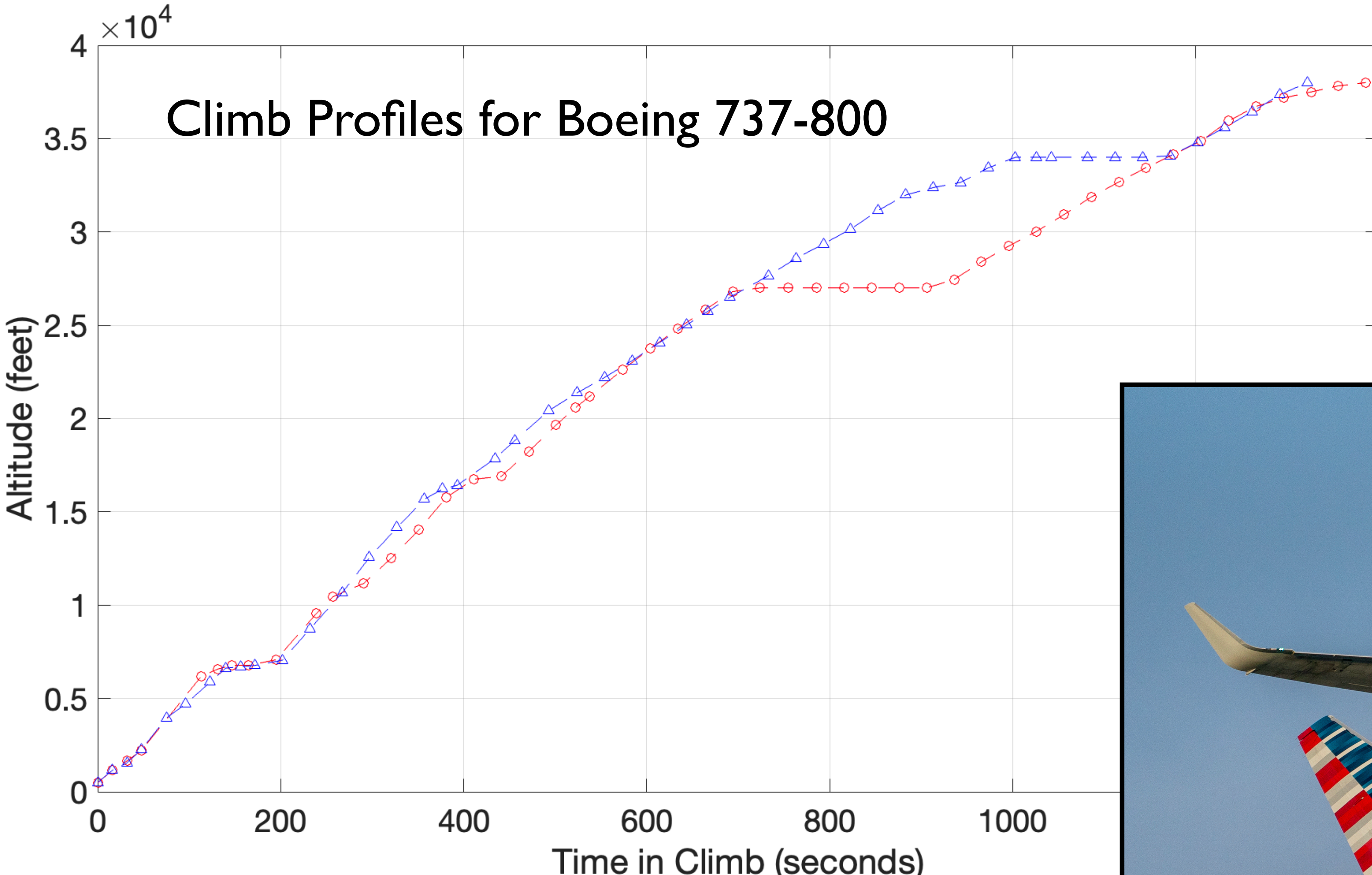
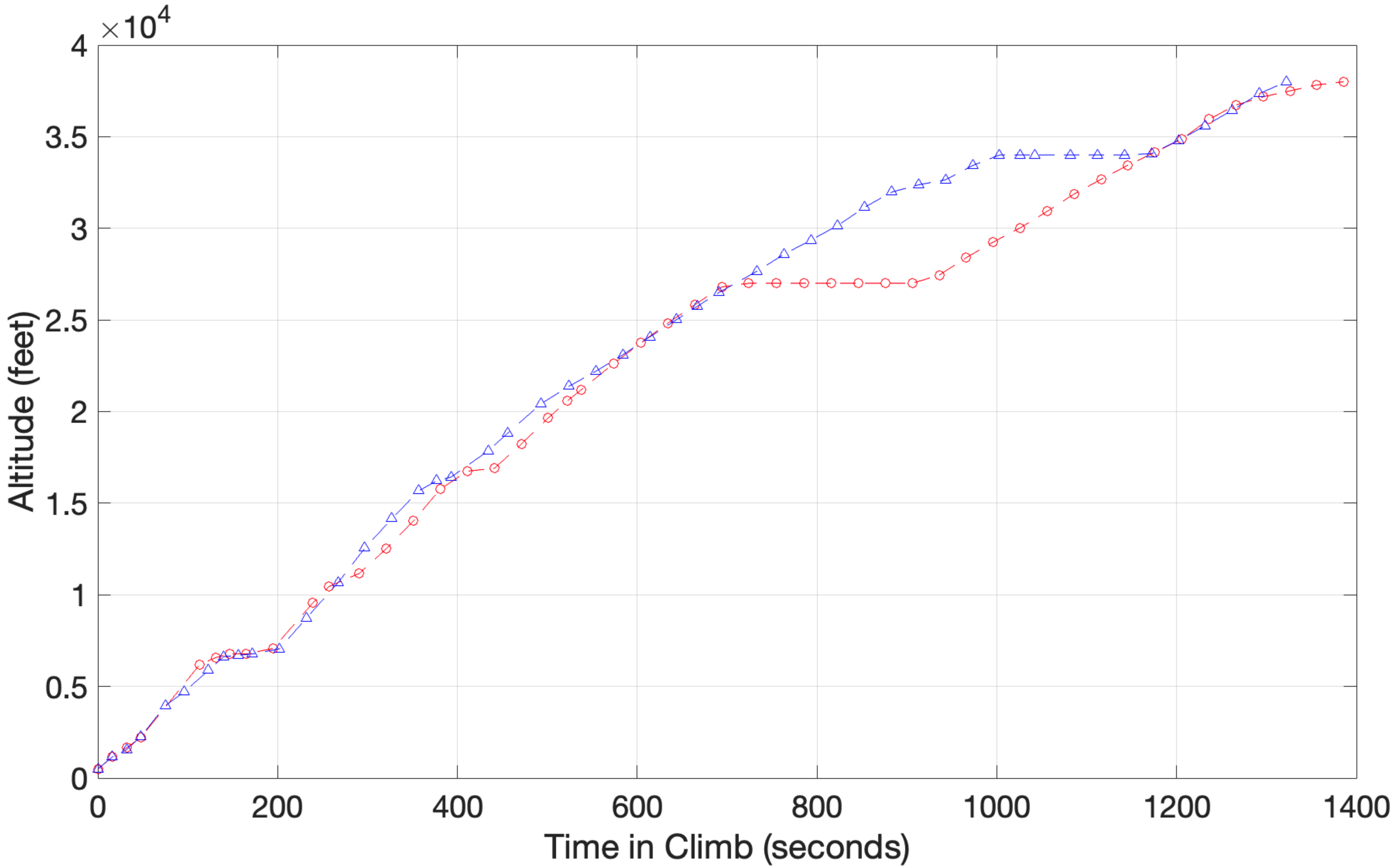
CEE 5614 - Analysis of Air Transportation Systems
Aircraft Performance Notes 2
Spring 2024



