Analysis of Air Transportation Systems

Fundamentals of Aircraft Performance (3)

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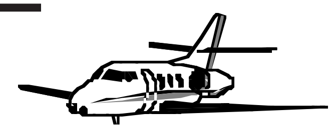
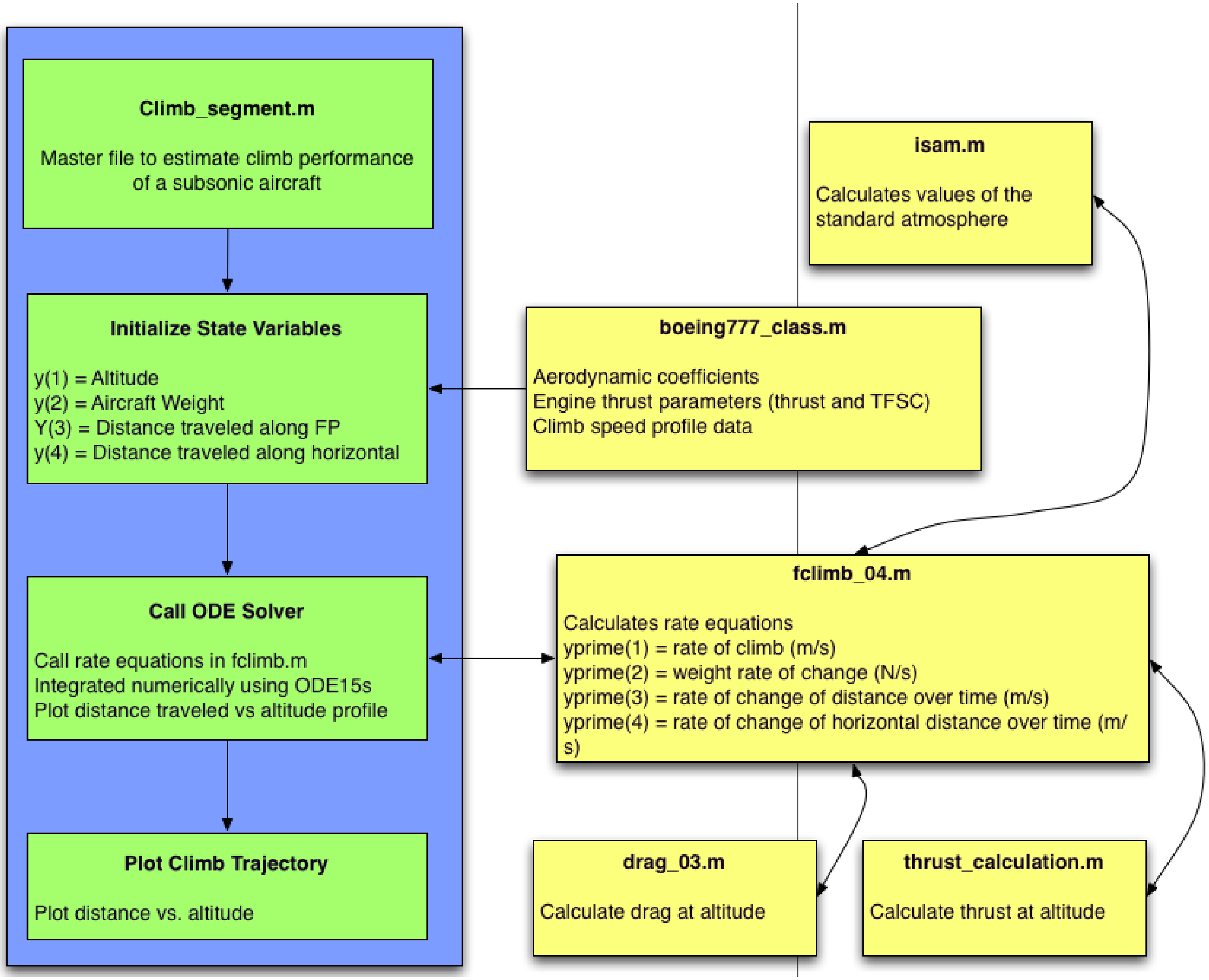
Computer Program to Estimate Aircraft Climb Performance

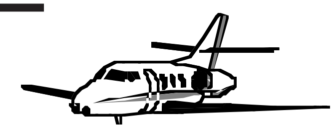
- A computer model to estimate the climb performance of subsonic aircraft
- Assumes a parabolic drag polar
- Assumes a known velocity profile during climb
- A separate program file is created to specify the aircraft performance characteristics

Climb Performance Computer Program





File: Climb_segment_2003.m

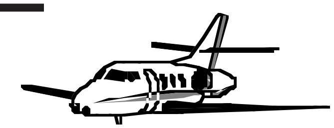


```
% Program: Climb Performance Analysis Program

% Calls: FUNCTION   fclimb_04
%                   aircraft file (e.g., eclipse_class.m or boeing777_class.m)
%                   drag_03.m
%                   thrust_calculation.m
%
% Assumes all engines operate normally
% Characteristics similar to a SATS Aircraft
%
% Programmer: Toni Trani
% Sep. 16, 2004
% Sep. 2004 - introduced function ISAM to simplify code

% Define global variables (to be shared across other functions and routines)

clear all

global g mass rhos hcruise

% Enter aircraft file desired

% eclipse500_class
boeing777_class

h_airport = 0;          % airport altitude (m)
rhos = 1.225;           % sea level density (kg/m-m-m)
hcruise =11000;         % cruise altitude (m)

Mass_init = mass*g;     % Initial weight (Newtons)
```

