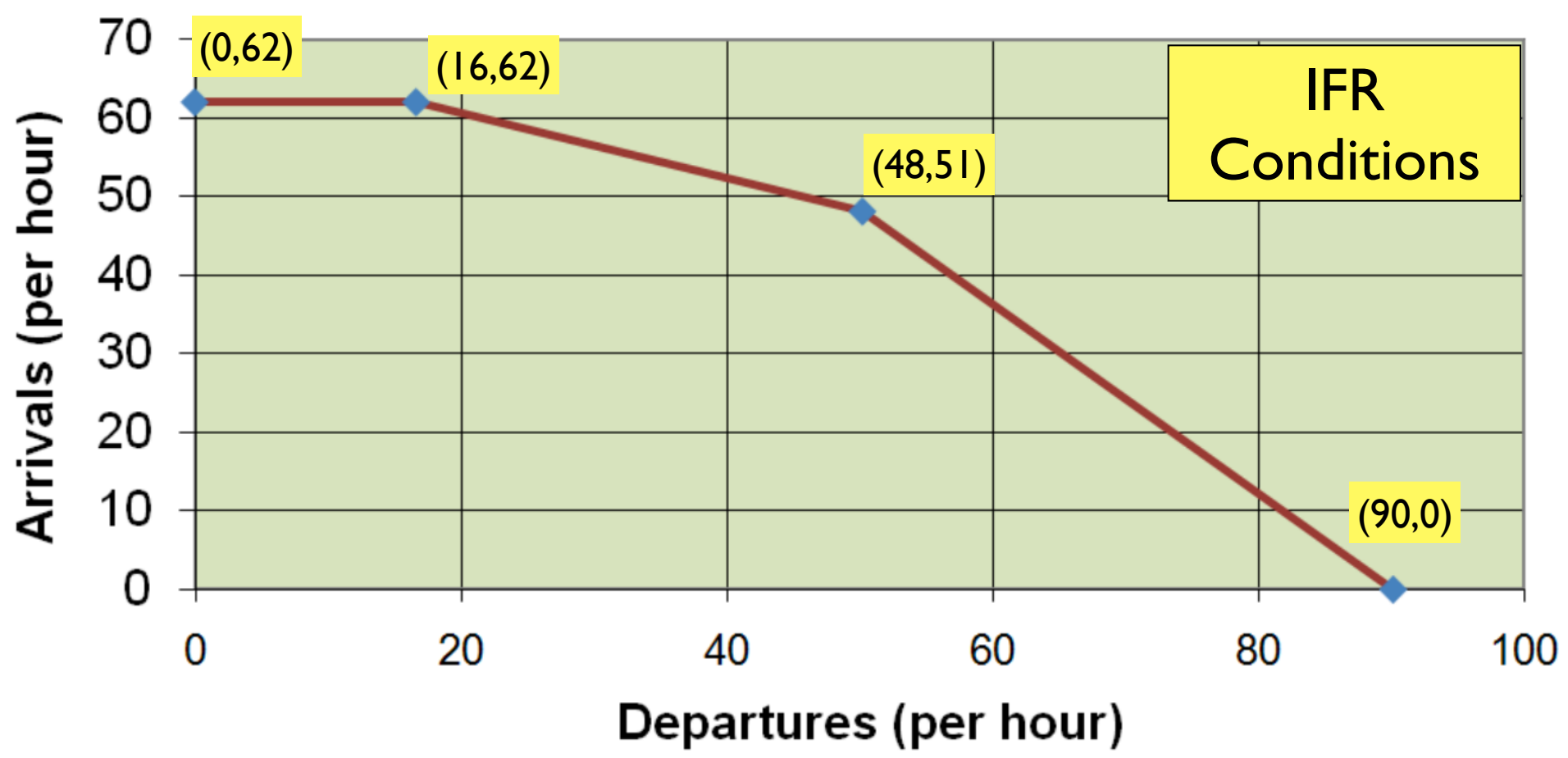
# IFR Capacity Pareto Diagram (Single Runway Analysis)

Saturation Capacity for a Single Runway at the Airport under Various Operational Conditions. The diagram applies to one runway (either 09L-27R or 09R-27L)



# IFR Capacity Pareto Diagram (Two Parallel and Independent Runways)

Saturation Capacity for two runways at the Airport under Various Operational Conditions. The diagram applies to one runway (either 09L-27R or 09R-27L)



# Observations

- Arrivals on runways 09L and 09R are independent ( $> 4300$  ft separation) (radar available)
- The Pareto diagram found for one runway replicates for the second parallel runway (also used in mixed operations mode)
- The arrivals-only saturation capacity of the two-runway system is 62 per hour
- The departures-only saturation capacity for two parallel runways is 90 per hour