Boeing 737-800

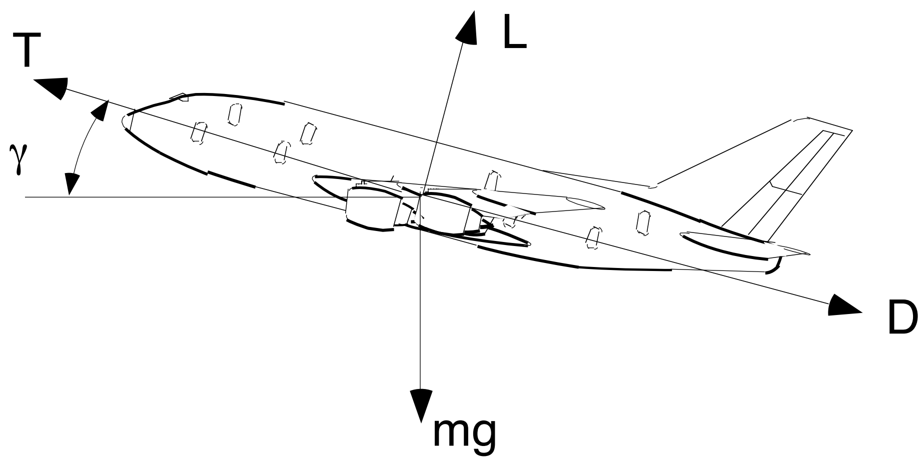
Differences in Climb Profiles

- Boeing 737-900 climb profiles from DCA to AUS



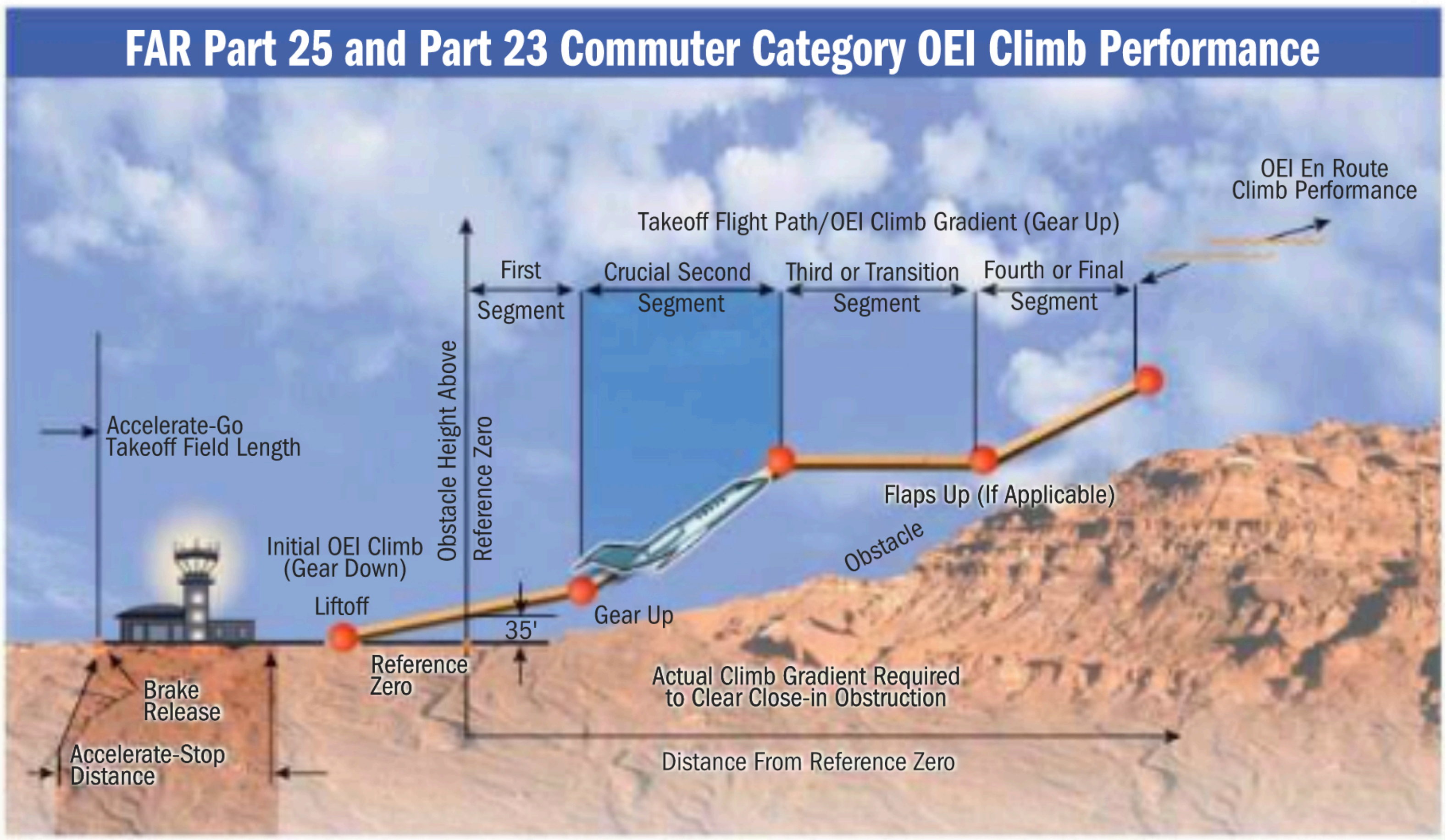
Climb Performance

Many airport and airspace simulation models employ simplified algorithms to estimate aircraft climb performance in the terminal area.



Climb Segments

- Explain regulations that apply to 14 CFR Part 23 and 25 aircraft under One Engine Inoperative (OEI) conditions
- Explanation of climb segments used in aircraft certification

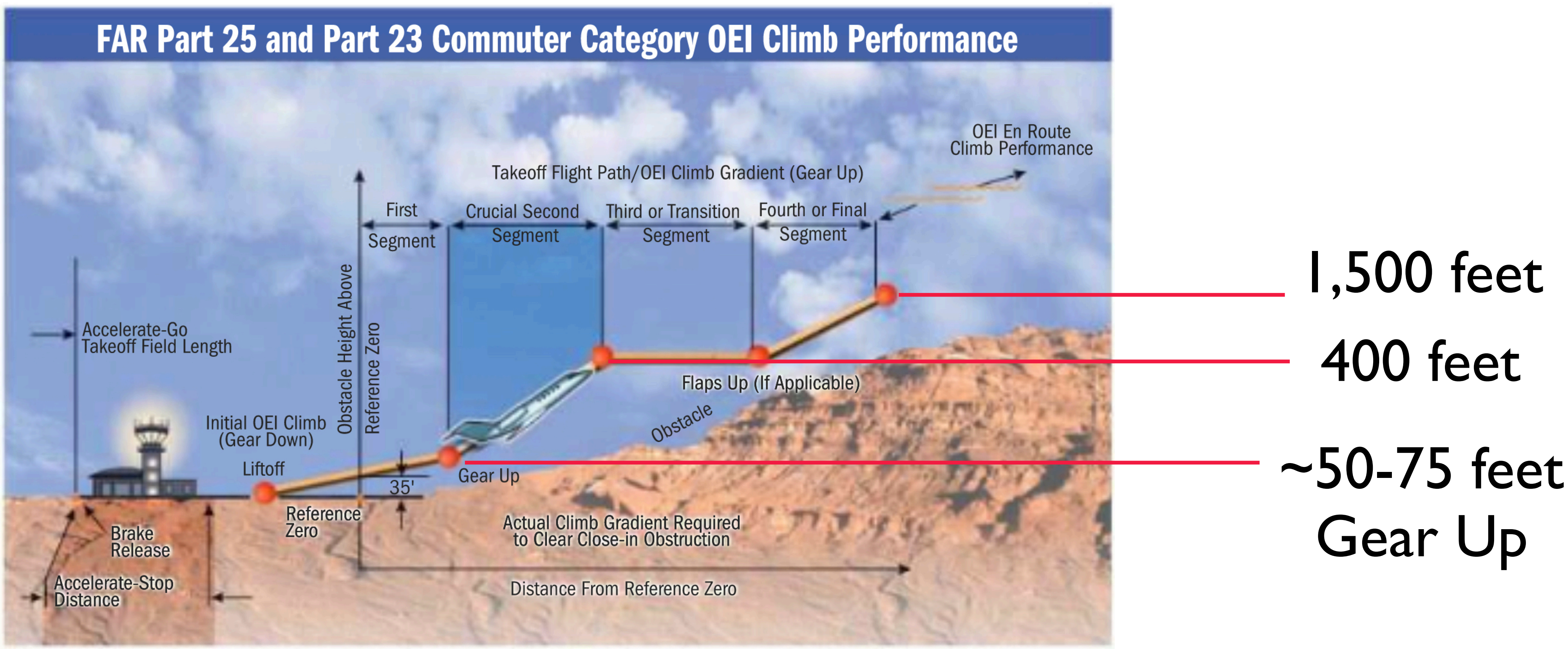


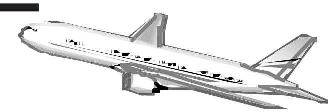
Recall:
14 CFR Parts 23 and 25 are regulations that apply in the certification of aircraft

Source: Business and Commercial Aviation

Climb Segment Requirements

Type	4-Engine Aircraft	3-Engine Aircraft	2-Engine Aircraft
Minimum Climb Gradient (14 CFR Part 25.121	3.0%	2.8%	2.4%





Basic Climb Performance Analysis

The basic equations of motion along the climbing flight path and normal to the flight path of an air vehicle are:

$$m\frac{dV}{dt} = T - D - mg\sin\gamma \qquad (25)$$

$$m\frac{d\gamma}{dt}V = L - mg\cos\gamma \qquad (26)$$

where: m is the vehicle mass, V is the airspeed, T and D are the tractive and drag forces, respectively; γ is the flight path angle. L is the lift force and $mg\cos\gamma$ is the gravitational component normal to the flight path.

Climb Performance Model Simplifications

For small γ (flight path angle):

$$\sin\gamma = \frac{T - D}{mg} - \frac{1}{g}\frac{dV}{dt} \qquad (27)$$

where: the first term in the RHS accounts for possible changes in the potential state of the vehicle (i.e., climb ability) and the second terms is the acceleration capability of the aircraft while climbing. Further algebraic manipulation yields,

$$V\sin\gamma = \frac{dh}{dt} = \frac{V[T - D]}{mg} - \frac{V}{g}\frac{dV}{dt}$$

where: dh/dt is the rate of climb and V is the true airspeed. Note that if one neglects the second term (acceleration factor) assuming small changes in V as the vehicle climbs one can easily estimate the rate of the climb of the vehicle for a prescribed climb schedule.

Incorporation of a Parabolic Drag Polar Model

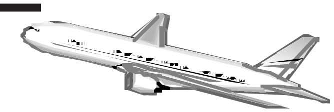
Let lift and drag be expressed in the simple parabolic form,

$$L = \frac{1}{2}\rho S C_L V^2 \qquad (28)$$

$$D = \frac{1}{2}\rho S C_D V^2 \qquad (29)$$

where: C_L and C_D are the lift and drag coefficients (nondimensional), V is the airspeed, S is the wing area (reference area) and ρ is the density of the air surrounding the vehicle.

Final Derivation of Climb Rate Expression



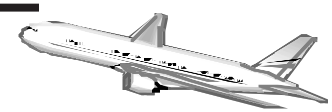
The functional form of the lift and drag coefficients (C_L, C_D) in its simplest form is,

$$C_D = C_{D0} + C_{Di} = C_{D0} + \frac{C_L^2}{\pi A R e} \qquad (30)$$

$$C_L = \frac{2mg}{\rho S V^2} \qquad (31)$$

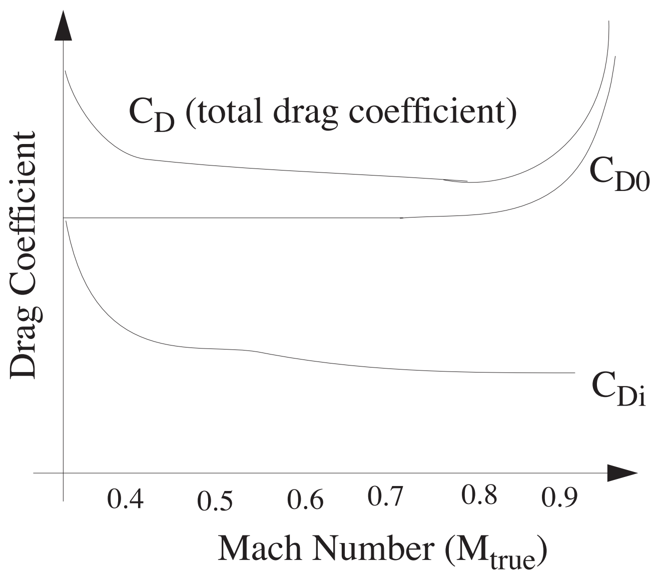
where: C_{D0} is the zero lift drag coefficient, and the second drag term accounts for drag due to lift generation (i.e., induced drag).Then the rate of climb function becomes,

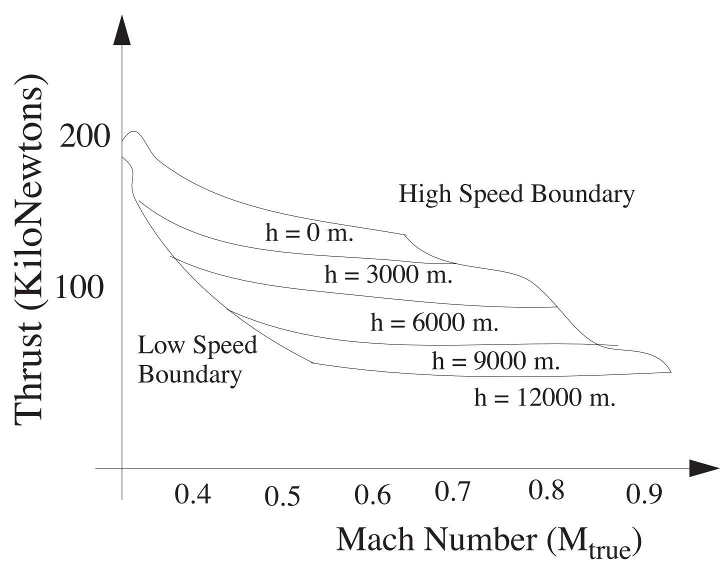
$$\frac{dh}{dt} = \frac{V\left[T(\rho, V) - \frac{1}{2}\rho V^2 S\left\{C_{D0}(M) + \frac{C_L^2(M, V)}{\pi A R e}\right\}\right]}{mg} \qquad (32)$$