Function: fclimb_04.m

```
% FUNCTION: fclimb_04
%
% Calculates the rates of change of aircraft state
% variables during climb profile

% programmer: A. Trani
% date: Sept. 17, 2004


function yprime = fclimb_04(t,y)

global  g tsfc macht Cdoct thrust_table mach_table lapse_rate_factor rhos Vclimb altc hcruise

V = interp1 (altc,Vclimb,y(1));                 % speed schedule - speed is calibrated airspeed (CAS

% Computes standard atmosphere values

% Speed of sound in m/s at different cruise altitudes in meters

[mtrue,a,rho,temp] = isam(y(1),V);          % calculates the true mach number of the aircraft at ev

vtas = mtrue * a;                   % true airspeed (m/s)
q = 0.5 * rho * vtas^2;              % dynamic pressure (N)
phi = 0.0;                          % bank angle (radians)

% Compute drag and L/D ratio (call drag_new)
[drag,LD,cl,cd]=drag_03(mtrue,rho,a,y(2)/g,macht,Cdoct,phi);

% Estimates the thrust during climb
```

Function: drag_03.m

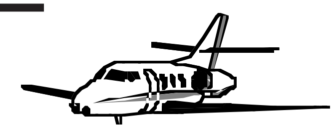


```
% Function to estimate the drag of an aircraft
% with quadratic (parabolic) drag polar
% Can be used stand-alone or with various performance programs

function [drag,LD,cl,cd]=drag_03(mach,rho,a,mass,macht,Cdoct,phi)

global A e S g neng tsfc

% starts computations
   rhos = 1.225;
   vtas = mach .* a;
   q = 0.5 * rho .* vtas.^2;

   % Estimates the cruise CL (lift coefficient) at half point of the cruise profile

   cl = 2*g*mass ./ (rho .* vtas.^2 .* S .* cos(phi));

   % Estimates drag coefficient

   kf     = 1/(pi.* A.*e);

   cdo =   interp1(macht,Cdoct,mach);             % zero lift drag coefficient
   cd  =   cdo + kf * cl.^2;                      % total drag coefficient

   % Estimate the total vehicle drag
```

Function: thrust_calculation.m

```
% Funtion to estimate the thrust characteristics of a subsonic aircraft
%
% programmer: A. Trani
% Sept. 15, 2004
%
function [thrust] = thrust_calculation(ttable,mach_table,lapse_rate,true_mach,density)

global neng rhos

tmach =  neng * interp1 (mach_table, ttable, true_mach);
thrust = tmach * (density/rhos)^lapse_rate;
```

Aircraft Data File: boeing777_class.m



```
global A e S neng tsfc macht Cdoct mass thrust_table mach_table lapse_rate_factor Vclimb altc

S = 440;            % wing area (square m)
A = 8.7;            % Wings aspect ratio
e = 0.87;           % Oswald's efficiency factor
g = 9.81;
neng = 2;           % Number of engines
tsfc = 1.6e-4;      % thrust specific fuel consumption (N/N/s)
mass = 320000;      % mass at operating point (kg)

% Drag characterictics - CDO function (zero lift drag function)

Cdoct   = [0.018   0.018   0.0192    0.023   0.037   0.038   0.040];
macht   = [0.0     0.75    0.80      0.85    0.90    0.95    1.00];

% Thrust parameters for Eclipse aircraft (at sea level)

thrust_table = [370000 240000];     % Thrust limits (sea level in Newtons)
mach_table = [0.0  0.9];                    % mach number limits to bound thrust
lapse_rate_factor = 0.9;                    % thrust lapse rate factor

% Computes the aircraft profile given altitude (Vcas given) - typical for
% twin engine aircraft similar to the Boeing 777-200 aircraft

Vclimb = [ 200 200  230 250 270  300  300  300  300  300  300  300 300  300  300];
```

Example of Aircraft Climb Performance

The following example gives an idea of the typical procedures in the estimation of the aircraft climbing performance. Assume that a heavy transport aircraft has drag polar of the form,

$$C_D = C_{Do} + \frac{C_L^2}{\pi A R e}$$

where: AR = 8.7, e =0.87 and C_{DO} (the zero lift drag coefficient) varies according to true airspeed (TAS) according to the following table:

Mach Number	**C_{DO} (nondimensional)**
0.0 to 0.75	0.0180
0.80	0.0192

Mach Number	C_{DO} (nondimensional)
0.85	0.023
0.90	0.037
0.95	0.038
1.00	0.040

The engine manufacturer supplies you with the following data for the engines of this aircraft:

True Mach Number	Sea Level Thrust (Newtons)
0	370,000
0.90	240,000

For simplicity assume that thrust variations follow a linear behavior between 0 and 0.90 mach. The thrust also decreases with altitude according to the following simple thrust lapse rate equation,

$$T_{altitude} = T_{Sea\ Level} (\rho/\rho_o)^{.90} \quad (1)$$

where ρ is the density at altitude h and ρ_o is the sea level standard density value (1.225 kg./ m^3).