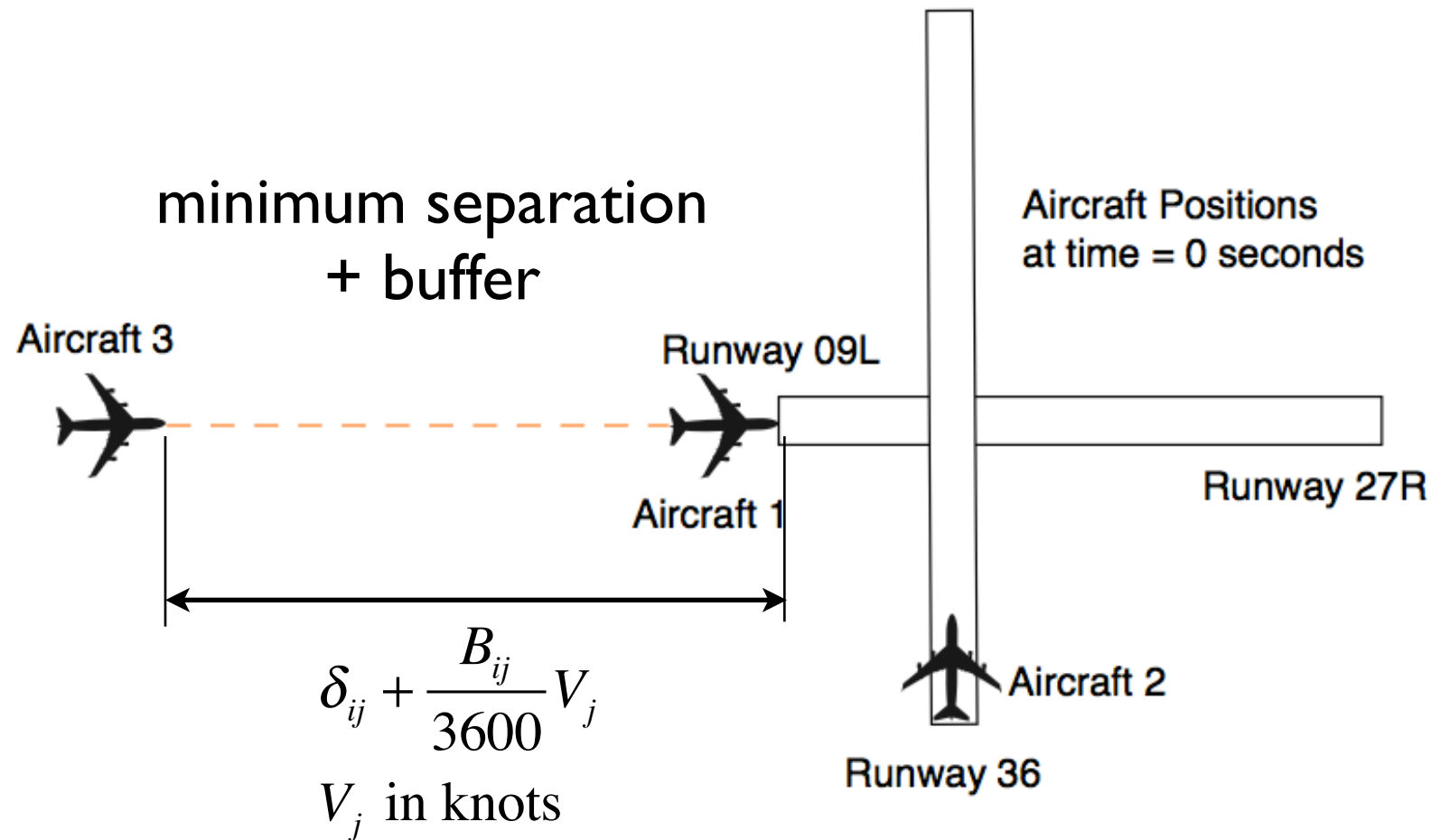
# Detailed Analysis for Intersecting Runways

- The intersecting runway is treated as another asset at the airport
- Need to answer the fundamental questions:
- Are there any gaps left by successive arrivals (do nothing) allowing departures from runway 36?
- Quantify the capacity benefit for IFR conditions

# Approach

- Visualize the situation by drawing various operations
- Determine the added number of departures on runway 36 allowed with the “natural” arrival gaps on runway 09L
- Assume that departures on runway 09L are not processed since runway 36 offers clear advantages
- The diagrams that follow illustrate various steps in the sequence of events likely to happen at the airport **as “closing” case, pairwise arrival sequences**

# Aircraft Positions at Time $t = 0$ s



$$\delta_{ij} + \frac{B_{ij}}{3600} V_j$$

$V_j$  in knots

$B_{ij}$  in seconds

Aircraft 1 crosses runway 09L threshold . Aircraft 3 follows in-trail at the required separation behind aircraft 1

# Calculations of Travel Time for Landing Aircraft to Cross Runway Intersection

- Calculation of the travel times from threshold crossing point to runway intersection point
- The travel times to cross the intersection of runway 18-36 (as the aircraft lands on runway 09L) are: 5.8, 5.0 and 4.6 seconds for small, large and heavy aircraft, respectively
- These travel times influence the ATC tower controller (i.e. local controller) decision on when to clear a departure on the crossing runway