Mathematical Approximation for Aircraft Thrust and Drag

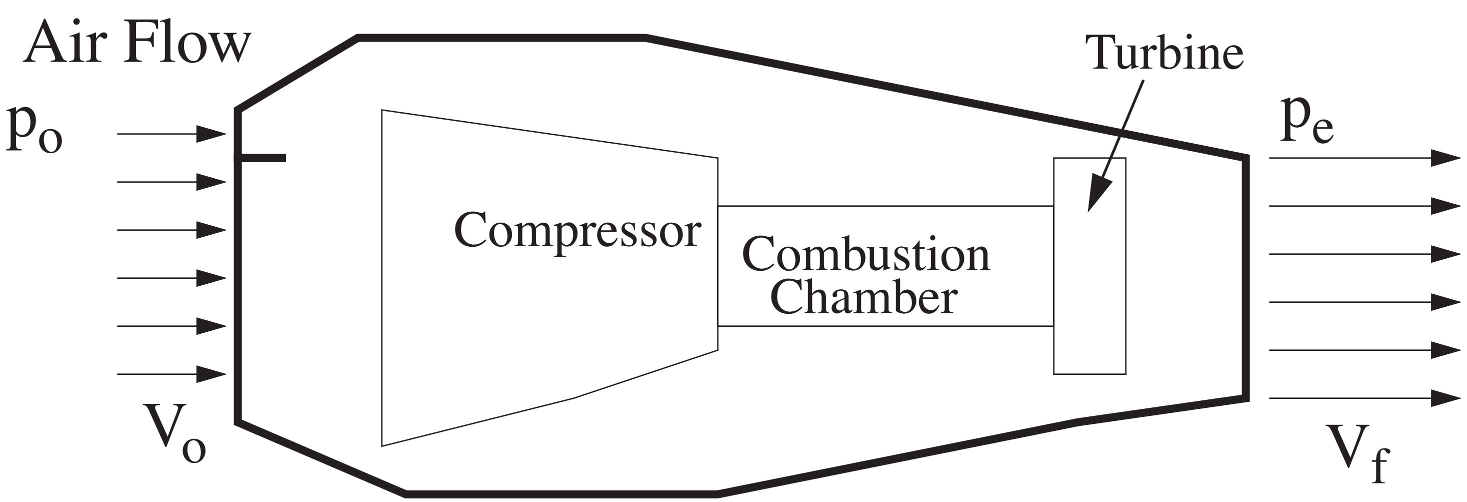
Thrust and drag are two fundamental variables extracted from wind-tunnel and flight tests.



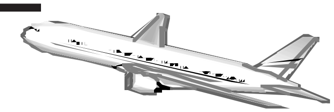


Modeling Aircraft Thrust

- Thrust is a function of aircraft speed and altitude
- Basic thermodynamics dictates that thrust is the net result of the speed differential between inlet and outlet of the engine



$$T = f(V_f - V_o) \text{ and } f(p_o - p_e)$$

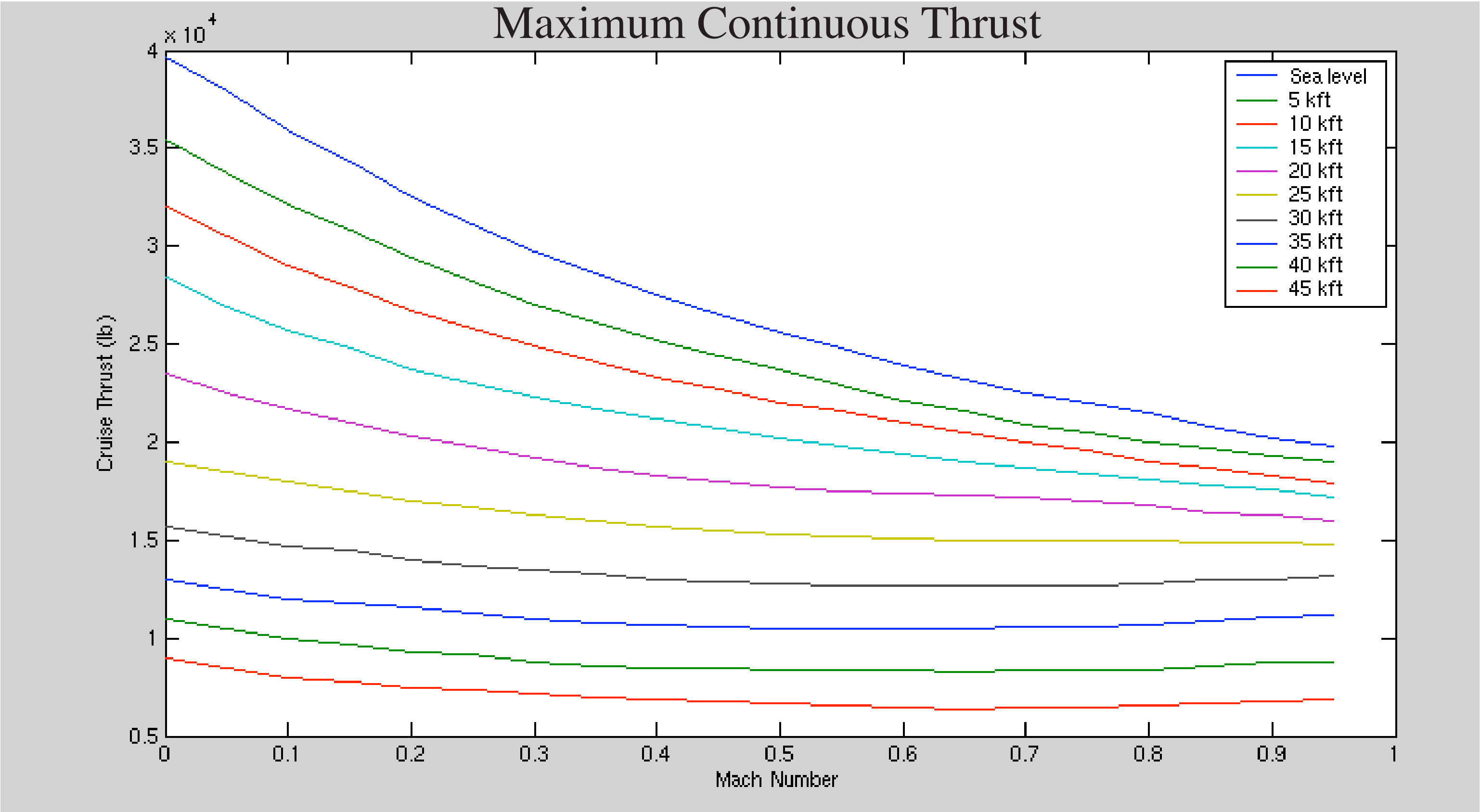


Basic Propulsion Forces Modeling Ideas

- Thrust is a function of altitude (or density)
  - A general thrust lapse function can be obtained using real engine data (empirical data)
- Thrust is a function of aircraft speed
  - A more complex function can be obtained using real engine data (empirical data)
  - The thrust losses during takeoff troll are significant as illustrated in the figures below
- Thrust functions are provided by the engine manufacturer in terms of tables (thrust vs altitude and mach number and thrust specific fuel consumption vs altitude and mach number)

Sample Thrust Variations (PW JT9D Engine)

Observe the large variations of thrust with respect to aircraft altitude



Modeling Thrust Using a Thrust Lapse Gradient

A simple way to model thrust as a function of altitude is presented below:

$$T_h = T_0 \left(\frac{\rho_h}{\rho_0} \right)^m \qquad (33)$$

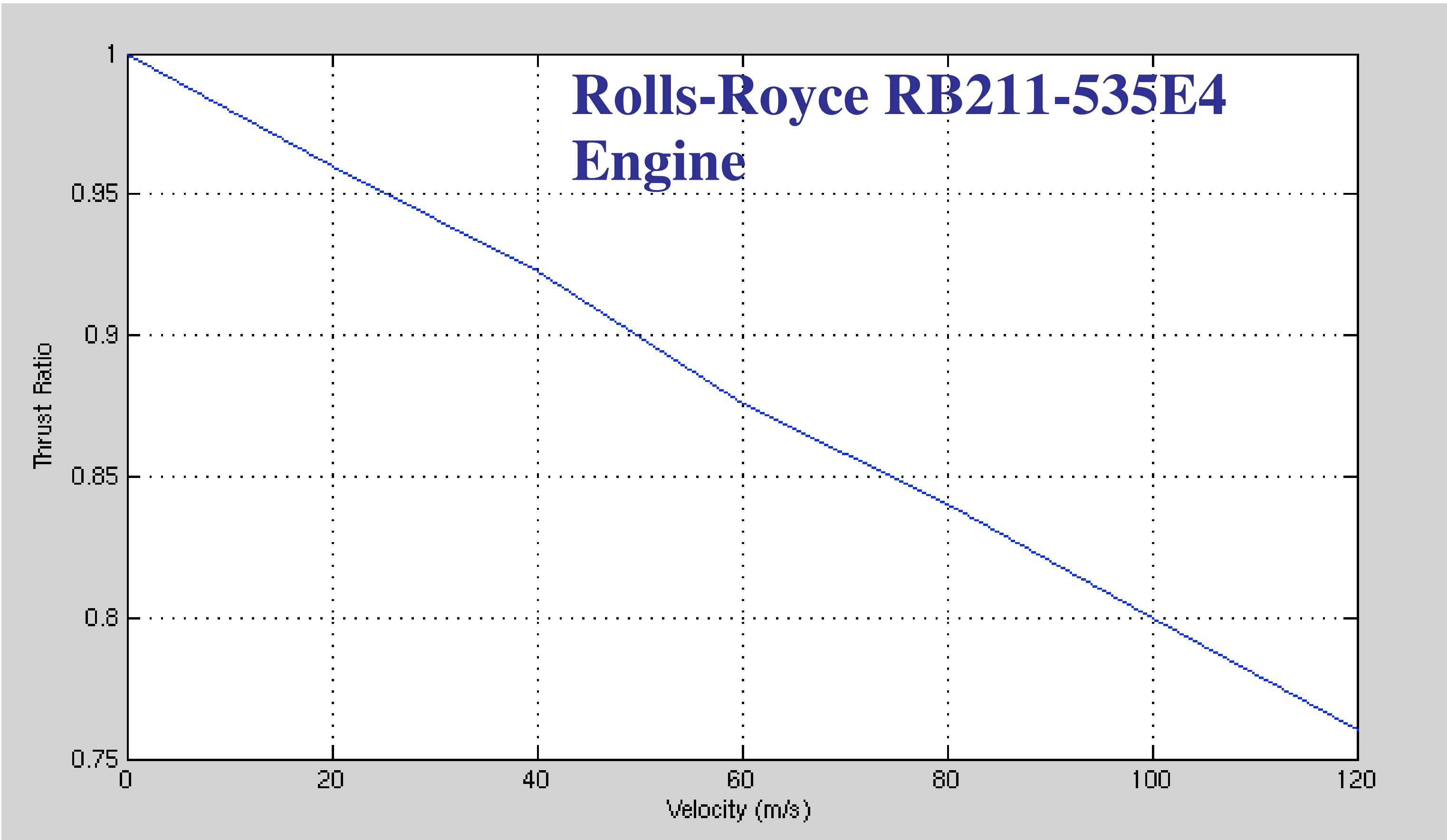
where:

T_h is the thrust at altitude, T_0 is the sea level static thrust,

ρ_h and ρ_0 are the density values at altitude and at sea level, respectively

m is an empirical coefficient derived from real data

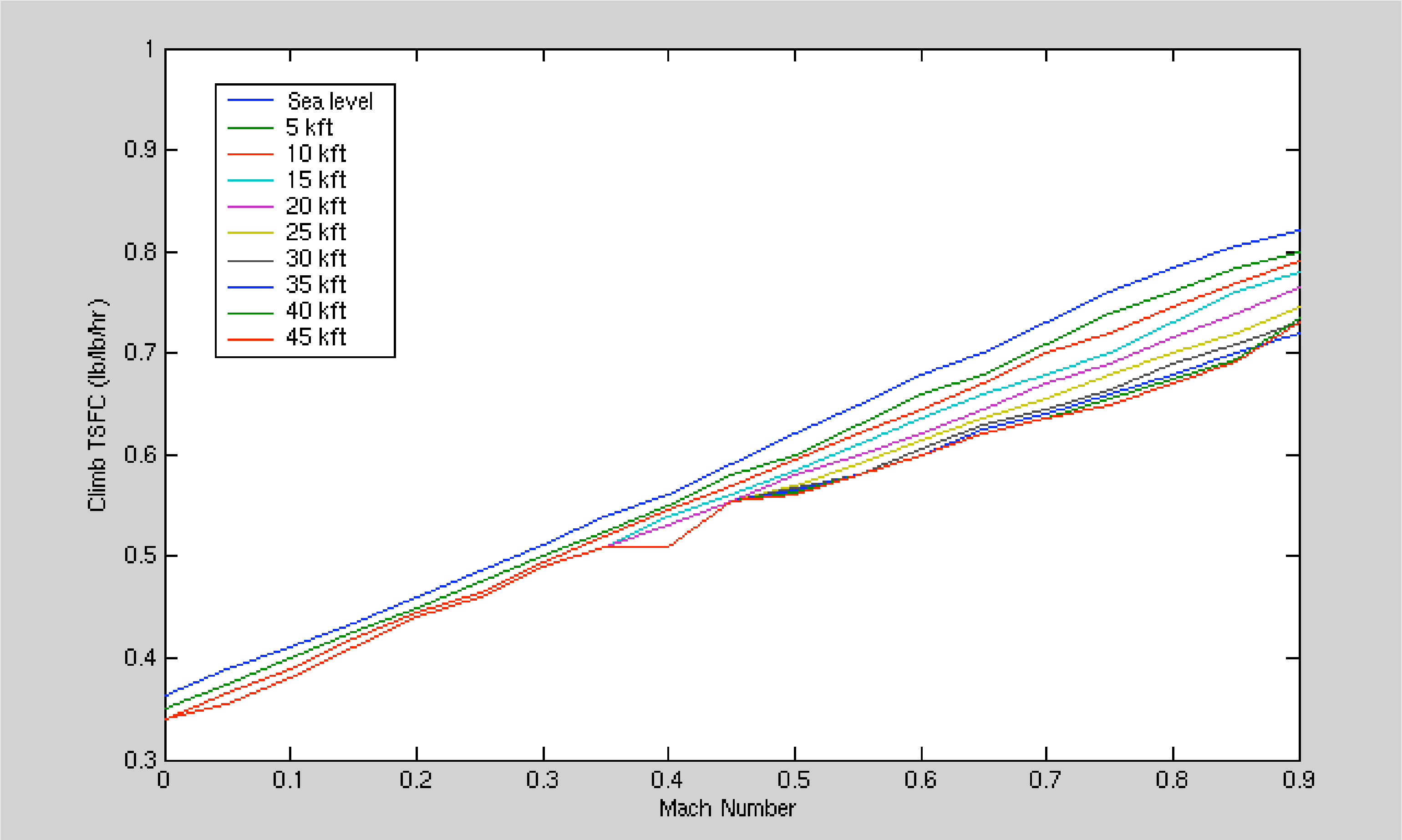
Takeoff Thrust Variations with Speed



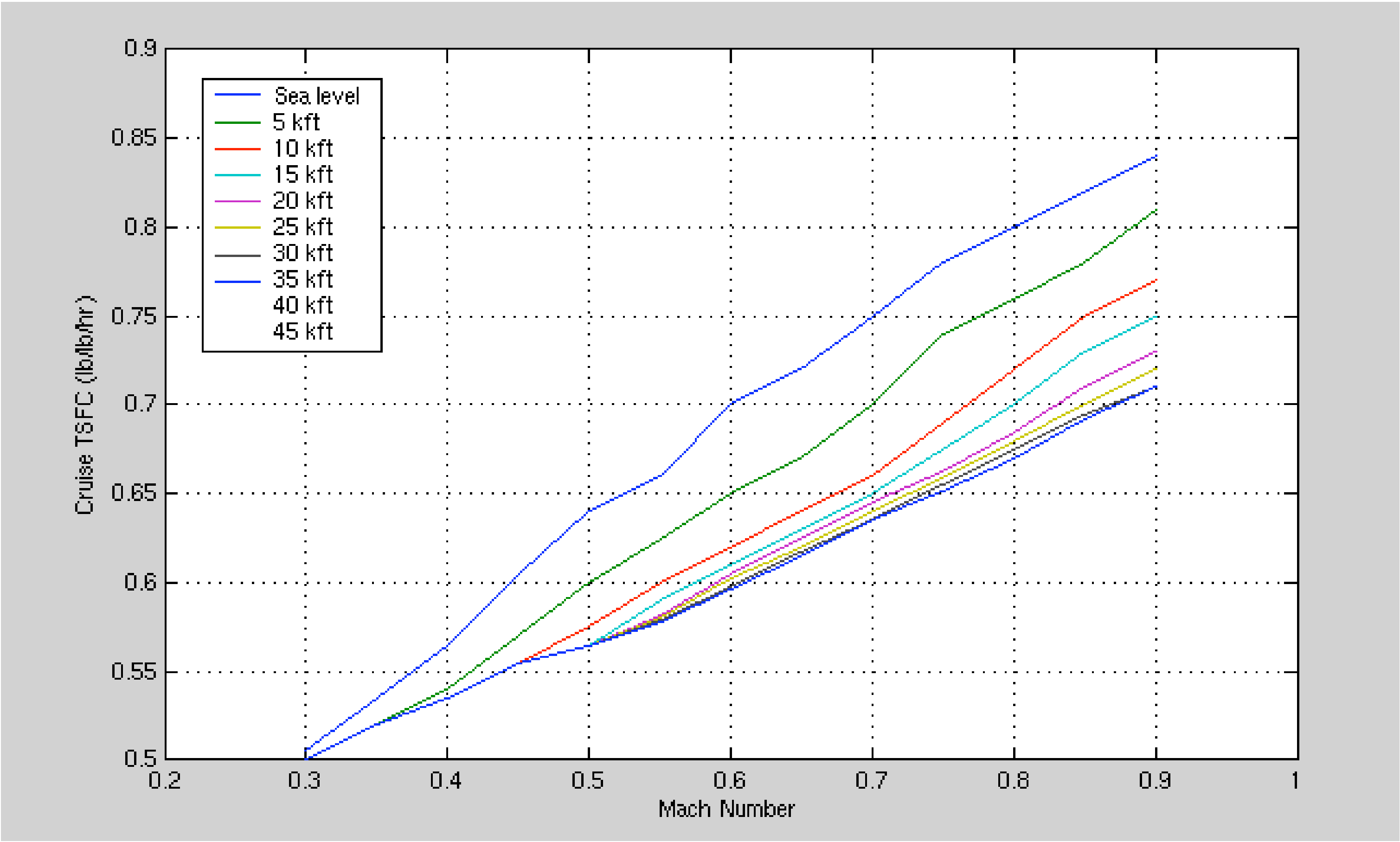
Thrust ratio = T_v / T_{static}

Source: Mair and Birdsall (1992)

Variations of Climb TSFC (PW JT9D Engine)



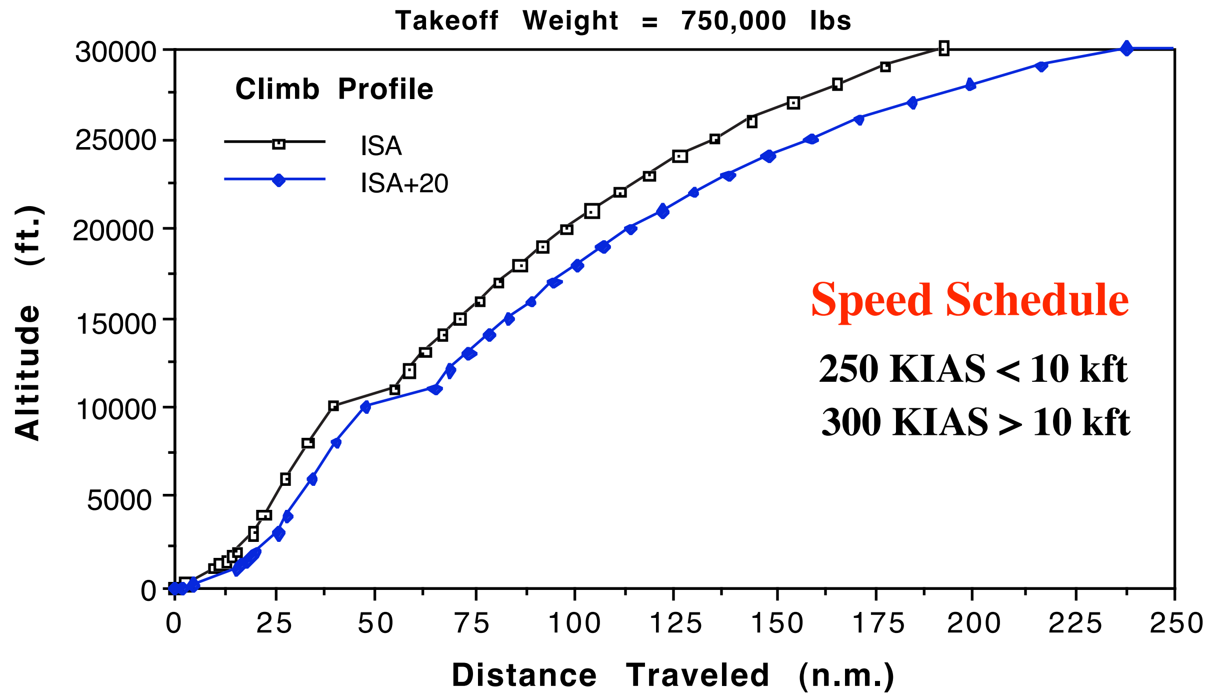
Variations of Cruise TSFC (PW JT9D Engine)

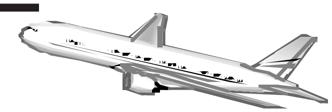


Sample Climb Trajectory Results

Numerical integration of equation (30) for a given flight speed schedule (speed time history) yields the following climb profiles.