The aircraft in question has four engines and has a wing area of 440 m^2.

A) Calculate the thrust and drag for this vehicle while climbing from sea level to 10,000 m. under standard atmospheric conditions with a speed profile shown below. Simulate the climb performance equation of motion assuming that the takeoff weight is 320,000 kg.

Vclimb = [200 200 230 250 270 300 300 300 300 300 300 300 300 300 300]; % speed in knots (IAS)

altc = [0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000]; % altitude for IAS in meters

The fuel consumption is approximately proportional to the thrust as follows,

F_c = TSFC (T)

where: tsfc = 1.6e-4 is the thrust specific fuel consumption (N/N/s)

B) Find the time to climb and the fuel consumed to 10,000 m.

C) What is the approximate horizontal distance traveled to reach 10,000 m. altitude?

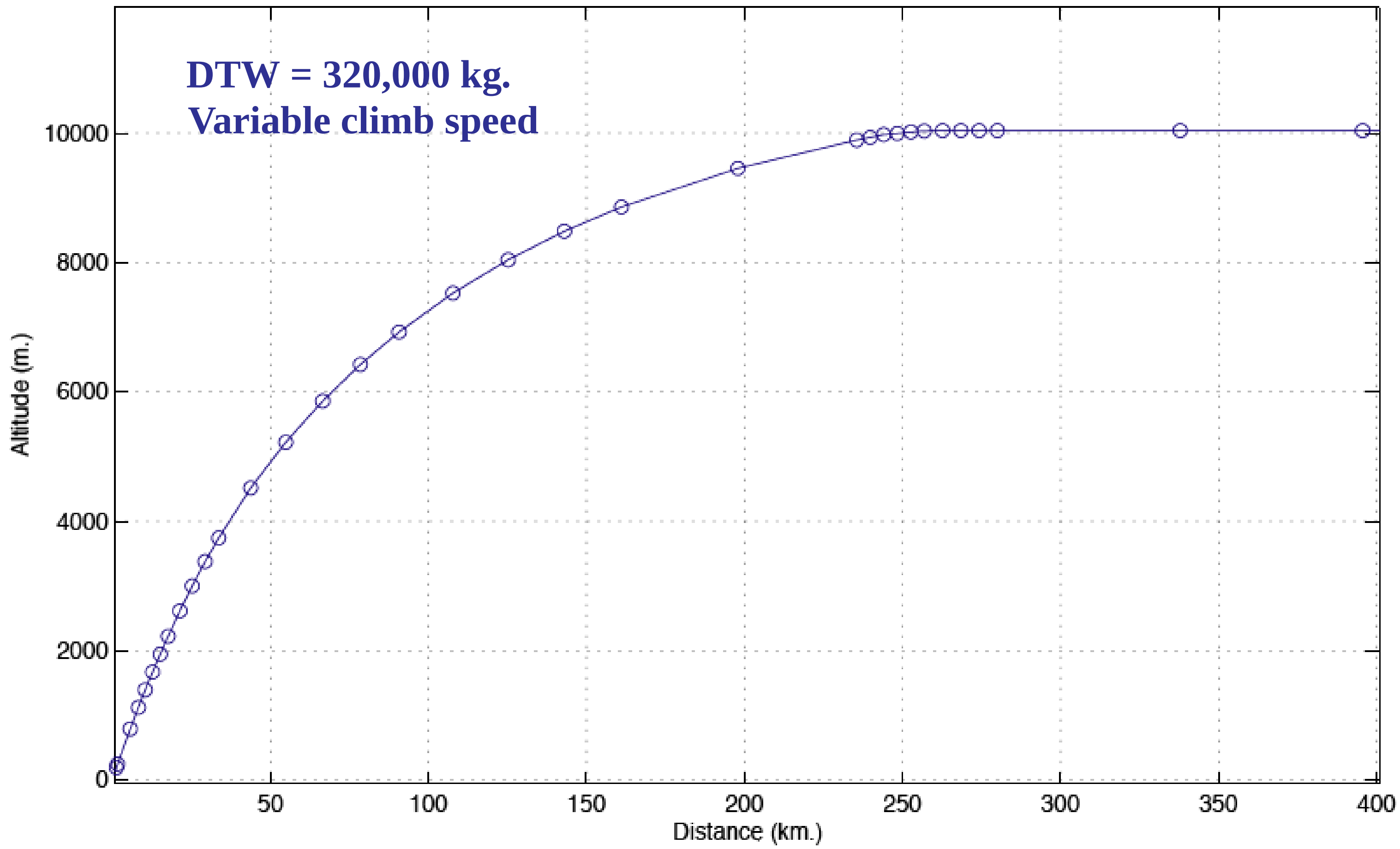
Solution

- Set the initial conditions in the main program (climb_segment_2003.m) as follows:

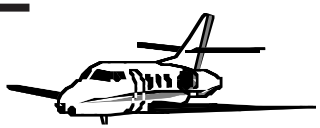
```
global g mass rhos hcruise
% Enter aircraft file desired
boeing777_class
h_airport = 0;          % airport altitude (m)
rhos = 1.225;           % sea level density (kg/m-m-m)
hcruise =10000;         % cruise altitude (m)
Mass_init = mass*g;     % Initial weight (Newtons)
```