# Calculations of Travel Time to Cross Runway Intersection for Departing Aircraft on Runway 36

$$S = V_i t + \frac{1}{2} a t^2$$

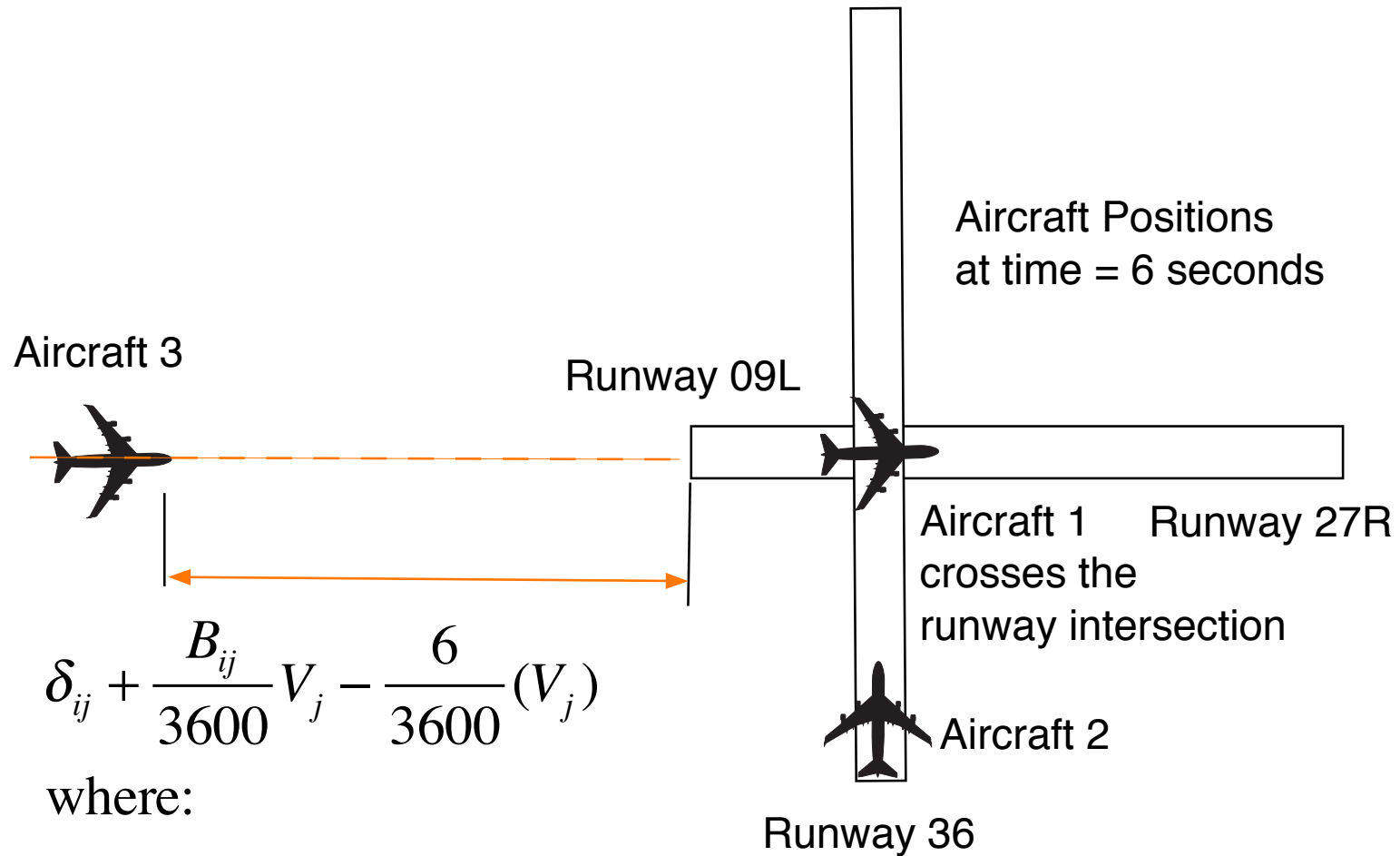
$$t^2 = \frac{2S}{a}$$

$$t = \sqrt{\frac{2S}{a}}$$

$$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2(555.6 \text{ m})}{2.2 \text{ m/s}^2}} = 22.5 \approx 23 \text{ seconds}$$

Aircraft departing runway 36 take ~23 seconds to cross the runway intersection

# Aircraft Positions at Time $t=6$ s



$$\delta_{ij} + \frac{B_{ij}}{3600} V_j - \frac{6}{3600} (V_j)$$

where:

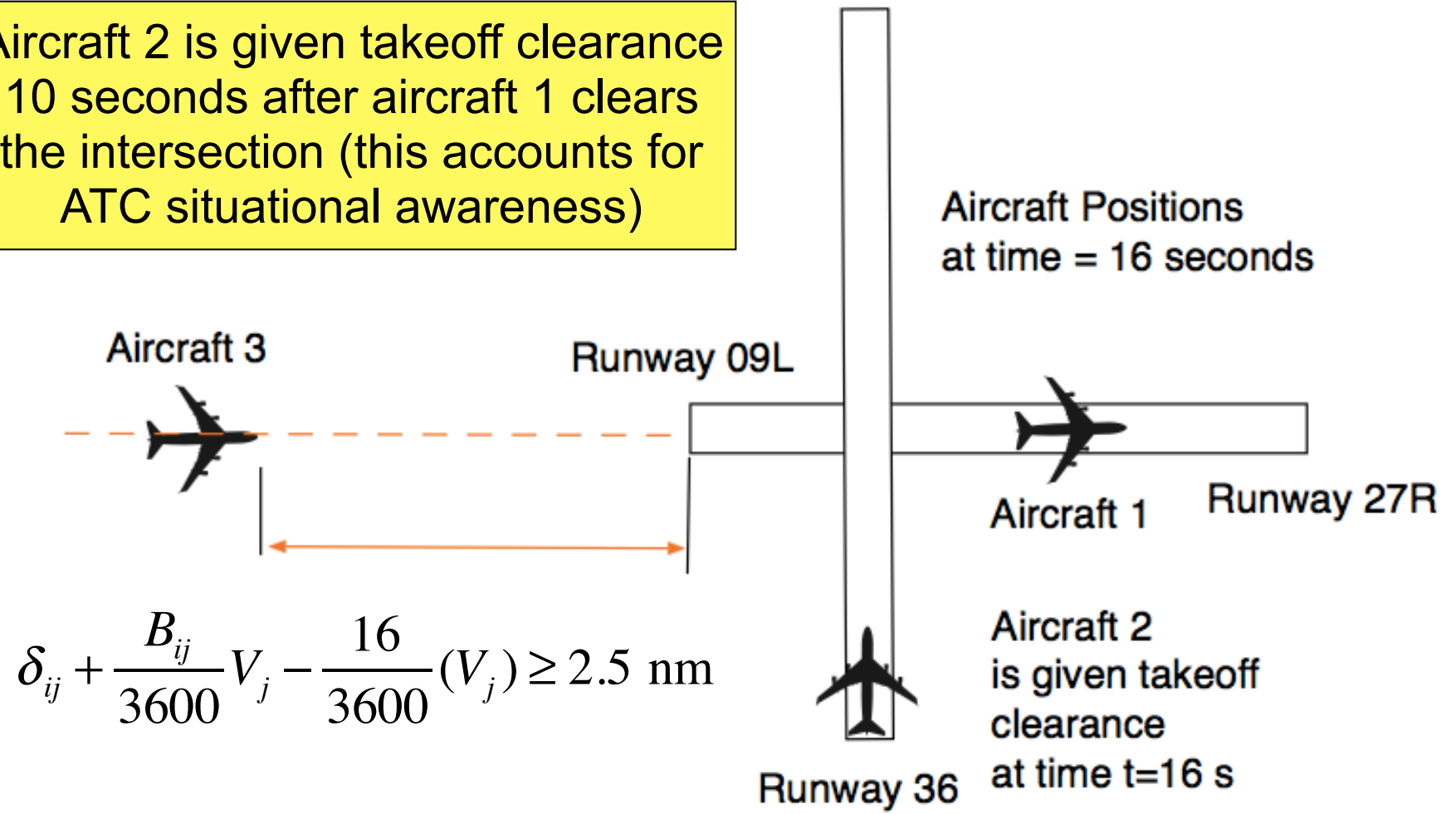
$B_{ij}$  is the buffer time in seconds

$V_j$  is the following aircraft speed

in knots

# Aircraft Positions at Time t=16 s

Aircraft 2 is given takeoff clearance 10 seconds after aircraft 1 clears the intersection (this accounts for ATC situational awareness)