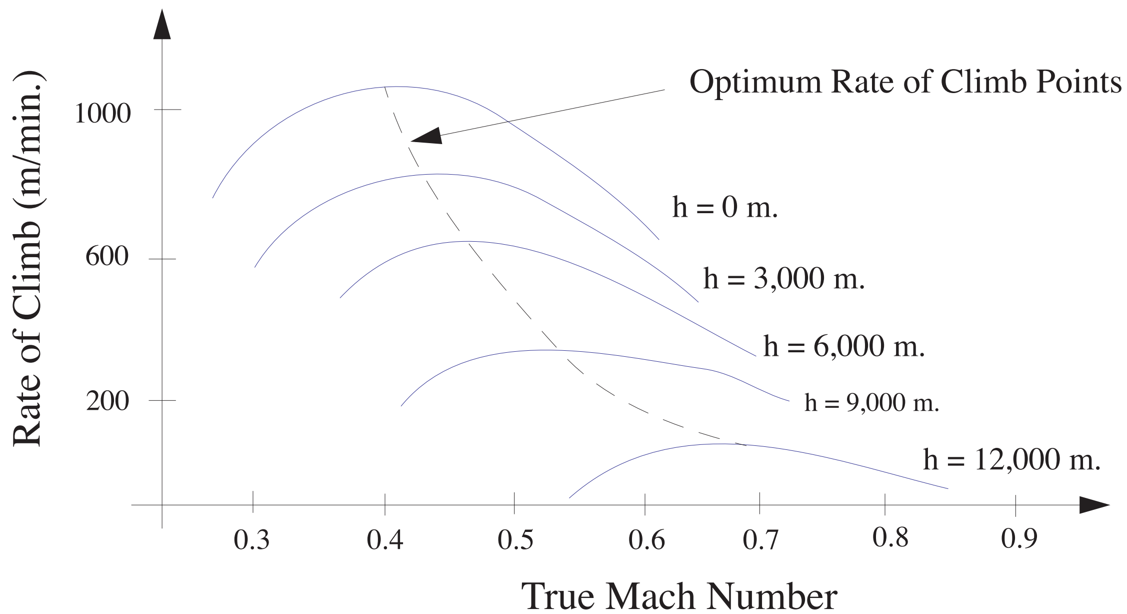
Four engine, turbofan-powered aircraft





Typical Rate of Climb Envelope

Iterative analysis of the rate of climb equation yields the following results across the complete flight envelope.



Example of Aircraft Climb Performance

The following example gives an idea of the typical procedures in the estimation of the aircraft climbing performance. Assume that a heavy transport aircraft has drag polar of the form,

$$C_D = C_{Do} + \frac{C_L^2}{\pi A R e}$$

where: AR = 8.0, e =0.87 and C_{DO} (the zero lift drag coefficient) varies according to true airspeed (TAS) according to the following table:

Mach Number	**C_{DO} (nondimensional)**
0.0 to 0.75	0.0180
0.80	0.0192

Mach Number	C_{DO} (nondimensional)
0.85	0.023
0.90	0.037
0.95	0.038
1.00	0.040

The engine manufacturer supplies you with the following data for the engines of this aircraft:

True Airspeed (m/s)	Sea Level Thrust (Newtons)
0	250,000
300	150,000

For simplicity assume that thrust variations follow a linear behavior between 0 and 300 m/s. The thrust also decreases with altitude according to the following simple thrust lapse rate equation,

$$T_{altitude} = T_{Sea\ Level} \, (\rho/\rho_o)^{.90} \qquad (1)$$

where ρ is the density at altitude h and ρ_o is the sea level standard density value (1.225 kg./ m^3).

The aircraft in question has four engines and has a wing area of 525 m^2.

A) Calculate the thrust and drag for this vehicle while climbing from sea level to 10,000 m. under standard atmospheric conditions at a constant indicated airspeed of 280 knots. Simulate the climb performance equation of motion assuming that the takeoff weight is 360,000 kg.

B) Estimate the rate of climb of the vehicle if the fuel consumption is approximately proportional to the thrust as follows,

$F_c = TSFC\ (T)$

where $TSFC = 2.1 \times 10^{-5}$ (Kg/second)/Newton

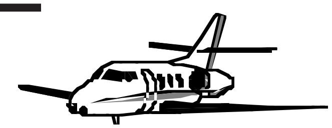
C) Find the time to climb and the fuel consumed to 10,000 m.

D) What is the approximate distance traveled to reach 10,000 m. altitude?

Solution

- The process to estimate the complete climb profile for the aircraft is best done in a computer. There are numerous computations that need to be repeated for each altitude.
- A suitable algorithm to solve the equations of motion of the aircraft over time is presented in the following pages.

Computational Algorithm Flowchart



Initial Aircraft States
Mass (W_o), Altitude (h_o)
and Distance Traveled (S_o)

Given: Speed profile
Typically V as a
function of Altitude (h)

1) Compute Atmospheric values for a given altitude (density, speed of sound, etc.) — From Table

2) Compute lift coefficient (C_L)
C_L = f (mass, density, wing area, etc.) — Equation (31)

3) Compute the drag coefficient (C_D)
C_D = f (C_L, Mach, AR, e, etc.) — Equation (30)

Computational Algorithm (contd.)

Iterate

4) Compute total drag (D)

$D = f\ (C_D, V, S, \text{density})$

Equation (29)

5) Compute the thrust produced (T)

$T = f\ (\text{Mach and density})$

Equation (33)

6) Compute the rate of climb (*dh/dt*)

$dh/dt = f\ (T, D, V)$

Equation (32)

7) Compute fuel burn (*dW/dt*)

$dW/dt = -\ TSFC\ (T)$

ΔT is a suitable step size (say 5 seconds)

Computational Algorithm (contd.)

8) Compute the distance traveled (dS/dt)

$$dS/dt = V * \Delta T$$

9) Update new aircraft altitude (h)

$$h_t = h_{t-1} + \Delta T\,(dh/dt)$$

10) Update new aircraft mass (W)

$$W_t = W_{t-1} + \Delta T\,(dW/dt)$$

11) Update new aircraft distance (S)

$$S_t = S_{t-1} + \Delta T\,(dS/dt)$$

ΔT is a suitable step size (say 5 seconds)

Solution Using Numeric Software Packages

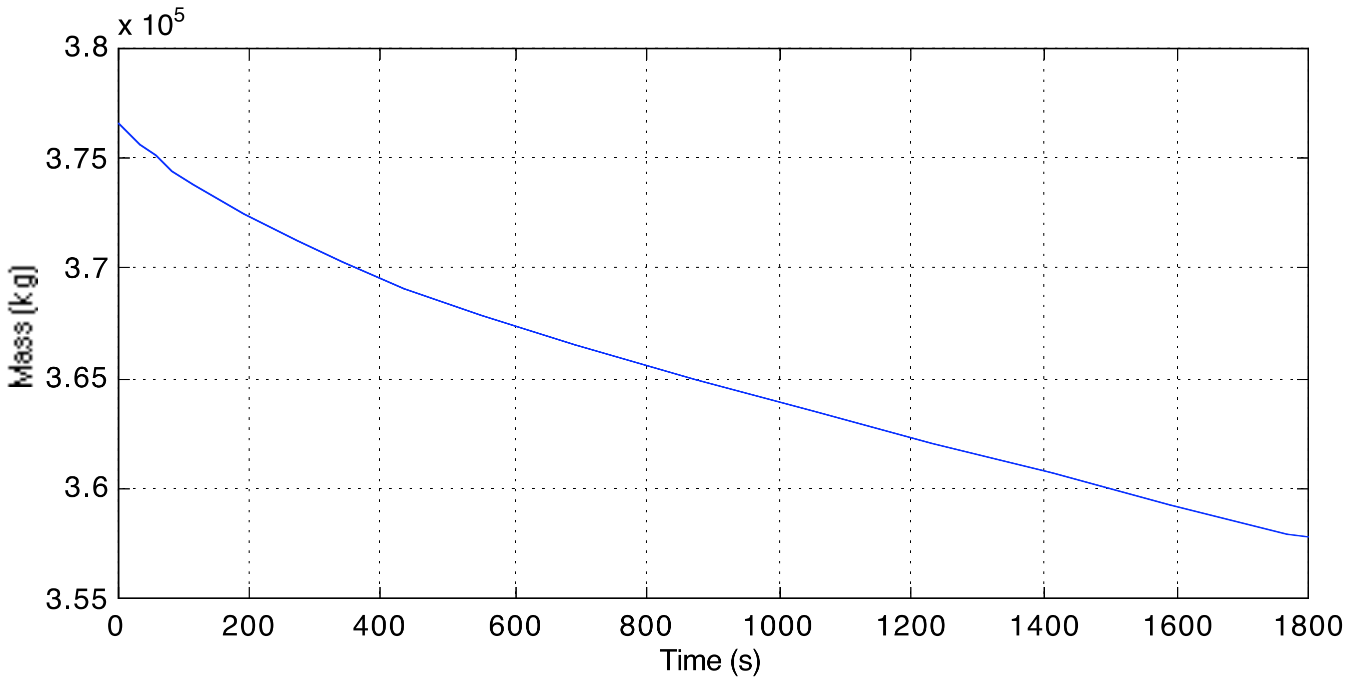
- Several engineering packages can perform these computations quickly and easily (Matlab, Mathematica, Mathcad, etc.)
- All of them have differential equation solvers that can be used in this analysis
- The source code to solve this problem is presented in Matlab at the course web site: http://128.173.204.63/courses/cee5614/syllabus_ce_5614.html
- The process can also be implemented in a standard Spreadsheet application like Excel

Computational Results

- Note that the aircraft takes about 25 minutes to climb to 10,000 m. and that the rate of climb is near zero at that altitude.
- The time solution for fuel consumption indicates that this aircraft consumes about 20 metric tons in the climb segment as shown in the figure.
- Note that this amount is reasonable considering that a the four engine aircraft carries up to 175 metric tons of fuel.

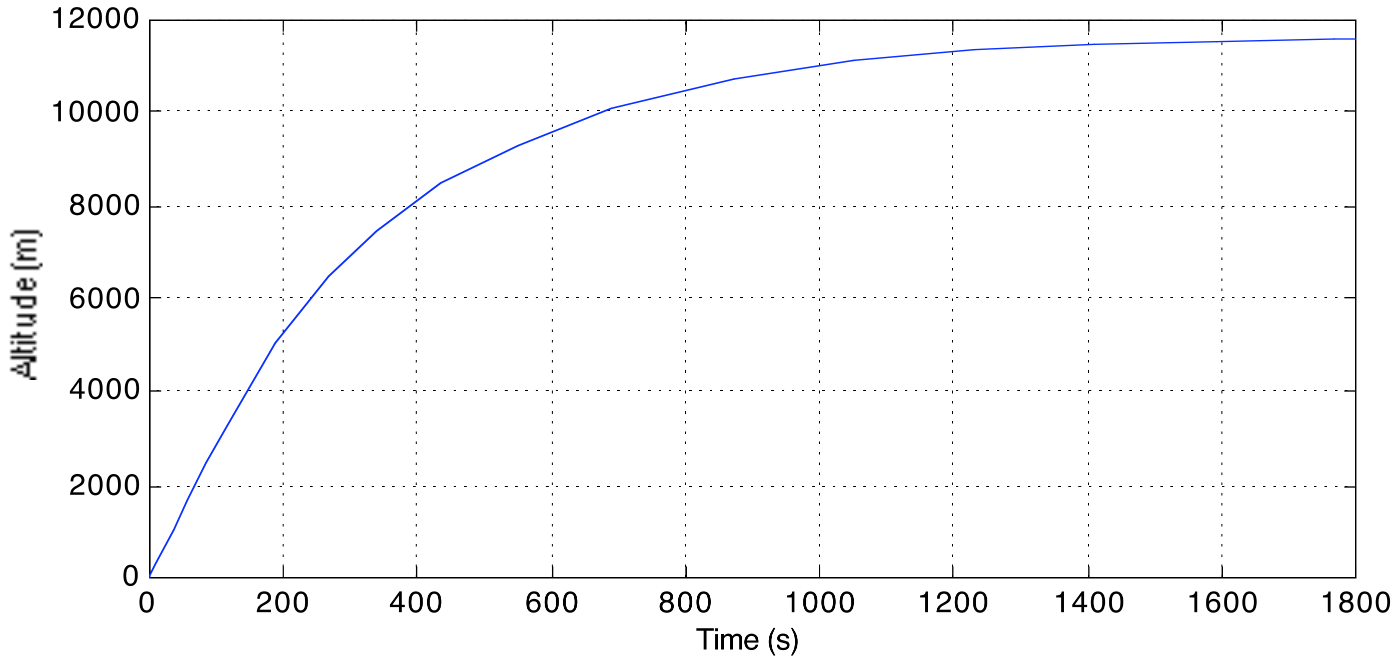
Climb Performance Estimation Results

The diagram illustrates the changes to aircraft mass as a function of time (dW/dt) for the hypothetical four-engine transport aircraft modeled