- The aircraft file called boeing777_class.m is used
- The climb profile is shown in the next pages

Climb Profile Solution

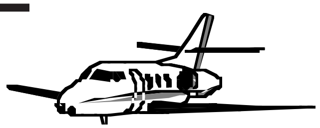


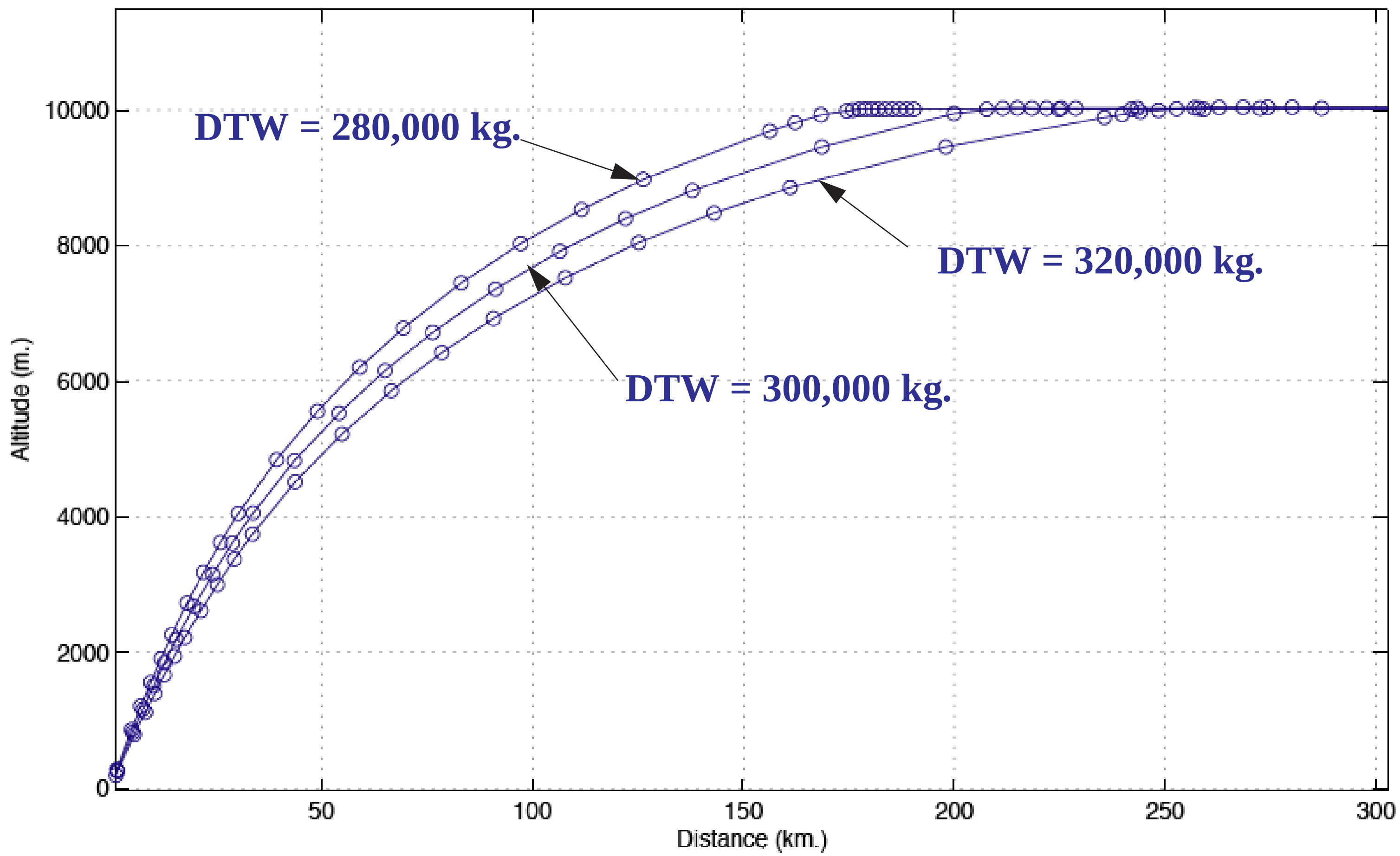
Analysis (Distance vs. Altitude Profile)



- The aircraft climb rate is high at low altitudes because the thrust developed by the engines is high compared with the drag forces
- As the vehicle climbs higher, engine thrust developed is reduced and so does climb rate
- The aircraft covers 250 kilometers to climb to 10,000 meters (33,000 ft.).
- The ODE solver takes a few more iterations near the Top of Climb (TOC) to solve the differential equations of motion because the rates of change of the state variables change more drastically (specially in the neighborhood of the TOC when the aircraft levels off)

Climb Profile Comparison (DTW Changes)