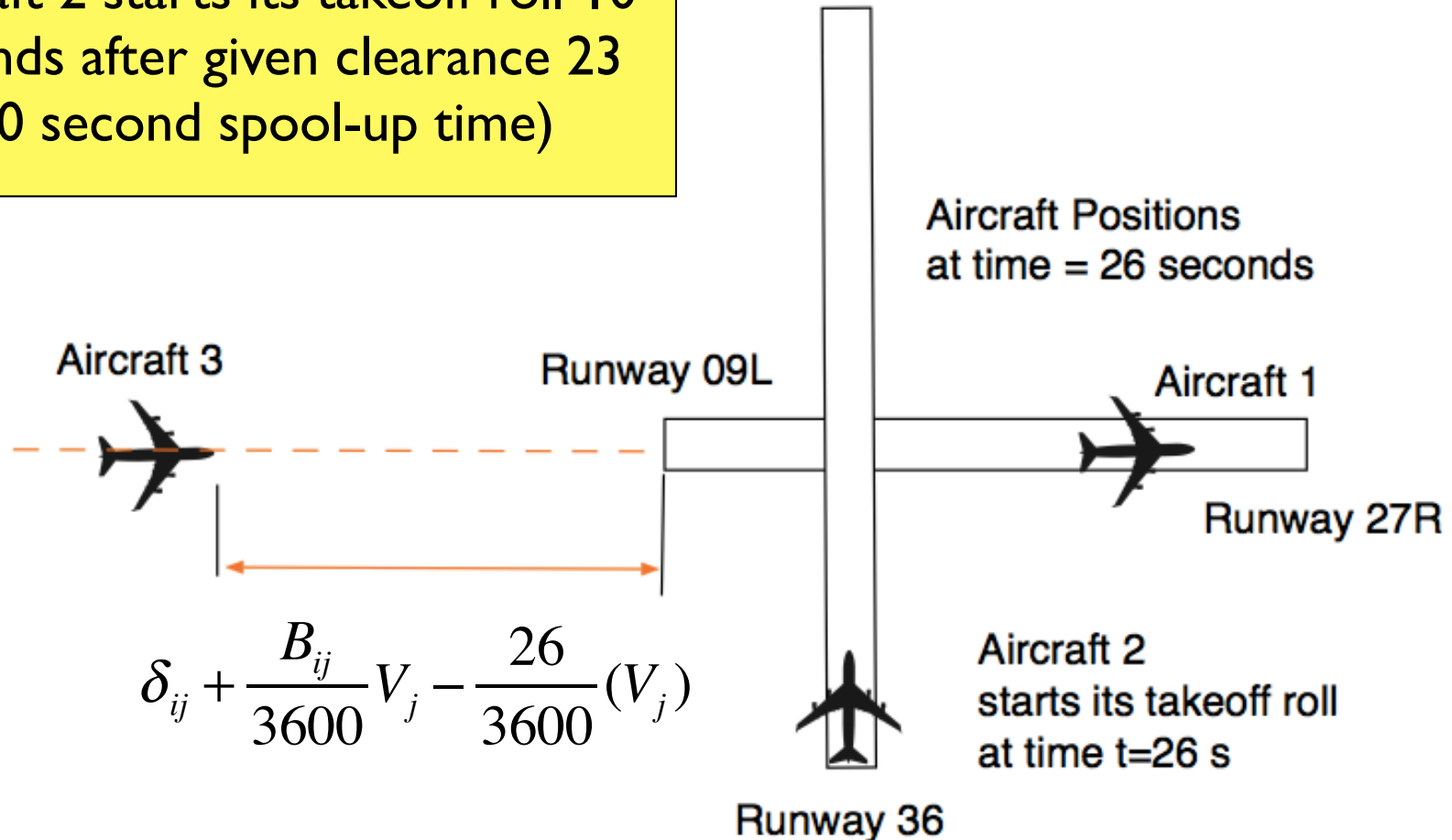


$$\delta_{ij} + \frac{B_{ij}}{3600} V_j - \frac{16}{3600} (V_j) \geq 2.5 \text{ nm}$$

For departures on runway 36, it is desired that when the departing aircraft (aircraft 2) is cleared for takeoff, the next arrival to runway 09L (aircraft 3) be no less than 2.5 nm from the arrival threshold to 09L