Climb Performance Estimation Results

The diagram illustrates the changes to aircraft altitude as a function of time for the hypothetical four-engine transport aircraft modeled



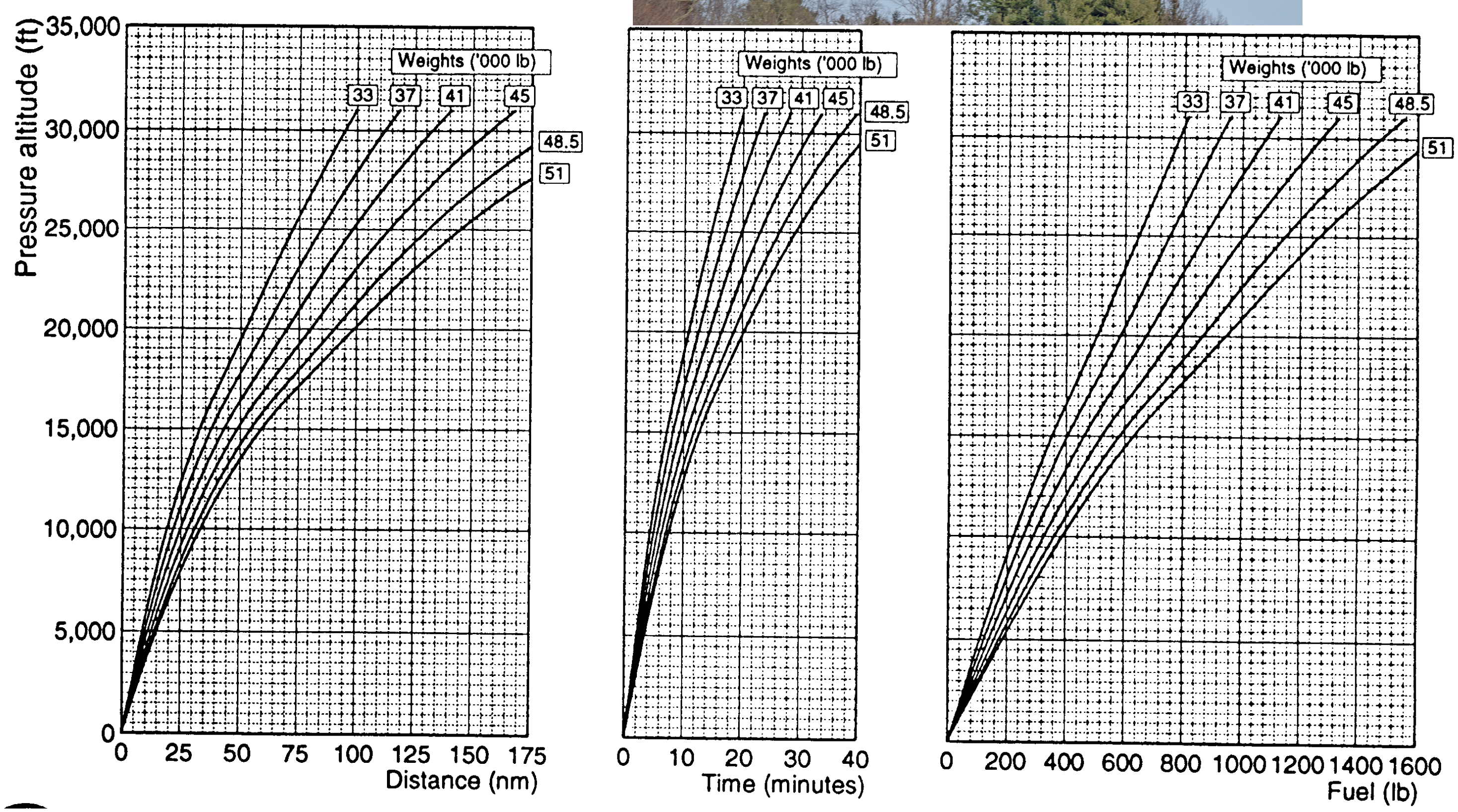
Climb Performance Presentation Charts

The previous discussion presented the foundations of the theoretical climb performance. In practice aircraft manufacturers and airlines present climb performance in graphical and tabular format. The figure below presents climb information for a Swedish-made Saab 2000 - a commuter aircraft powered by two turbo-propeller driven engines.

Sample Climb Estimation Presentation Charts

Saab 2000

ISA + 15^0 C
240 Knots IAS / 0.5 Mach
Anti-Ice Off



source: Saab 2000 aircraft flight manual

Detailed Example of Aircraft Performance Calculations: Climb Performance

CEE 5614
Analysis of Air Transportation Systems

Dr. Antonio A. Trani
Professor

Example - Aircraft Climb Performance

- Use the the vehicle characteristics for the very large capacity transport aircraft in the Matlab files for CEE 5614 to solve this problem (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)
- Estimate the rate of climb for this aircraft at two distinct points in the climb profile:
  - a) 600 meters (2,000 feet) and 210 knots - IAS
  - b) 8,000 meters (26,200 feet) and 290 knots - IAS
- Estimate the thrust produced by the engines under both conditions
- Find the Lift to Drag ratio for both conditions
- Assume the International Standard Atmosphere applies to both aircraft states

Example - Aircraft Climb Performance Data File

- Very large capacity transport aircraft (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)

```
% Aircraft file to support other computational modules
% Aircraft = Similar to Airbus A380 (heavy transport)

global A e S neng tsfc macht Cdoct mass thrust_table mach_table lapse_rate_factor Vclimb altc g

S = 858;              % wing area (square m)
A = 9.0;              % Wings aspect ratio
e = 0.84;             % Oswald's efficiency factor
g = 9.81;
neng = 4;                  % Number of engines
tsfc = 1.6e-4;             % thrust specific fuel consumption (N/N/s)
mass = 540000;            % mass at operating point (kg)
```

geometric, mass and specific fuel consumption

```
% Drag charactericties - CDO function (zero lift drag function)

Cdoct  = [0.020   0.020  0.0204    0.022  0.037  0.038  0.040];
macht  = [0.0    0.75   0.80      0.85   0.90   0.95   1.00];
```

drag data

```
% Thrust parameters for Eclipse aircraft (at sea level)

thrust_table = [338000 180000];    % Thrust limits (sea level in Newtons)
mach_table = [0.0  0.9];                % mach number limits to bound thrust
lapse_rate_factor = 0.96;               % thrust lapse rate factor
```

engine thrust data

```
% Computes the aircraft profile given altitude (Vcas given) - typical for
% four engine aircraft similar to the Airbus A380 aircraft

Vclimb = [180 200  220 250 270  280  290  300  300  300  300  300 300  300  300];     % knots IAS
Vdescent = [ 300 300  300 300 300  300  300  300  300  300  300  300 300  300  300];  % knots IAS
altc = [ 0 1000  2000 3000 4000  5000  6000  7000  8000  9000  10000  11000 12000  13000  14000]; % altitude (meters)
```

speed profile data

Example - Aircraft Climb Performance Controlling the Speed Profile

- Very large capacity transport aircraft (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)
- The aircraft speed is controlled by the last **three lines of data** in the aircraft data file
- Line 1 (see below) defines the climb speed in knots (indicated airspeed)
- Line 2 defines the descent speed in knots (IAS)
- Line 3 defines the altitudes at which each speed value is selected
- For example: The aircraft below climbs at 210 knots just after takeoff (zero altitude) and reaches 250 knots at 4000 meters

```
% Computes the aircraft profile given altitude (Vcas given) – typical for
% four engine aircraft similar to the Airbus A380 aircraft

Vclimb = [210 210  220 230 250  260  290  290  290  290  290  300 300  300  300];      % knots IAS
Vdescent = [ 300 300  300 300 300  300  300  300  300  300  300  300 300  300  300];   % knots IAS
altc = [ 0 1000  2000 3000 4000  5000  6000  7000  8000  9000  10000  11000 12000  13000  14000]; % altitude (meters)
```

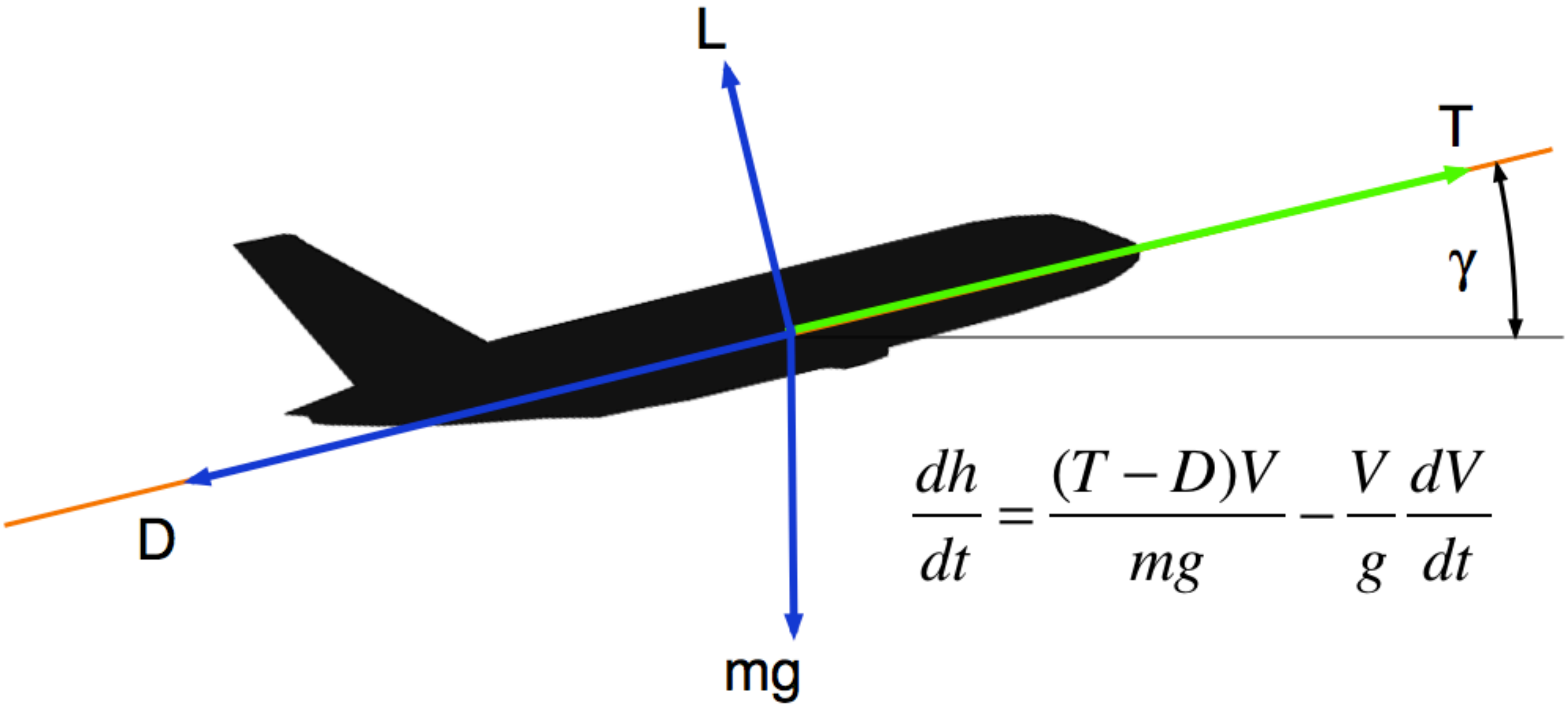
Example - Very Large Capacity Aircraft Data File