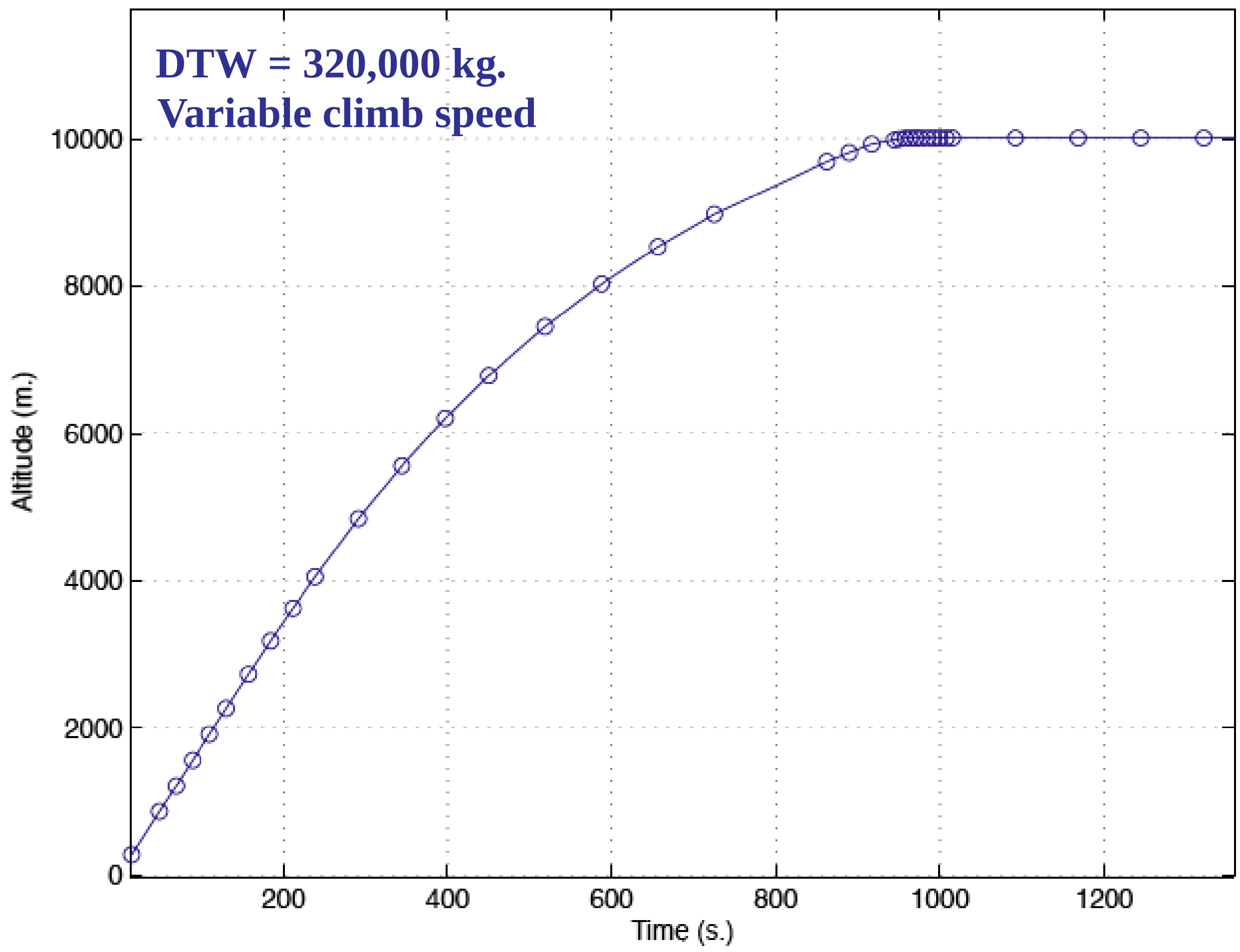




Analysis (Changes in Desired Takeoff Weight)

- A heavier aircraft (DTW = 320,000 kg) climbs slower than a lighter aircraft (DTW = 280,000 kg)
- The difference in climb distance is significant (175 km for DTW=280,000 versus 250 km when DTW=320,000 kg)
- The initial cruise altitude for a lighter aircraft is higher than that for a heavier aircraft (not shown in the diagram but can be easily demonstrated using the same program)