# Aircraft Positions at Time $t = 26$ s

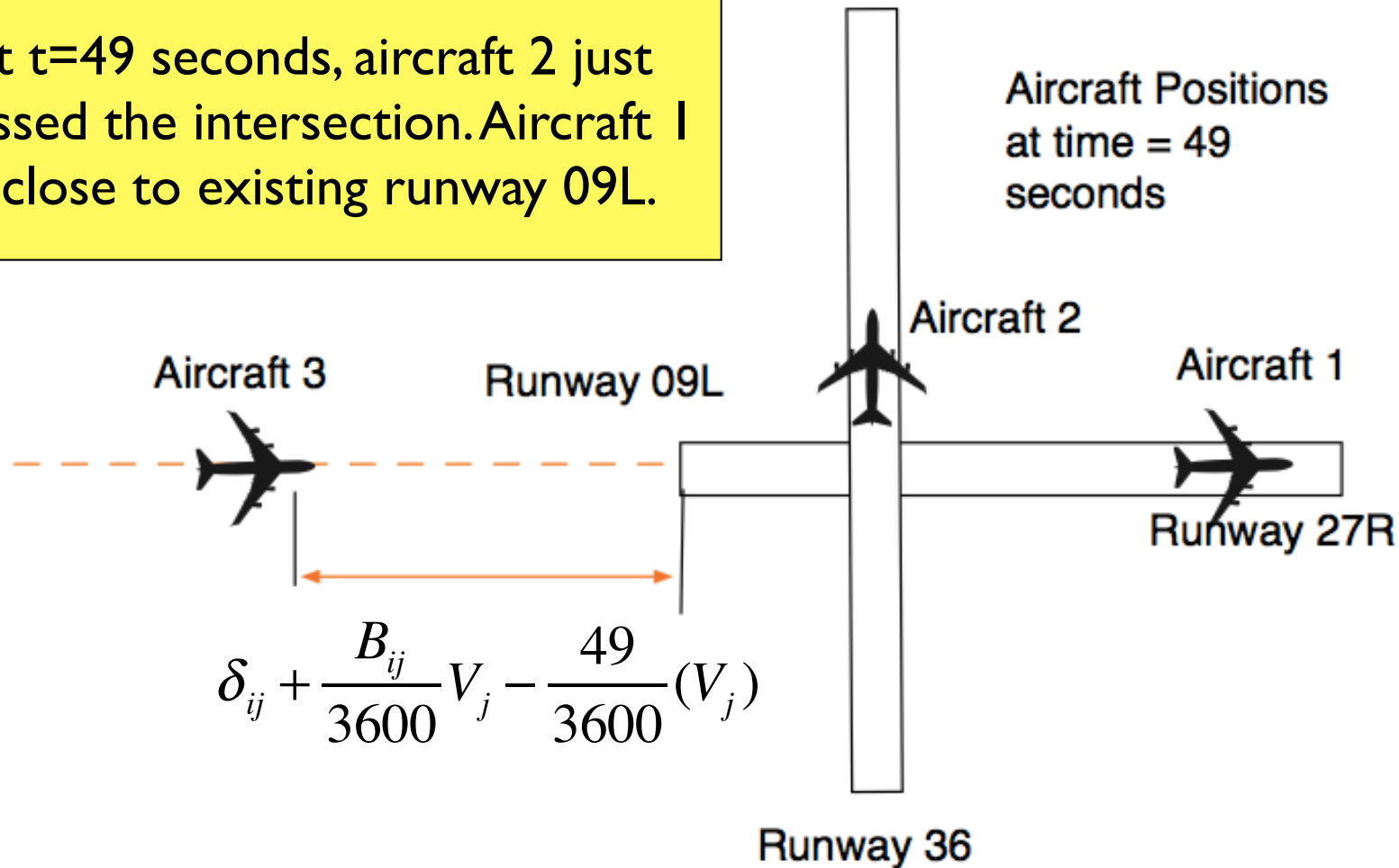
Aircraft 2 starts its takeoff roll 10 seconds after given clearance 23 (10 second spool-up time)





# Aircraft Situation at t = 49 s

At t=49 seconds, aircraft 2 just crossed the intersection. Aircraft 1 is close to existing runway 09L.



# Observations

- The interaction time period between the aircraft arrival (i) on runway 09L and a single departure on runway 36 is around 16 seconds to check for possible gaps allowing departures between successive arrivals.
- Note that in this problem solution we are using 2.5 nm as the critical distance to release departure considering that it takes another 10 seconds to spool up de engines and 23 seconds to cross the intersection
- In 33 seconds, (10+23 s) the approaching aircraft (3) has traveled around 0.89 nm
- Using 2.5 nm separation at takeoff clearance is therefore not as conservative as using 2.0 nm separation as aircraft 2 crosses the runway intersection point.

# General Condition to Release $n$ Departures on Runway 36

- Define:

$t_{n-36}$  = time for  $n$  departures on runway 36

$E(t_d)$  = expected time between successive departures on runway 36

For one departure to occur on runway 36

at  $t_{1-36} = 16$  seconds

We need to check that:

$$\delta_{ij} + \frac{B_{ij}}{3600} V_j - \frac{16}{3600} (V_j) \geq 2.5 \text{ nm}$$

$$T_{ij} - \frac{16}{3600} (V_j) \geq 2.5 \text{ nm}$$

For two successive departures on runway 36

$$t_{2-36} = (16 + E(t_d))$$

$$t_{2-36} = (16 + 80) = 96 \text{ seconds}$$

All conditions here are measured against the critical distance of 2.5 nm between the arriving aircraft to runway 09L and the runway threshold

# General Condition for Release of n Departures from Runway 36 (cont.)

For three successive departures on runway 36

$$t_{3-36} = (16 + 2E(t_d)) = 176 \text{ seconds}$$

For four successive departures on runway 36

$$t_{3-36} = (16 + 3E(t_d)) = 256 \text{ seconds}$$

Note that for n departures we can generalize:

$$t_{n-36} = 16 + E(t_d)(n - 1) \text{ seconds}$$

where:

$n$  = number of departures on runway 36 per arrival gap on runway 09L

$E(t_d)$  = expected value of time between successive departures on runway 36

All conditions here are measured against the critical distance of 2.5 nm between the arriving aircraft to runway 09L and the runway threshold

# Analysis of Crossing Runway Operations (IFR Case)

Augmented Matrix ( $T_{ij} + B_{ij}$ ) (seconds)				
		Trailing		
	Small	Large	Heavy	
Small	112.80	100.88	96.08	
Large	178.34	100.88	96.08	
Heavy	211.82	153.74	96.08	

Arrival-arrival  
matrix ( $T_{ij}+B_{ij}$ )

- 16 seconds

Time remaining on following aircraft approach segment (seconds)				
$n=1$		Trailing		
	Small	Large	Heavy	
Small	96.80	84.88	80.08	
Large	162.34	84.88	80.08	
Heavy	195.82	137.74	80.08	

Time left  
for following  
aircraft to reach  
runway 09L  
threshold

# Analysis of Crossing Runway Operations (IFR Case)

Distance left between following aircraft and runway threshold (nm)				
n=1	Trailing			
	Small	Large	Heavy	
Small	3.36		3.42	3.45
Large	5.64		3.42	3.45
Heavy	6.80		5.55	3.45

Distance between following aircraft on runway 09L to runway threshold

verify

$$\delta_{ij} + \frac{B_{ij}}{3600} V_j - \frac{16}{3600} V_j \geq 2.5 \text{ nm}$$

Number of Departures on runway 36 per arrival gap on 09L				
n	Trailing			
	Small	Large	Heavy	
Small	1.00		1.00	1.00
Large	2.00		1.00	1.00
Heavy	2.00		1.00	1.00