- An aircraft similar in size and performance as the Airbus A380
- Four turbofan engines each developing 34,400 kg (338,000 N) at sea level
- Maximum takeoff mass is 540,000 kg. (1.188 million pounds)

Airbus A380 taxies to the gate at LAX (A.A. Trani)

Example - Aircraft Climb Performance
Picture the Situation

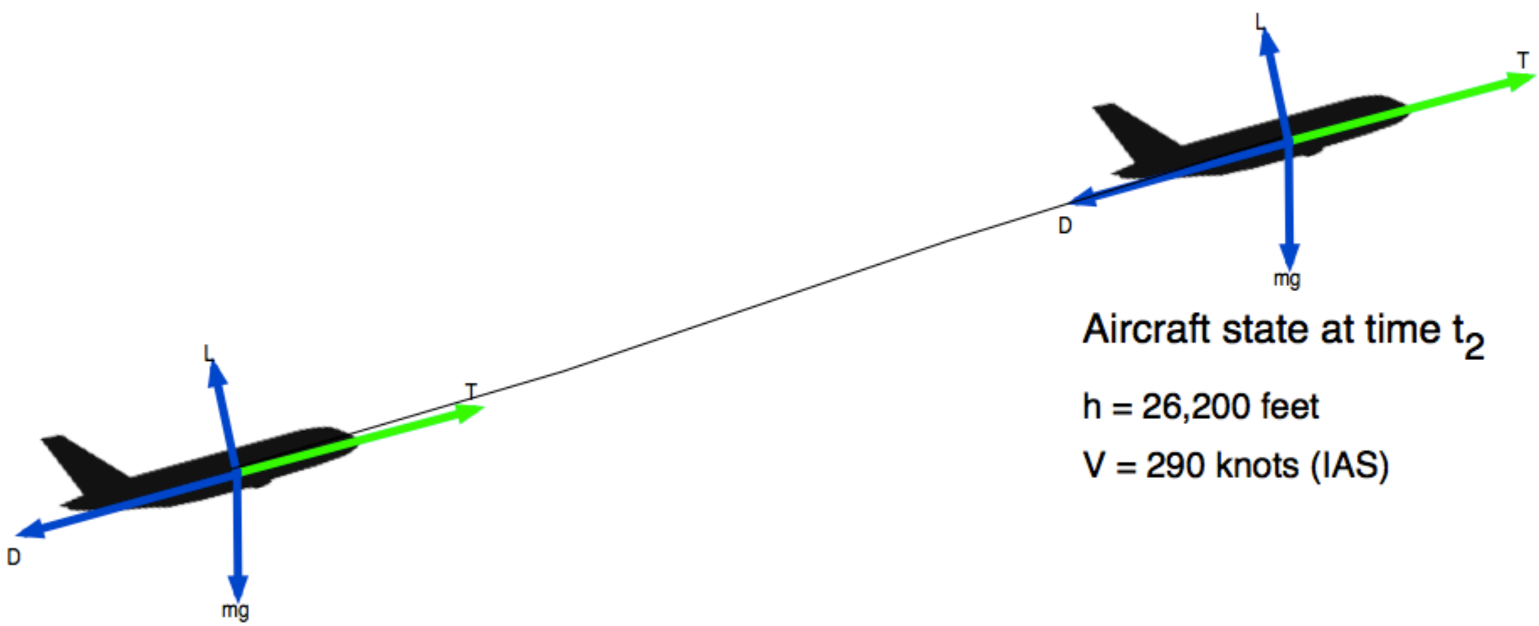
- Always picture the situation and sketch a free body diagram of the system
- For this analysis we will ignore the second term in the Right Hand Side (RHS) of the differential equation (acceleration term)
- This simulates that the pilot is interested in climbing as fast as possible and thus using all the engine thrust to climb



$$\frac{dh}{dt} = \frac{(T-D)V}{mg} - \frac{V}{g}\frac{dV}{dt}$$

Calculation of Performance at two points in the Climb Profile

- The analysis assumes the aircraft is studied as a point mass system. We evaluate the performance at two discrete points
- The analysis can be repeated many times to study time to climb, fuel used, and other metrics



Aircraft state at time t_2

h = 26,200 feet

V = 290 knots (IAS)

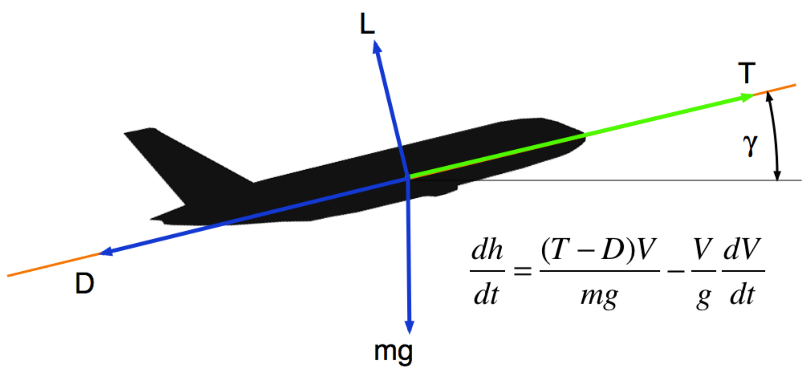
Aircraft state at time t_1

h = 2,000 feet

V = 210 knots (IAS)

Calculation Procedure

- Step 1: Estimate true airspeed using atmospheric model
- Step 2: Estimate the lift coefficient needed to sustain flight using the basic lift equation
- Step 3: Estimate drag coefficient
- Step 4: Estimate total drag (D)
- Step 5: Estimate the thrust produced by the engines at altitude (T)
- Step 6: Find the rate of climb (dh/dt)



$$\frac{dh}{dt} = \frac{(T-D)V}{mg} - \frac{V}{g}\frac{dV}{dt}$$

Aircraft Climb Performance
2,000 feet and 210 knots IAS

- Using the standard expression to estimate the true mach number of the aircraft at altitude,

$$M_{true} = \sqrt{5\left[\left\{\frac{\rho_0}{\rho}\left(\left[1+0.2\left(\frac{V_{IAS}}{661.5}\right)^2\right]^{3.5}-1\right)+1\right\}^{0.286}-1\right]}$$

- The true mach number is 0.3267, the speed of sound at 600 meters is 337.96 m/s and the density of air is 1.156 k/cu. m.
- The true airspeed (TAS) is 110.41 m/s or 214.6 knots
- Use the fundamental lift equation to estimate the lift coefficient under the known flight condition

$$L = mg = \frac{1}{2}\rho V^2 S C_l \longrightarrow C_l = \frac{2mg}{\rho V^2 S}$$

Aircraft Climb Performance
2,000 feet and 210 knots IAS

- The lift coefficient needed to maintain flight is,

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2*(540{,}000)(9.81)}{(1.1560)(110.42)^2(858)} \frac{(kg)(m/s^2)}{(kg/m^3)(m/s)(m^2)} = 0.8761$$

- The lift coefficient is non-dimensional
- The drag coefficient can be calculated using the standard parabolic drag polar model

$$C_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi A \mathrm{Re}} = 0.020 + \frac{0.8761^2}{\pi(9.0)(0.84)} = 0.0523$$

- Note that the value of C_{do} is found by interpolation in the table function relating C_{do} and Mach number (C_d is non-dimensional)

Aircraft Climb Performance
2,000 feet and 210 knots IAS

- The total drag is,

$$D = \frac{1}{2}\rho V^2 S C_d = \frac{1}{2}(1.156)(110.42)^2(858)(0.0523) = 316{,}340\,N$$

- The calculated drag has units of Newtons (verify by yourself)

- The thrust produced by all 4 engines in the very large capacity transport is estimated using the simple linear model

$$T_{0,M} = T_{0,M=0} - \lambda M_{true}$$

$$T_{h,M} = T_{0,M}\left(\frac{\rho_h}{\rho_0}\right)^m$$

- The first expression estimates the thrust at sea level (hence subscript 0) at any mach number

- The second expression corrects the thrust developed for any altitude (h)

Definition of Terms to Estimate Engine Thrust

$T_{0,M}$ = Thrust at sea level and at Mach number M (Newtons)

$T_{0,M=0}$ = Thrust at sea level and at Mach = 0 (zero speed) (Newons)

λ = Rate of change of thrust vs. Mach number (lapse rate) (Newton/Mach)

M_{true} = True mach number (dimensionless)

$T_{h,M}$ = Thrust at altitude h and Mach number M

ρ_h = Air density at altitude h (kg/m^3)

ρ_o = Air density at sea level (zero altitude) (kg/m^3)

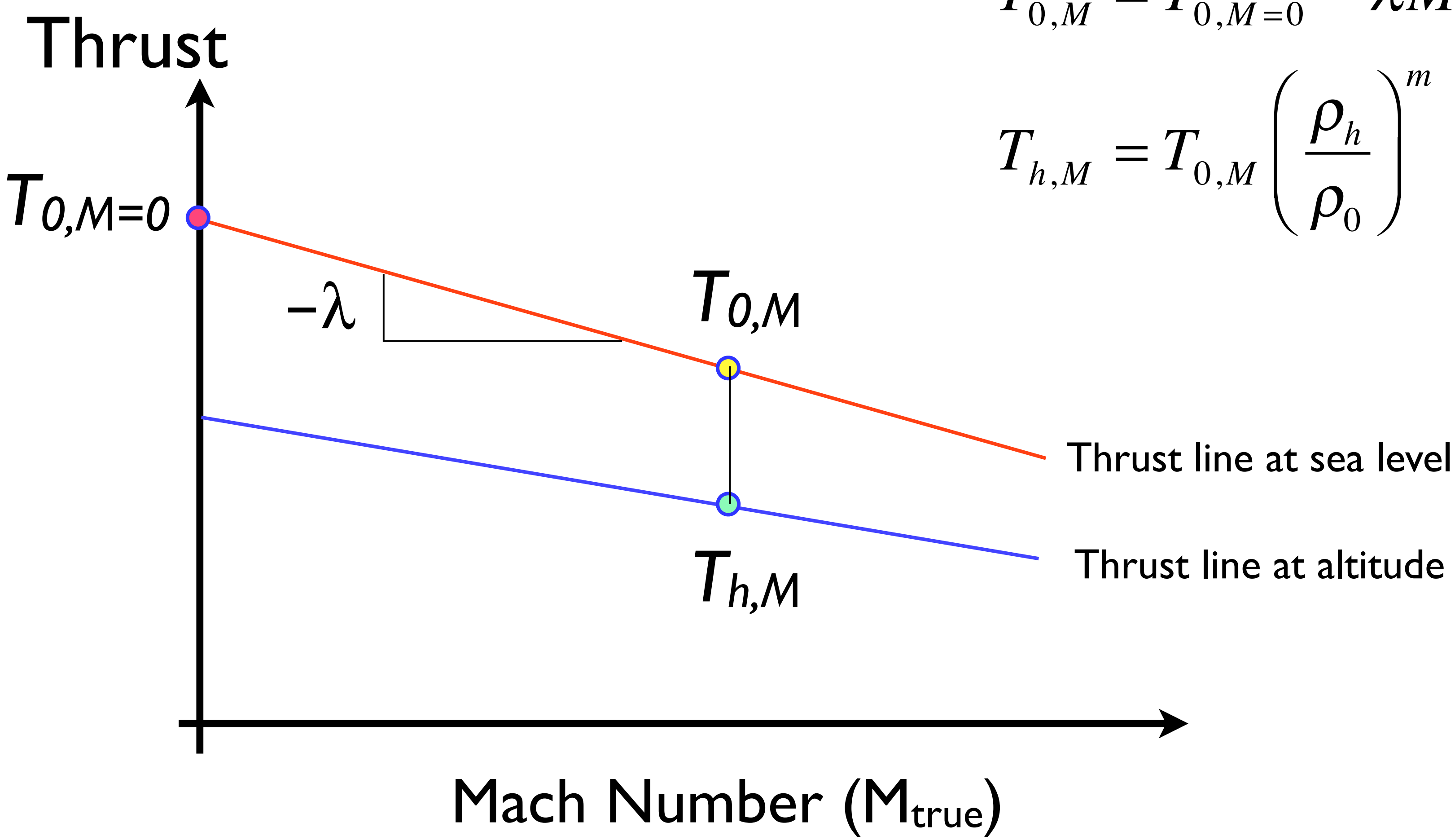
m = Thrust lapse rate (dimensionless)