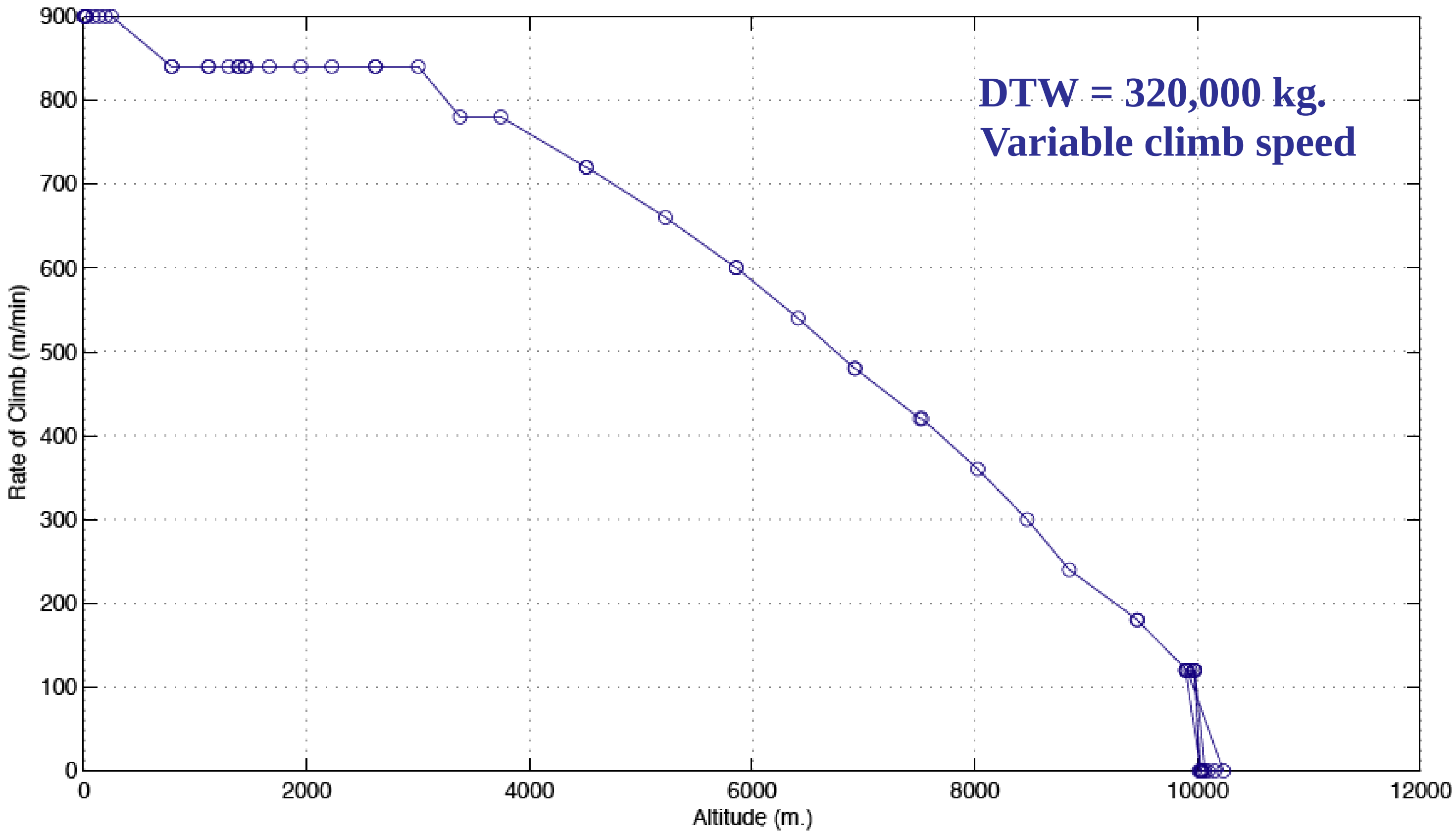
Climb Time Profile



Analysis (Climb Time Profile)

- The aircraft takes 1,000 seconds (17 minutes) to climb to 10,000 meters
- A lighter aircraft reaches the TOC faster (using time as the metric) than a heavier aircraft

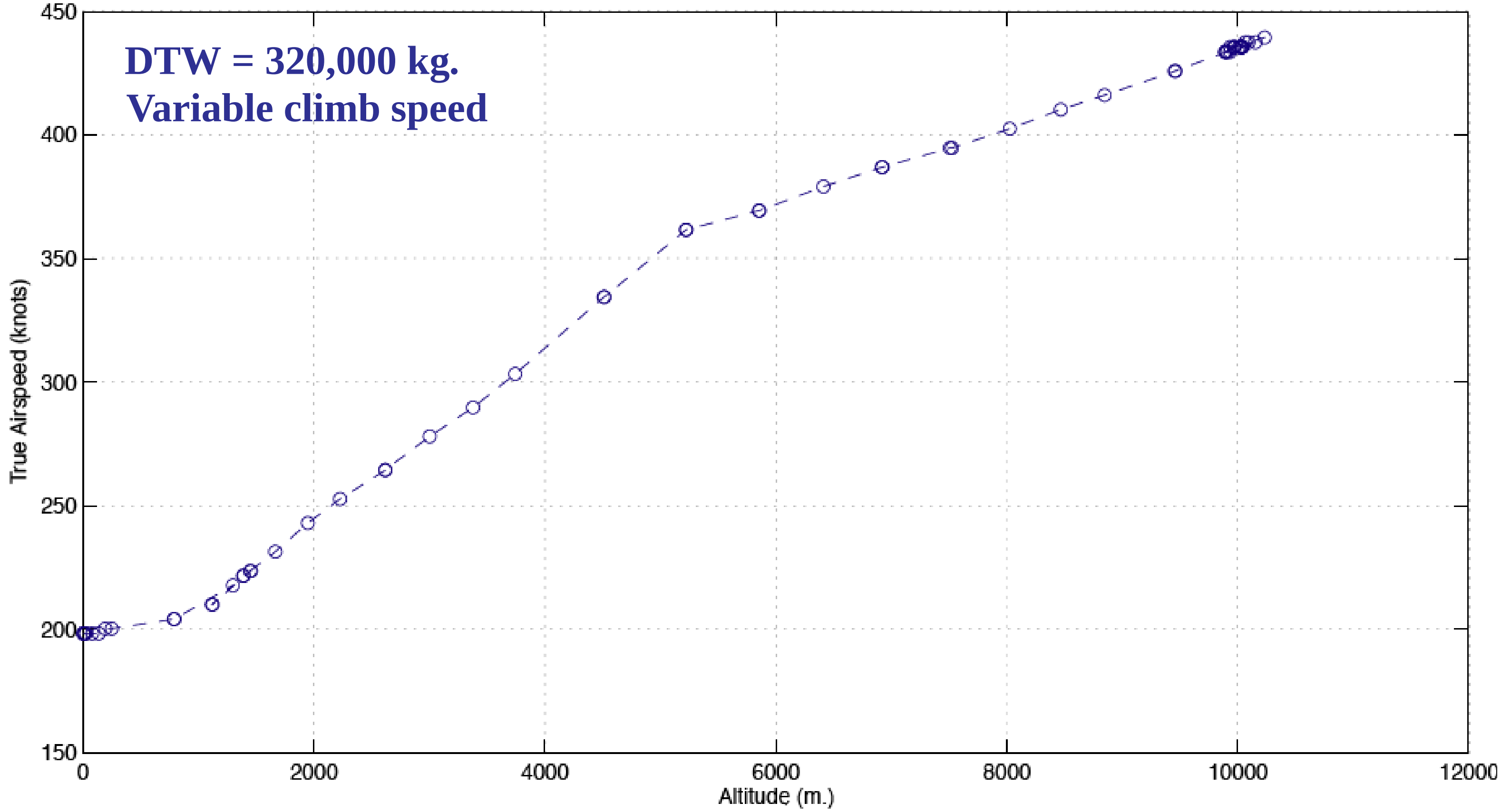
Aircraft Climb Profile (Rate of Climb)