Potential departures on runway 36 per arrival gap on runway 09L

# Analysis of Crossing Runway Operations (IFR Case)

Pij Matrix (dim)		Trailing		
	Small	Large	Heavy	
Small	0.010	0.065	0.025	
Large	0.065	0.423	0.163	
Heavy	0.025	0.163	0.063	

$$ED_{g-ij} = P_{ij} DG_{ij} TG$$

$ED_{g-ij}$  = equivalent departures per gap between aircraft i and j

$P_{ij}$  = probability of i following j

$DG_{ij}$  = Departures per gap between i and j

$TG$  = total gaps per hour

Number of Departures on runway 36 per arrival gap on 09L		Trailing		
n	Small	Large	Heavy	
Small	1.00	1.00	1.00	
Large	2.00	1.00	1.00	
Heavy	2.00	1.00	1.00	

Sample calculation

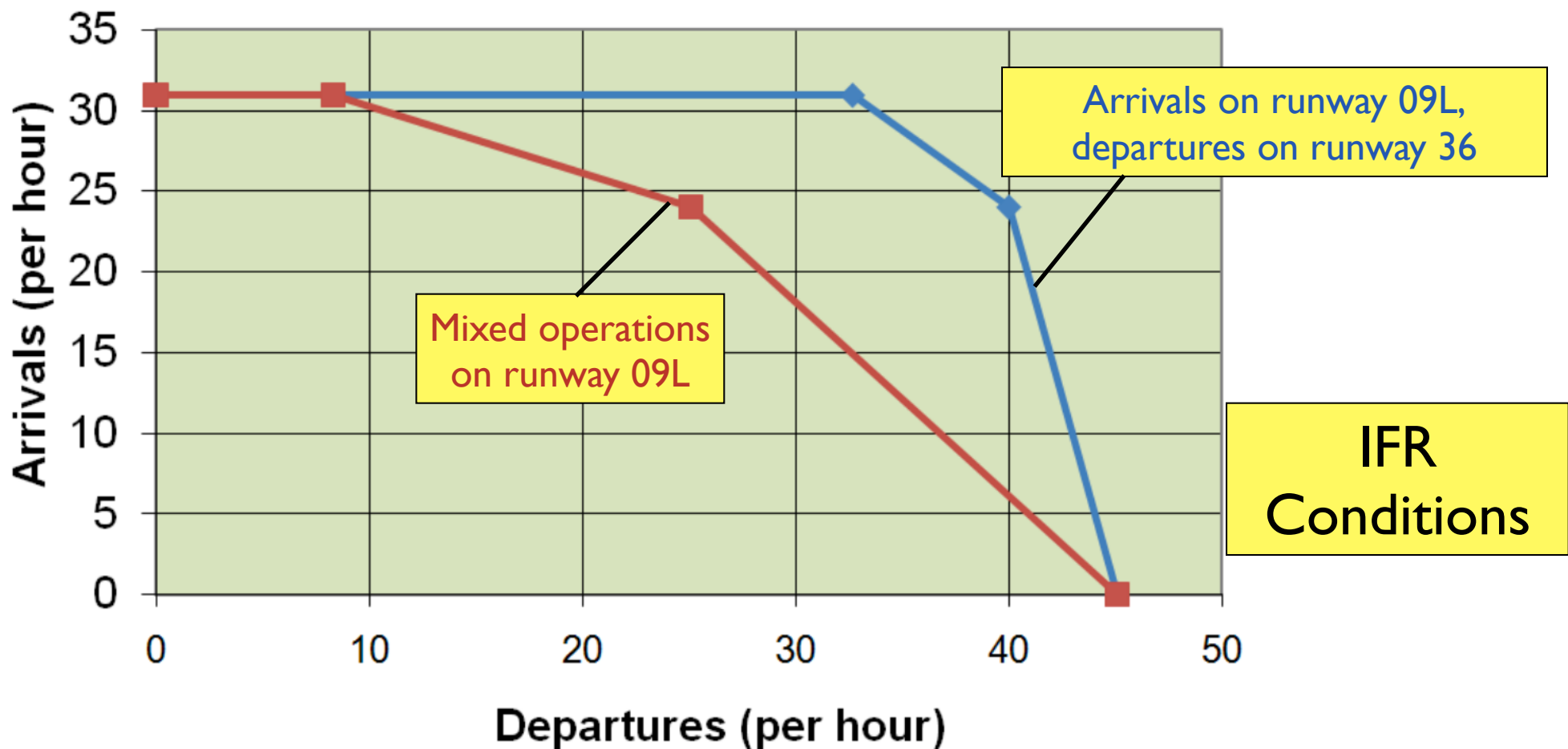
$$ED_{s-s} = 0.010 * 1.0 * (30.97 - 1) = 0.3$$

Number of departures on runway 36		Trailing		
n	Small	Large	Heavy	
Small	0.30	1.95	0.75	
Large	3.90	12.67	4.87	
Heavy	1.50	4.87	1.87	
Sum of departures on runway 36				32.68

Total departures on runway 36 considering all arrival gaps on runway 09L

# IFR Capacity Pareto Diagram (Runways 09L and 36 as Coupled Pair)

Saturation capacity for two runways operated with dependent operations. Arrivals on runway 09L, departures on runway 36.





