## Assignment 5: Air Transportation Systems Analysis

## Solution

## Problem 1

For the new generation long-range transport aircraft provided in the class web site (http:// 128.173.204.63/courses/cee5614/cee5614_pub/boeing787_class.m ) to answer the following questions.
a) Use the unrestrictedClimbAnalysis.m Matlab script to estimate the mass of the aircraft at the Top of Climb (TOC) point. The aircraft takeoff weight is $225,000 \mathrm{~kg}$. with OEW of $117,700 \mathrm{~kg}$., $71,300 \mathrm{~kg}$. of fuel and $36,000 \mathrm{~kg}$ of payload (passengers and belly cargo). The pilot climbs to 35,000 feet restricted by Air Traffic Control. Use the default climb speed profile provided in the aircraft data file. Use ISA atmospheric conditions in your calculations. The departure airport is located at sea level conditions.


Figure 1. Climb Analysis. Twin-engine Transport Aircraft. Mass $=225,000 \mathrm{~kg}$ (Takeoff). 127 nm to Climb to 35,000 feet. $4,067 \mathrm{~kg}$ of Fuel Used in the Climb Profile to 35,000 feet. Mass at TOC $=220,933 \mathrm{~kg}$.
b) Use the unrestrictedDescendAnalysis.m Matlab script to estimate the fuel used from the Top of Descent (TOD) point (at 35,000 feet) to the destination airport. The destination airport is located at sea level conditions (assume ISA conditions in the decent as well). In this calculation assume the mass at the TOD point is $158,000 \mathrm{~kg}$. This provides a $4,000 \mathrm{~kg}$. of fuel reserve allowance at the end of the long flight.


Figure 2. Descent Profile for Large Twin-Engine Transport Aircraft. 185 nm used in the Descent Profile.


Figure 3. Descent Profile from 35,000 feet. 1,374 kg of Fuel Used in the Descent Profile.
c) Using the Breguet range equation, find the still-air (no wind) range of the aircraft with the parameters provided in parts (a) and (b) with the aircraft flying at 36,000 feet and Mach 0.82. To calculate the range parameter use the mass of the aircraft at a mid-point between TOC and TOD.

At the TOC point:
Flying at 36,000 feet and ISA conditions, Mach 0.82 equates to $241.9 \mathrm{~m} / \mathrm{s}$ and 312.6 knots IAS. Density at $36,000 \mathrm{ft}$ is $0.3653 \mathrm{~kg} / \mathrm{cu} . \mathrm{m}$.
$C_{d}=0.0274$
$C_{l}=0.5269$
$D=112,740 N$
$L / D=19.22$
Using the aircraft L/D ratio at the TOC point, the Range equations is:
$R=\frac{V}{\operatorname{TSFC}}\left(\frac{L}{D}\right) \ln \left(\frac{W_{i}}{W_{f}}\right)$
$R=\frac{241.9 \mathrm{~m} / \mathrm{s}}{1.53 e-4 \mathrm{~N} / \mathrm{s} / \mathrm{N}}(19.22) \ln \left(\frac{220993 * g \mathrm{~N}}{158000 * g \mathrm{~N}}\right)$
$R=1.0196 e 7 \mathrm{~m}$
$R=10,196 \mathrm{~km}$
$R=5,506 \mathrm{~nm}$
This calculation is pessimistic because the aircraft L/D ratio is higher than 19.22 during the cruise profile. Use the mid point weight to estimate the value of L/D.


Figure 4. Range Parameters for Lang-Range Transport. Mass at TOCX $=220,993 \mathrm{~kg}$. Mass at $T O D=158,000 \mathrm{~kg}$.
d) Verify that the best speed for maximum range is Mach 0.81 while the aircraft cruises at 12,000 meters ( 39,000 feet).

To verify this part we can make a plot of the range parameter vs. Mach number and also for multiple altitudes. The plot shown in Figure 5 demonstrates that this aircraft maximizes its range when flown at Mach 0.81 and 12,000 meters.


