## Assignment 4: Air Transportation Systems Analysis

Solution

## Problem 1

Use the new generation twin-engine transport aircraft to answer this question (http://128.173.204.63/ cee5614/cee5614_pub/B787_class.m).
a) Estimate the aircraft rate of climb after departing New York Kennedy International airport while the aircraft flies at 350 meters above sea level and 220 knots indicated. Assume atmospheric conditions are ISA. Assume the wing flaps are fully retracted and use the regular table lookup function to estimate Cdo provided in the aircraft file. The aircraft departs New York with a mass of 215,000 kilograms.

Modeling the aircraft climb using the six steps discussed in class.

Cruise Mach Number (dim) : 0.338
True airspeed (knots) : 222.8
Altitude (m) : 350
Altitude (feet) : 1148
Aircraft Mass (kg) : 215000
Lift Coefficient (dim) : 0.70386
Drag Coefficient (dim) : 0.033881
Drag (N) : 101526.949
Thrust (N) : 492097.8436
Rate of climb (m/s) : 21.23
Rate of climb (ft/min) : 4177.96
Rate of climb (m/min) : 1273.77
b) Estimate the rate of climb after departing New York Kennedy International airport while the aircraft flies at 350 meters above sea level conditions and 220 knots indicated airspeed and with one engine failed (assume one engine fails just as the aircraft climbs through 350 meters above sea level conditions). Assume the wing flaps are fully retracted and use the regular table lookup function to estimate Cdo provided in the aircraft file. The aircraft departs New York with a mass of 215,000 kilograms.

[^0]| 10 | e = 0.87; \% Oswald's efficiency factor |  |
| :---: | :---: | :---: |
| 11 | tcRatio $=0.09 ; \quad \%$ thickness to chord ratio (dimansionless) |  |
| 12 | sweepAngle = 33; \% sweep angle of the wing (degrees) |  |
| 13 | $\mathrm{g}=9.81$; |  |
| 14 | neng $=1 ; \quad$; | \% Number of engines |
| 15 | tsfc $=1.53 \mathrm{e}-4$; | \% thrust specific fuel consumption (N/N/s) |
| 16 |  |  |
| 17 | \% Aircraft mass limitations |  |
| 18 |  |  |
| 19 | OEW = 117700; | \% Operating empty mass (kg) |
| 20 | Max_fuel = 101323; | \% Maximum fuel mass (kg) |
| 21 | Max_payload $=43300$; | 00; \% Maximum payload (kg) |
| 22 | MTOGW = 227930; | \% Maximum gross takeoff mass (kg) |
| 23 | mass $=215000$; | \% mass at operating point (kg) |

36
\% Computes the aircraft profile given altitude (Vcas given) - typica \% twin engine aircraft similar to the Boeing 787-800 aircraft

```
Vclimb = [220 220270290 310 310]; % knots
Vdescent = [210250 250310320 320]; % knots
altc =[ 0 400010000 1500033000 43000]/3.28; % units
```

Figure 1. Changes to Aircraft File to Simulate an Engine Failure.

```
37 B787_A4_2023
38
39
40
41
```

```
h_airport = 350; % airport altitude (m) - departing field elevation
```

h_airport = 350; % airport altitude (m) - departing field elevation
rhos = 1.225; % sea level density (kg/m-m-m)
rhos = 1.225; % sea level density (kg/m-m-m)
deltaTemp = 0; % ISA + deltaTemp conditions for analysis (deg. Kelvin)

```
deltaTemp = 0; % ISA + deltaTemp conditions for analysis (deg. Kelvin)
```

Figure 2. Changes to the Unrestricted Climb File to Simulate an Engine Failure.

Cruise Mach Number (dim) : 0.338
True airspeed (knots) : 222.8
Altitude (m) : 350
Altitude (feet) : 1148
Aircraft Mass (kg) : 215000
Lift Coefficient (dim) : 0.70386
Drag Coefficient (dim) : 0.033881
Drag (N) : 101526.95
Thrust (N) : 246048.92

## Rate of climb (m/s) : 7.86

Rate of climb (ft/min) : 1545.96
Rate of climb (m/min) : 471.33
c) Compare the rates of climb obtained in parts (a-b). Comment on the implications if the aircraft looses an engine at 350 meters.

The rate of climb with one engine out is $63 \%$ less than with all engines operating. This is a significant loss in performance.

## Problem 2

Use the new generation twin-engine transport aircraft to answer this question (http://128.173.204.63/ cee5614/cee5614_pub/B787_class.m). This problem is a continuation of Problem 1. The aircraft departs New York with a mass of 215,000 kilograms.
a) Run the unrestricted climb Matlab code demonstrated in class for the aircraft in question. Simulate the climb profile for 3600 seconds (one hour) and estimate the time to climb and distance traveled to reach FL 330 (33,000 feet). The aircraft departs JFK at ISA conditions.

Time to TOC $=880$ seconds (14.7 minutes)
Distance to TOC $=90.6$ nautical miles
b) Using the results obtained in part (a), estimate the fuel burned during the climb to reach FL 330.