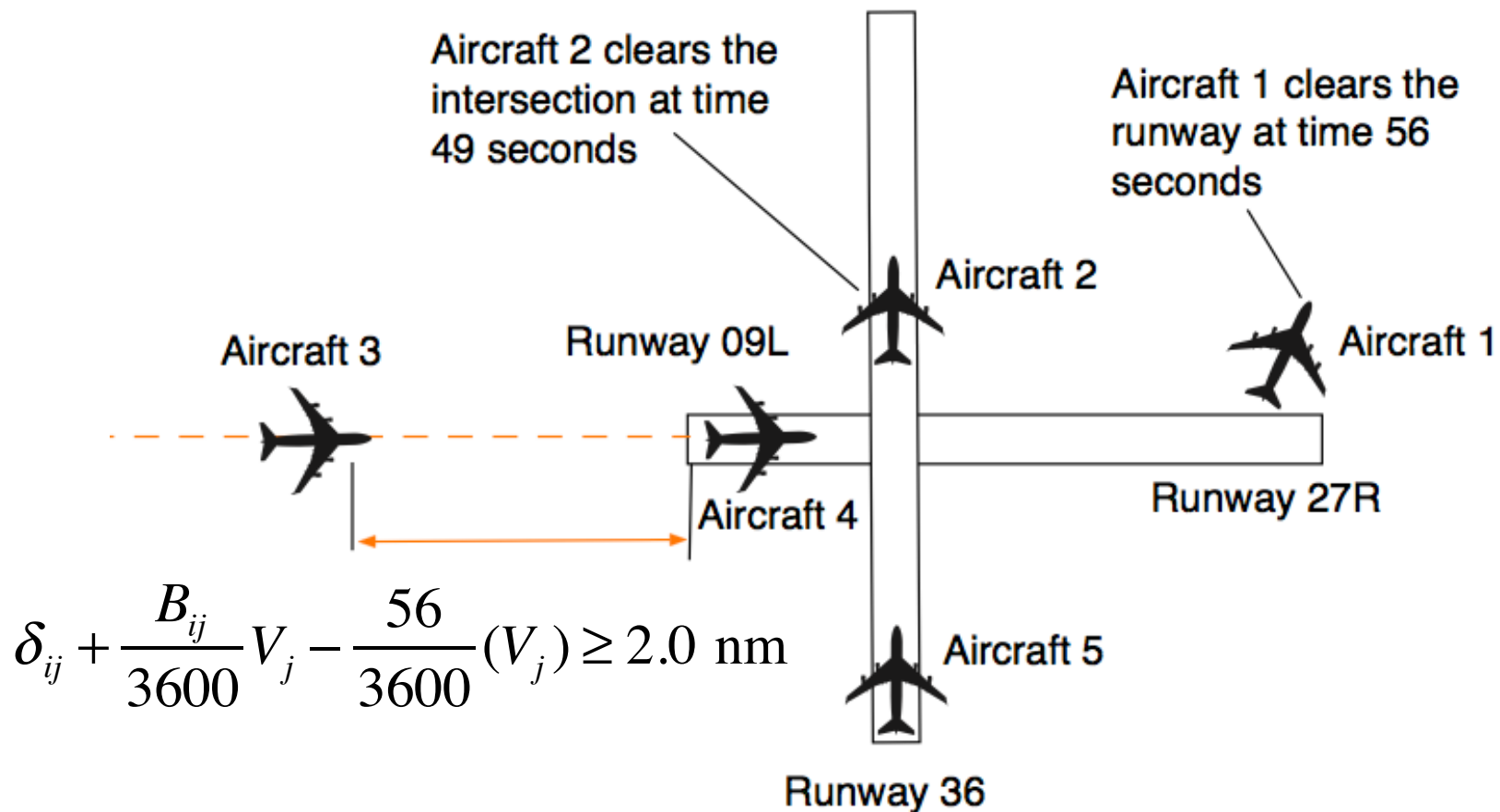
# Preliminary Conclusions

- The total number of departures on runway 36 is estimated to be **33 per hour**
- This is slightly more than the number of arrivals on the primary runway (09L)
- Processing departures on runway 36 is advantageous:
  - 8 departures on runway 09L-27R per hour
  - 33 departures on runway 36-18 per hour
  - Both results assume arrival priority on runway 09L-27R

# Considering Departures on Runways 09L and 36

# Critical Time to Release a Departure on Runway 09L (t=56 or t=49)

At t=56 s Aircraft 1 clears the runway  
 At t=49 s Aircraft 2 clears the runway intersection  
 Aircraft 4 can be released for takeoff if both conditions are met



# General Condition to Release $n$ Departures on Runway 09L subject to Release of Takeoff on Runway 36

- Define:

$t_{09L}$  = time for  $n$  departures on runway 09L subject to departures on runway 36

$E(t_d)$  = expected time between successive departures on runway 36

For one departure to occur on runway 09L subject to takeoffs on runway 36 we have to check two conditions

at  $t_{1-09L} = 56$  seconds

We need to check that:

$$\delta_{ij} + \frac{B_{ij}}{3600} V_j - \frac{56}{3600} (V_j) \geq 2.0 \text{ nm}$$

$$T_{ij} - \frac{56}{3600} (V_j) \geq 2.0 \text{ nm}$$

All conditions here are measured against the critical distance of 2.0 nm between the arriving aircraft to runway 09L and the runway threshold

# General Condition to Release $m$ Departures on Runway 09L subject to Release of Takeoff on Runway 36

For two successive departures on runway 09L between arrivals on 09L and departures on runway 36 we need to check two conditions

$$t_{2-09L} = (56 + E(t_d))$$

$$t_{2-36} = (56 + 80) = 136 \text{ seconds}$$

$$T_{ij} - \frac{136}{3600}(V_j) \geq 2.0 \text{ nm}$$

and

$$t_{2-09L} = t_{2-36} + (33)$$

$$T_{ij} - \frac{96 + 33}{3600}(V_j) \geq 2.0 \text{ nm}$$

$$T_{ij} - \frac{129}{3600}(V_j) \geq 2.0 \text{ nm}$$

All conditions here are measured against the critical distance of 2.0 nm between the arriving aircraft to runway 09L and the runway threshold

# Analysis of Crossing Runway Operations (IFR Case)

Augmented Matrix ( $T_{ij} + B_{ij}$ ) (seconds)				
		Trailing		
	Small	Large	Heavy	
Small	112.80	100.88	96.08	
Large	178.34	100.88	96.08	
Heavy	211.82	153.74	96.08	

Arrival-arrival  
matrix ( $T_{ij}+B_{ij}$ )  
  
- 56 seconds

Time remaining on following aircraft approach segment (seconds)				
$n=2$		Trailing		
	Small	Large	Heavy	
Small	56.80	44.88	40.08	
Large	122.34	44.88	40.08	
Heavy	155.82	97.74	40.08	

Time left (seconds)  
for following  
aircraft to reach  
runway 09L  
threshold after  
arrival on runway  
09L has cleared the  
runway

# Analysis of Crossing Runway Operations (IFR Case)

Distance left between following aircraft and runway threshold (nm)				
n=1		Trailing		
	Small	Large	Heavy	
Small	1.97	1.81	1.73	
Large	4.25	1.81	1.73	
Heavy	5.41	3.94	1.73	