Figure 5. Still-Air Range for Three Altitudes and various Mach Numbers. Black line $=12,000$ meter cruise, red line $=10,000$ meter cruise and Blue line $=8,000$ meter cruise. At Mach 0.81 and 12,000 meters the range is maximum.

## Problem 2

An airline wants to fly the route CLT-FRA with a new advanced twin engine aircraft (http://128.173.204.63/ courses/cee5614/cee5614_pub/boeing787_class.m). The route of this flight is one of the many routes optimized daily for North Atlantic operations using the North Atlantic Organized Track System - NAT OTS (see Figure 1). Read about the NAT at: http://en.wikipedia.org/wiki/North Atlantic Tracks. On the day of the flight there are 6 tracks designated by letters from $U$ to $Z$ as shown in Figure 1. You can check the daily tracks at: http://www.turbulenceforecast.com/atlantic_eastbound tracks.php.

The flight plan for this aircraft indicates a takeoff weight of $217,700 \mathrm{~kg}$. with OEW of $117,700 \mathrm{~kg}$., 60,000 kg . of fuel and $40,000 \mathrm{~kg}$ of payload (passengers and belly cargo). The aircraft is expected to climb to FL 360 directly after departing CLT and fly the first leg (CLT to NAT Track W) at Mach 0.82 (assume ISA conditions). The aircraft reaches the entry point of the North Atlantic Organized Track System (NAT OTS) near St. Pierre, Canada (see Figure 1). Just before entering the NAT the pilot requests FL 380 and Mach 0.83 for the North Atlantic leg crossing. Canadian controllers accept the request for both speed and cruise altitude. The final leg of the flight (NAT TRack W to FRA) is also expected to be flown at 38,000 feet as shown in Figure 1.


Figure 2.1. CLT-FRA Flight Using the Organized North Atlantic System (OTS).
a) Calculate the total fuel used in the flight given the parameters shown in Figure 1. In your calculations make sure to also include the fuel consumed in the climb phase from FL 360 to 380 before the aircraft enters the NAT OTS system. Assume the pilot performs the climb at Mach 0.82 and uses maximum continuous thrust.

For the cruise portion of the flight, approximate the fuel burn using the mid-point mass of every leg flown. Recall that fuel consumption in cruise is the product of drag and TFSC as shown below.

Solution: The total distance to be traveled from CLT to FRA is: $1480+1690+760=3,930$ nautical miles.

## Climb Profile:

For the climb profile we use DTW $=217,700 \mathrm{~kg}$.
Climb Fuel (kg) 5653.0627
Climb Time (minutes) 34.2945
Climb Distance (nm) 243.0136

## Cruise Profile to NAT:

Cruise distance at 36,000 feet is 1237 nm
Starting mass $=217,700-5,653 \mathrm{Kg}$
Cruise Fuel (kg) 15716.7784
Cruise Time (minutes) 157.7859
Cruise Distance (nm) 1237

## Climb Profile from 36,000 feet to $\mathbf{3 8 , 0 0 0}$ feet

At Mach 0.82 the aircraft does not have a good rate of climb capability to climb to 38,000 feet. The procedure to climb to 38,000 feet requires slowing down a bit to perhaps Mach 0.79 and then climbing to $38,000 \mathrm{ft}$. Assume climb is done at Mach 0.79 ( $\mathrm{V}=233 \mathrm{~m} / \mathrm{s}$ or 300 knots IAS). The plot in Figure 2.2 illustrates a climb at Mach 0.79 from 36,000 to 38,000 feet starting at a mass of $196,330 \mathrm{~kg}$.

Mass at NAT point $=217700-5653-15717=196,330 \mathrm{~kg}$.
Fuel to Climb $=196,330-195,780=546 \mathrm{~kg}$.
Distance in climb $(36-38 \mathrm{~K})=39 \mathrm{~nm}$
Mass at new TOC inside NAT $=195,780 \mathrm{~kg}$.


Figure 2.2 Climb form 36000 to 38000 feet at Mach 0.79 (300 knots IAS). The Climb Requires 39 nm and uses 546 kg . of fuel.