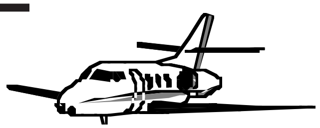
Analysis (Rate of Climb)

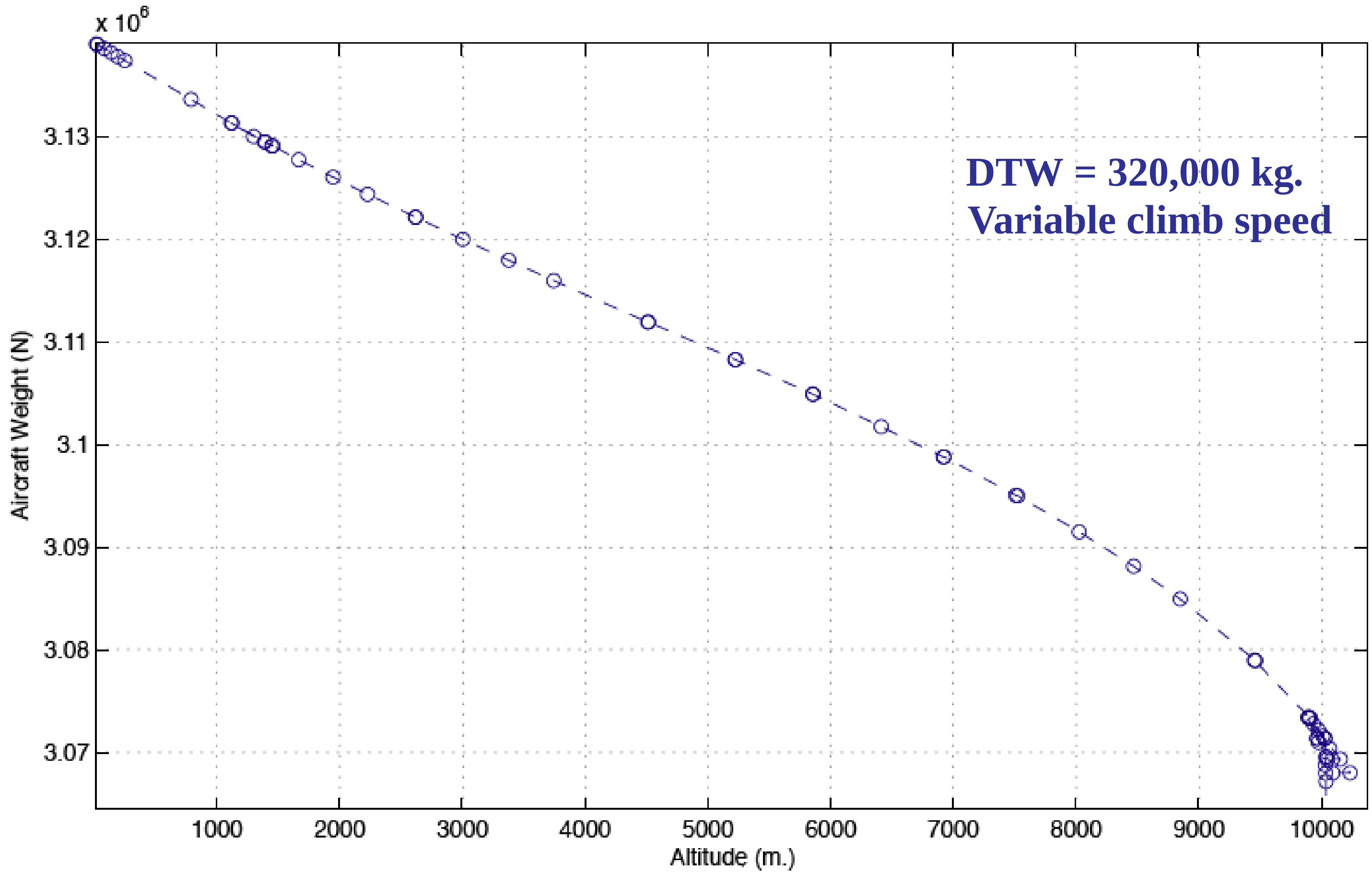
- The rate of climb diminishes with altitude gains
- At higher altitudes the thrust produced by the engine is greatly reduced
- The hysteresis observed near the TOC point is due to the small overshoot in altitude
- The true airspeed during climb (in knots) is observed to increase with altitude. Note that by the time the aircraft reaches TOC the speed is near 440 knots
- The aircraft weight changes substantially during climb (from 3,200,000 N at takeoff to near 3,070,000 N at TOC)