Distance between following aircraft on runway 09L to runway threshold

verify

$$\delta_{ij} + \frac{B_{ij}}{3600} V_j - \frac{56}{3600} V_j \geq 2.5 \text{ nm}$$

Number of Departures on runway 09L per arrival gap on 09L				
n		Trailing		
	Small	Large	Heavy	
Small	0.00	0.00	0.00	
Large	1.00	0.00	0.00	
Heavy	1.00	1.00	0.00	

Potential departures on runway 36 per arrival gap on runway 09L

# Analysis of Crossing Runway Operations (IFR Case)

Pij Matrix (dim)				
		Trailing		
		Small	Large	Heavy
Small		0.010	0.065	0.025
Large		0.065	0.423	0.163
Heavy		0.025	0.163	0.063

$$ED_{g-ij} = P_{ij} DG_{ij} TG$$

$ED_{g-ij}$  = equivalent departures per gap between aircraft i and j

$P_{ij}$  = probability of i following j

$DG_{ij}$  = Departures per gap between i and j

$TG$  = total gaps per hour

Number of Departures on runway 09L per arrival gap on 09L				
n		Trailing		
		Small	Large	Heavy
Small		0.00	0.00	0.00
Large		1.00	0.00	0.00
Heavy		1.00	1.00	0.00

Sample calculation

$$ED_{L-s} = 0.065 * 1.0 * (30.97 - 1) = 1.95$$

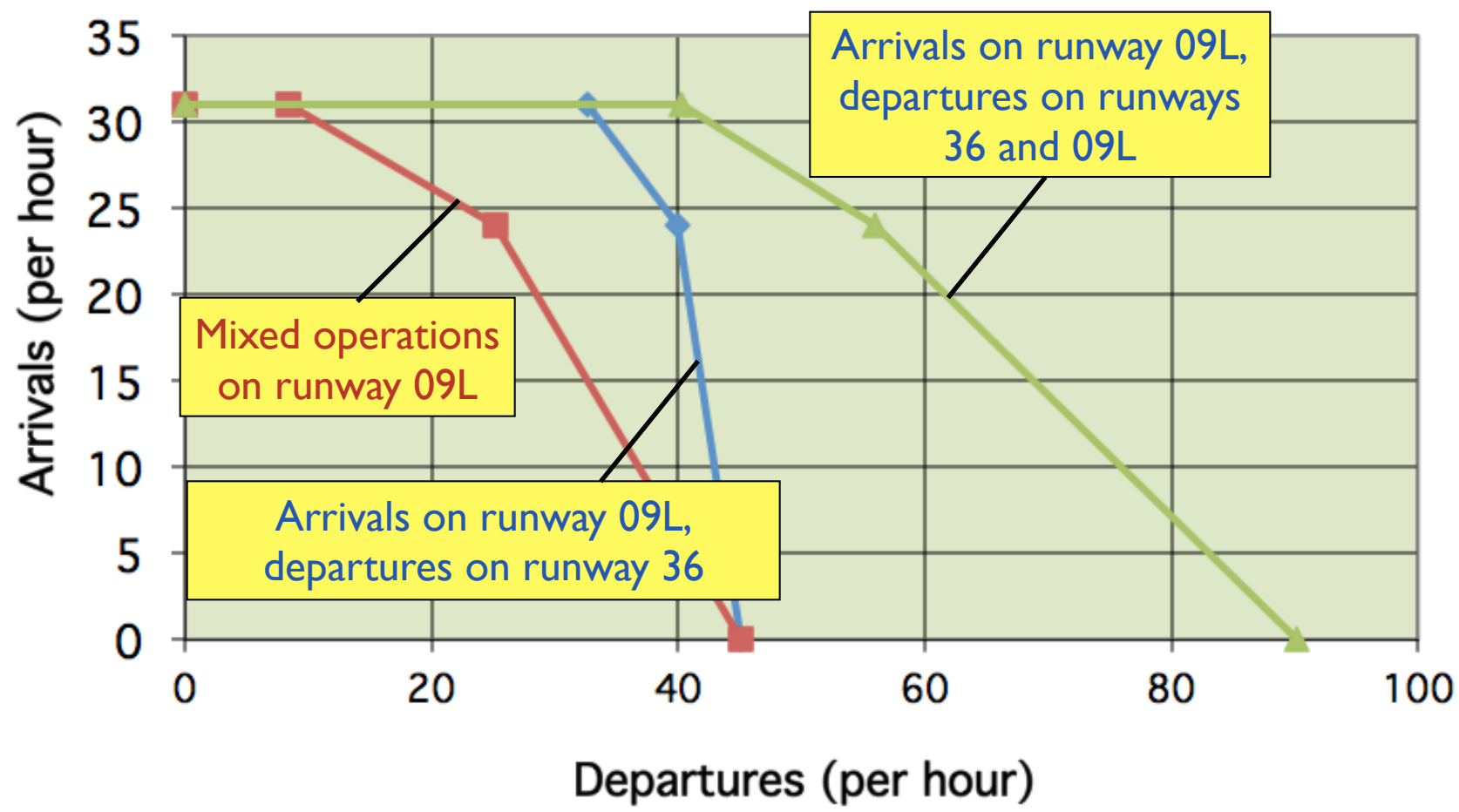
Number of departures on runway 09L subject to departures on 36				
n		Trailing		
		Small	Large	Heavy
Small		0.00	0.00	0.00
Large		1.95	0.00	0.00
Heavy		0.75	4.87	0.00
Sum of departures on runway 09L				7.57

Total departures on runway 09L considering departures on 36 and all arrival gaps on runway 09L



# Departure Capacity Benefits of Using Runway 09L

- The Pareto diagram expands compared to the solution allowing runway 36 operations



# Benefits of Using Runways 09L and 36

- In the limit, both runways produce 45 departures per runway per hour (i.e., 90 departures per hour)
- When arrival are given priority on runway 09L, 33 departures can be serviced on runway 36 and up to 7 departures on runway 09L

# IFR Capacity Pareto Diagram (Coupled Runway Pair 09L / 36 + Runway 09R)

Departures on runways 09L, 09R and 36  
Arrivals on runways 09L and 09R