Fuel to TOC (33,000 feet) $=3,372$ kilograms (7,418 lbs.)
c) On the return flight from Mumbai, the temperature conditions are ISA + $\mathbf{2 6}$ deg. C. Mumbai is at sea level conditions. Run the computer simulation (similar to part a) using the same initial takeoff mass of 215,000 kilograms and estimate the time to climb, distance traveled, and fuel used to reach FL330. Compare the results obtained in parts (a-c) and comment on the effect of temperature on rate of climb and time to climb.

Distance to TOC 126.7 nautical miles
Time to TOC 1159.2 seconds
Fuel to TOC $3,759.7 \mathrm{~kg}(8,271 \mathrm{lbs})$
Departing Mumbai, the distance to climb increase by $39 \%$. The fuel required to climb increases $11.5 \%$. The time to climb increases $32 \%$.
d) Another day, the flight departs Denver International Airport at 200,000 kilograms and ISA + 15 deg. C conditions. Find the initial cruise altitude selected by the pilot if the crew wants to have a 500 $\mathrm{ft} /$ minute climb rate capability at the initial cruise altitude. Normally, pilots want to have a small climb rate margin for maneuvering at altitude.

Altitude at TOC 10414.57 meters
Altitude at TOC 34159.79 feet
Distance to TOC 103.16 nautical miles
Time to TOC 923.58 seconds
Fuel to TOC 2848.54 kg
We conclude that depending upon the direction of flight, the pilot will select FL 340 (traveling West) or FL 330 (traveling East) as the initial cruise altitude.


Figure 3. Rate of Climb from Denver International Airport.

## Problem 3

The new generation twin-engine transport aircraft to answer this question (http://128.173.204.63/cee5614/ cee5614_pub/B787_class.m).
a) Estimate the value of Specific Air Range (SAR) for the aircraft after reaching the Top of Climb (TOC) point at 35,000 feet and with a mass of 205,000 kilograms. The aircraft cruises at Mach 0.83 .

Instantaneous SAR is the number of nautical miles per pound of fuel used. The process is to calculate the instantaneous fuel burn (lbs/s) and the nautical miles per second flown. The ratio of these two numbers is the instantaneous SAR.

Nautical Mlles per Second (nm/s) : 0.133
Nautical Mlles per minute ( $\mathrm{nm} / \mathrm{min}$ ) : 7.95
Fuel Burn (kgs/second) : 1.77
Fuel Burn (kgs/min) : 106.1
SAR (nm/lbs) : 0.165
SAR (nm/kgs) : 0.075
Alternative way:
$S A R=\frac{V}{T S F C * T}$
Since $T=D$ in cruise,
$S A R=\frac{V}{T S F C * D}$
$S A R=\frac{V}{T S F C * D}=\frac{245.6}{1.53 e-4^{*} 113356.8}=14.16$ meters $/ \mathrm{N}$ of fuel used

In more traditional units,
SAR = 138.9 meters/kilogram
SAR $=0.075$ nm/kilogram
b) Use the Range equation, to estimate the maximum range for the aircraft if the aircraft reaches the TOC point ( 35,000 feet) at 205,000 kilograms. The aircraft cruises at Mach 0.83 . The pilot estimates the aircraft carries 90,000 kilograms of fuel remaining at the TOC point. In the range calculation, assume the range is calculated using the aircraft mass at the mid-point between TOC and TOD. Also, assume that 6500 kilograms of fuel are needed in the descent plus a possible diversion to an alternate airport.


Figure 4. Range for Aircraft at FL 350 and Various Mach Numbers.
Range at Mach 0.83 and FL 350 is 13,350 kilometers ( 7,208 nautical miles).
c) Solve the problem in part (b) using a piecewise numerical analysis method (as explained class). Use the range value obtained in part (b) and refine the answer obtained by dividing the range into six segments in the numerical solution. Comment on the obtained in parts (b) and (c).

Using the numerical calculations using 500 steps we obtain the following:
Computational Steps 500
Cruise Mach Number (dim) 0.83
Cruise Range Distance (nm) 7,033
Cruise Fuel (kg) 83490.03
Cruise Time (minutes) 883.69
Cruise Time (hrs) 14.73

Initial Mass (kg) 205,000
Final Mass (kg) 121,510
The range estimated using the numerical procedure indicates that using the Breguet range equation at midpoint, is a reasonable approximation. The numerical procedure indicates that the range from TOC to TOD is 7,030 nautical miles using $83,500 \mathrm{kgs}$ of usable fuel (subtracting $6,500 \mathrm{kgs}$ of reserve fuel). The Breguet equation estimated $7,200 \mathrm{~nm}$ using the aerodynamic characteristics at the mid-point.


[^0]:    The setup of the problem involves changing parameters in the aircraft file and in the initial conditions of the unrestricted file. Figure 1 shows the changes made to the aircraft file. Specifically, we changed the number of engines from 2 to 1. The velocity profile is also changed from 230 knots to 220 knots to simulate the departure conditions at JFK.