Aircraft Climb Profile (Speed)

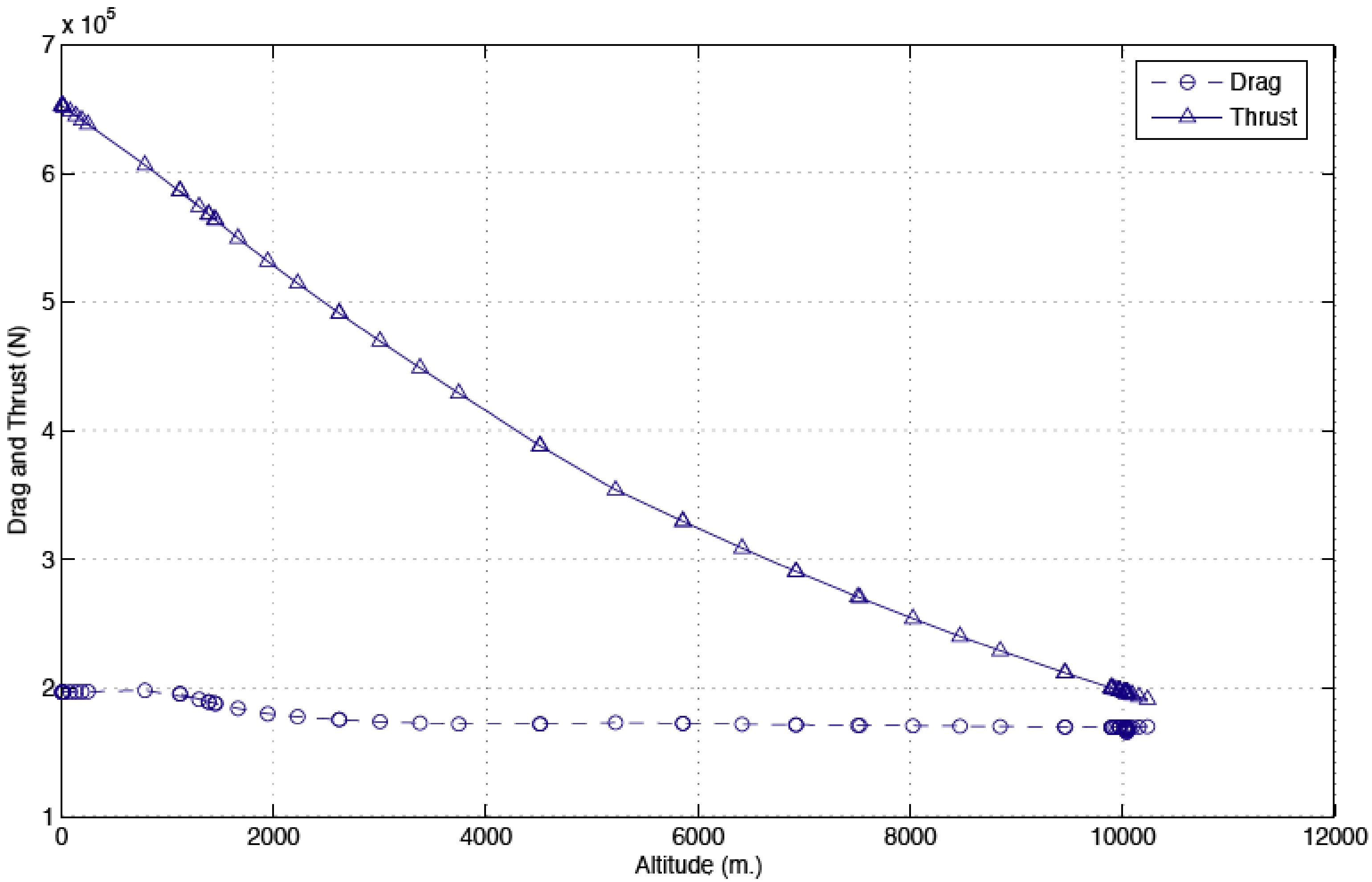


Aircraft Climb Profile (Aircraft Weight)





Aircraft Climb Profile (Thrust and Drag)



Analysis (Thrust and Drag)

- The thrust decrease substantially with altitude
- The total drag remains near constant compared with thrust
- At the TOC point, the aircraft has little climb capability because thrust and drag curves are close (little capability to climb)