## Cruise Profile Inside NAT:

Cruise distance at 38,000 feet is $(1,690-39)=1,651 \mathrm{~nm}$
Mach speed $=0.83$
Starting mass $=195,780 \mathrm{Kg}$

Cruise Fuel (kg) 19445.1247
Cruise Time (minutes) 208.0604
Cruise Distance (nm) 1651
Ending Mass $=195,780-19445=176,335 \mathrm{~kg}$.

## Cruise Profile from NAT to TOD point:

Cruise distance at 38,000 feet is (760-Descent distance) ~ 570 nm
I estimated a typical descent distance of 190 nm using the unrestricted descent profile and $165,000 \mathrm{~kg}$ as the mass at the TOD point.
Mach speed $=0.83$
Starting mass $=176,335 \mathrm{~kg}$.
Cruise Fuel (kg) 6413.586
Cruise Time (minutes) 71.8319
Cruise Distance (nm) 570

## Descent Profile:

Descent distance from 38,000 feet is 190 nm
Speed profile in the aircraft file provided.
Starting mass at TOD $=176,335-6,414=169,922 \mathrm{~kg}$.
Descent Fuel (kg) 1,305
Descent Distance (nm) 190

Table 2.1 Summary of Fuel Used in Flight.

| Phase of Flight | Fuel Used (kg) | Remarks |
| :--- | :---: | :--- |
| Climb to $36,000 \mathrm{ft}$ | 5,653 |  |
| Cruise to NAT | 15,717 | at 36,000 feet and Mach <br> 0.82 |
| Climb to $38,000 \mathrm{ft}$ | 546 | from 36-38 kft |
| Cruise inside NAT | 19,445 | at $38,000 \mathrm{ft}$ and Mach <br> 0.83 |
| Cruise from NAT to TOD | 6,414 | Mach 0.83 and 38 kft. |
| Descent | $\mathbf{1 , 3 0 5}$ | from 38000 feet |
| Total |  |  |

b) Based on the calculations performed, what is the fuel reserved carried? (fuel left after the flight is completed).

The flight carries plenty of extra fuel (more than 10 metric tons).
c) Calculate the fuel penalty to the airline if the aircraft receives an ATC clearance to fly the NAT OTS system at 36,000 feet instead (assume FL 360 for the NAT to FRA leg).

Using 3930 nm at 36,000 feet the following mission parameters are obtained:
Mission Fuel (kg) 49540.8952
Travel Time (minutes) 517.7856
Total Distance (nm) 3930
Average Speed (knots) 455.4009

Climb Fuel (kg) 5653.0627
Climb Time (minutes) 34.2945
Climb Distance (nm) 243.0136

Cruise Fuel (kg) 42356.4232
Cruise Time (minutes) 445.046
Cruise Distance (nm) 3489.0428

Descent Fuel (kg) 1531.4092
Descent Time (minutes) 38.4451
Descent Distance (nm) 197.9436

The additional fuel used flying at 36,000 feet is 461 kg . Note that the original flight was also doing Mach 0.83 in cruise inside the NAT up to the TOD point. Assuming no difference in wind conditions, the higher speed ( 5.7 knots difference) would have saved 3.41 minutes of travel time.
d) Calculate the additional cost to the airline per flight if the lower altitude is used. The fuel price today in large volumes is $\$ 3.15$ per gallon of Jet-A fuel. Comment if the cost differential would be significant if the airline makes 600 crossings per year in that route.

1 gallon $=6.7 \mathrm{lb}$. of jet-A fuel or 3.045 kg of jet-A fuel.
The airline saves $\mathbf{2 7 6 , 6 0 0} \mathrm{kg}$. $\mathbf{( 9 0 , 8 3 7}$ gallons) of fuel per year. This would translate into $\mathbf{\$ 2 8 6 , 1 4 0} \mathbf{~ i n}$ savings per aircraft.
e) What is the minimum longitudinal separation today in the OTS for aircraft with ADS-B and Datalink equipped aircraft?

10 minutes for all aircraft. 5 minutes in-trail if the aircraft are equipped with ADS-C and CPDLC datalink.