A Simple Aircraft Model for Engine Thrust

- The following picture provides a graphical presentation of the thrust model

$$T_{0,M} = T_{0,M=0} - \lambda M_{true}$$

$$T_{h,M} = T_{0,M} \left(\frac{\rho_h}{\rho_0} \right)^m$$



Aircraft Climb Performance
2,000 feet and 210 knots IAS

- The thrust developed by each engine is a linear function of Mach number. At sea level and Mach 0.3267 the thrust is,

$$T_{0,M} = T_{0,M=0} - \lambda M_{true}$$

$$T_{0,M} = 338{,}000 - 175{,}560 M_{true}$$

$$T_{0,M} = 338{,}000 - 175{,}560(0.3267)$$

$$T_{0,M} = 280{,}646 \text{ Newtons}$$

- The thrust at altitude (h) is then,

$$T_{h,M} = T_{0,M}\left(\frac{\rho_h}{\rho_0}\right)^m$$

$$T_{h,M} = 280{,}646\left(\frac{1.156}{1.225}\right)^{0.96}$$

$$T_{h,M} = 265{,}450 \text{ Newtons}$$

-

Aircraft Climb Performance
2,000 feet and 210 knots IAS

- The thrust developed by each engine has been calculated at the prescribed Mach number (0.3267) and climb speed (214.6 knots)

- The aircraft has four engines so the total thrust for the climb condition is,

$$T_{total} = T_{h,M} n$$

where

n is the number of engine

$T_{h,M}$ is the thrust at altitude and Mach number

$$T_{total} = 265{,}450(4) = 1{,}061{,}800 \text{ Newtons}$$

- This is the total thrust developed by four engines at 210 knots and 600 meters above sea level under ISA conditions

Aircraft Climb Performance
2,000 feet and 210 knots IAS

- The rate of climb of the aircraft can be calculated,

$$\frac{dh}{dt} = \frac{(T_{total} - D)V}{mg} =$$

$$\frac{dh}{dt} = \frac{(1{,}061{,}800 - 316{,}340)110.42}{540{,}000(9.81)} \frac{(N - N)}{kg(m/s^2)}$$

$$\frac{dh}{dt} = 15.53\ m/s$$

- This is equivalent to 932.3 meters per minute or 3,058 feet per minute
- This climb rate is typical of transport aircraft at low altitudes
- The process is now repeated for state 2

Aircraft Climb Performance
8000 m (26,200 feet) and 290 knots IAS

- The true mach number is 0.6512, the speed of sound is 308.0 m/s and the density of air is 0.524 k/cu. m.
- The true airspeed (TAS) is 200.8 m/s or 390 knots
- The lift coefficient needed to maintain flight at 200.63 m/s is,

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2 * (540{,}000)(9.81)}{(0.524)(200.8)^2(858)} \frac{(kg)(m/s^2)}{(kg/m^3)(m/s)(m^2)} = 0.5844$$

- The drag coefficient at 26,200 feet and 290 knots (IAS) can be calculated using the standard parabolic drag polar model

$$C_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi A \mathrm{Re}} = 0.020 + \frac{0.5844^2}{\pi(9.0)(0.84)} = 0.0344$$

- Note that the value of C_{do} at Mach 0.6512 is 0.020

Aircraft Climb Performance
8000 m (26,200 feet) and 290 knots IAS

- The total drag is,

$$D = \frac{1}{2}\rho V^2 S C_d = \frac{1}{2}(0.524)(200.8)^2(858)(0.0344) = 311{,}800N$$

- The thrust produced by all 4 engines in the very large capacity transport is estimated to be:

- The thrust developed by each engine at Mach 0.6512 and 8,000 meters is,

$$T_{0,M} = T_{0,M=0} - \lambda M_{true}$$

$$T_{0,M} = 338{,}000 - 175{,}560 M_{true}$$

$$T_{0,M} = 338{,}000 - 175{,}560(0.6512)$$

$$T_{0,M} = \ 235{,}110 \text{ Newtons}$$

- Now correct the thrust for altitude

Aircraft Climb Performance
8000 m (26,200 feet) and 290 knots IAS

- The thrust at altitude (h) is then,

$$T_{h,M} = T_{0,M}\left(\frac{\rho_h}{\rho_0}\right)^m$$

$$T_{h,M} = 235{,}110\left(\frac{0.524}{1.225}\right)^{0.96}$$

$$T_{h,M} = 104{,}040 \text{ Newtons}$$

- For four engines the total thrust is,

$$T_{total} = T_{h,M}\,n$$

$$T_{total} = 104{,}040(4) = 416{,}180 \text{ Newtons}$$

- Now we can calculate the rate of climb at Mach 0.6512 and 8,000 meters

Aircraft Climb Performance
8000 m (26,200 feet) and 290 knots IAS

- The rate of climb of the aircraft can be calculated,

$$\frac{dh}{dt} = \frac{(T_{total} - D)V}{mg} =$$

$$\frac{dh}{dt} = \frac{(411{,}180 - 316{,}800)(200.8)}{540{,}000(9.81)} \frac{(N - N)}{kg(m/s^2)}$$

$$\frac{dh}{dt} = 3.96\ m/s$$

- This is equivalent to 237 meters per minute or 779 feet per minute
- The rate of climb has ben reduced to about ~1/4 compared to near sea level conditions

Aircraft Climb Performance
Calculation of Lift-to-Drag Ratio

- The lift-to-drag ratio is calculated as the ratio of C_l and C_d,

- At h=600 meters and 210 knots (IAS)

$$L / D = \frac{C_l}{C_d} = \frac{0.8761}{0.0523} = 16.7$$

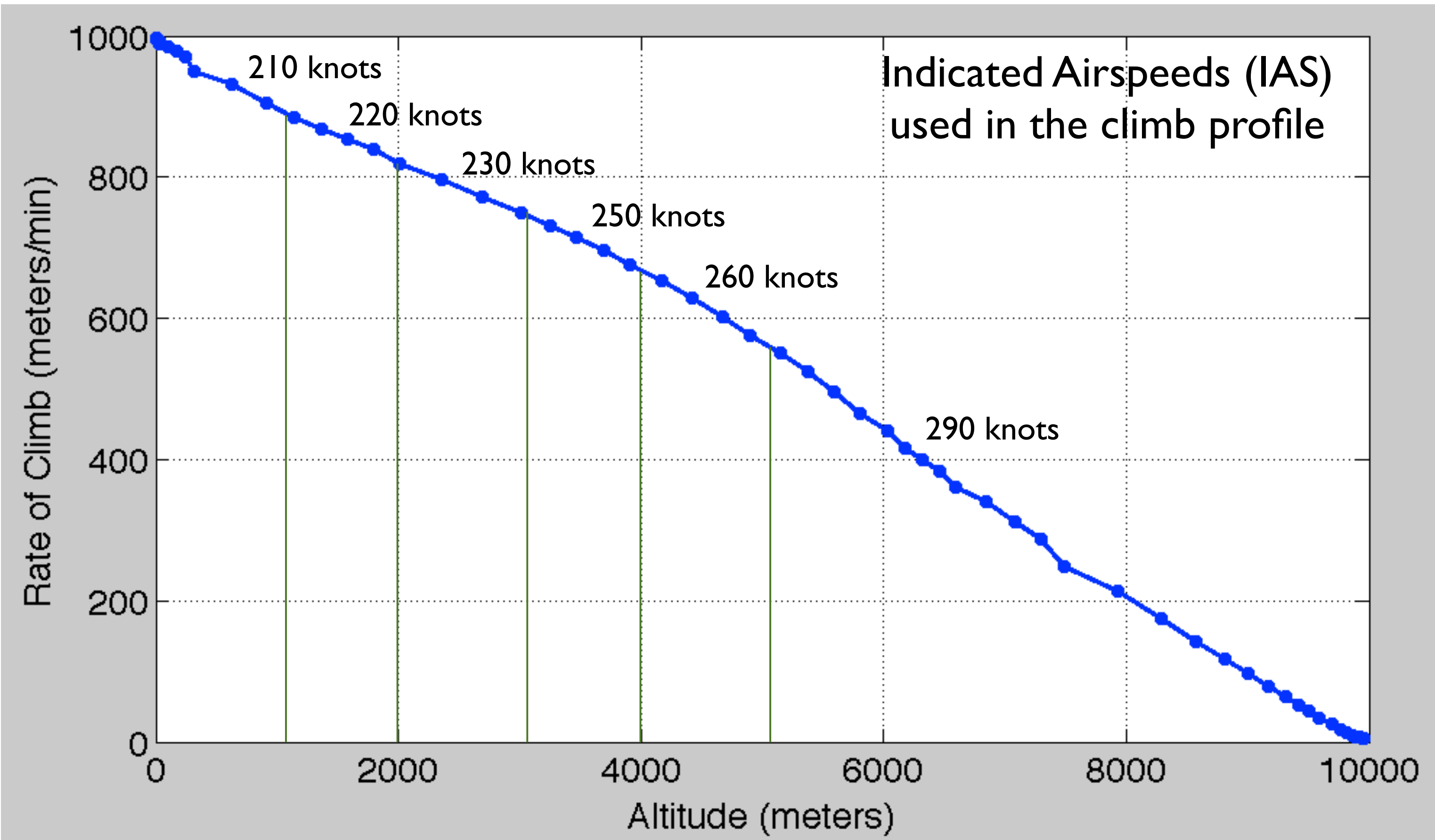
- At h=8,000 meters and 290 knots

$$L / D = \frac{C_l}{C_d} = \frac{0.5488}{0.0344} = 16.9$$

- The lift-to-drag ratio is a key parameter in the determination of range of the aircraft

Rate Of Climb Analysis

- Repeating the steps shown the previous pages we can estimate the rate of climb for the complete climb profile (i.e., at multiple altitude points)

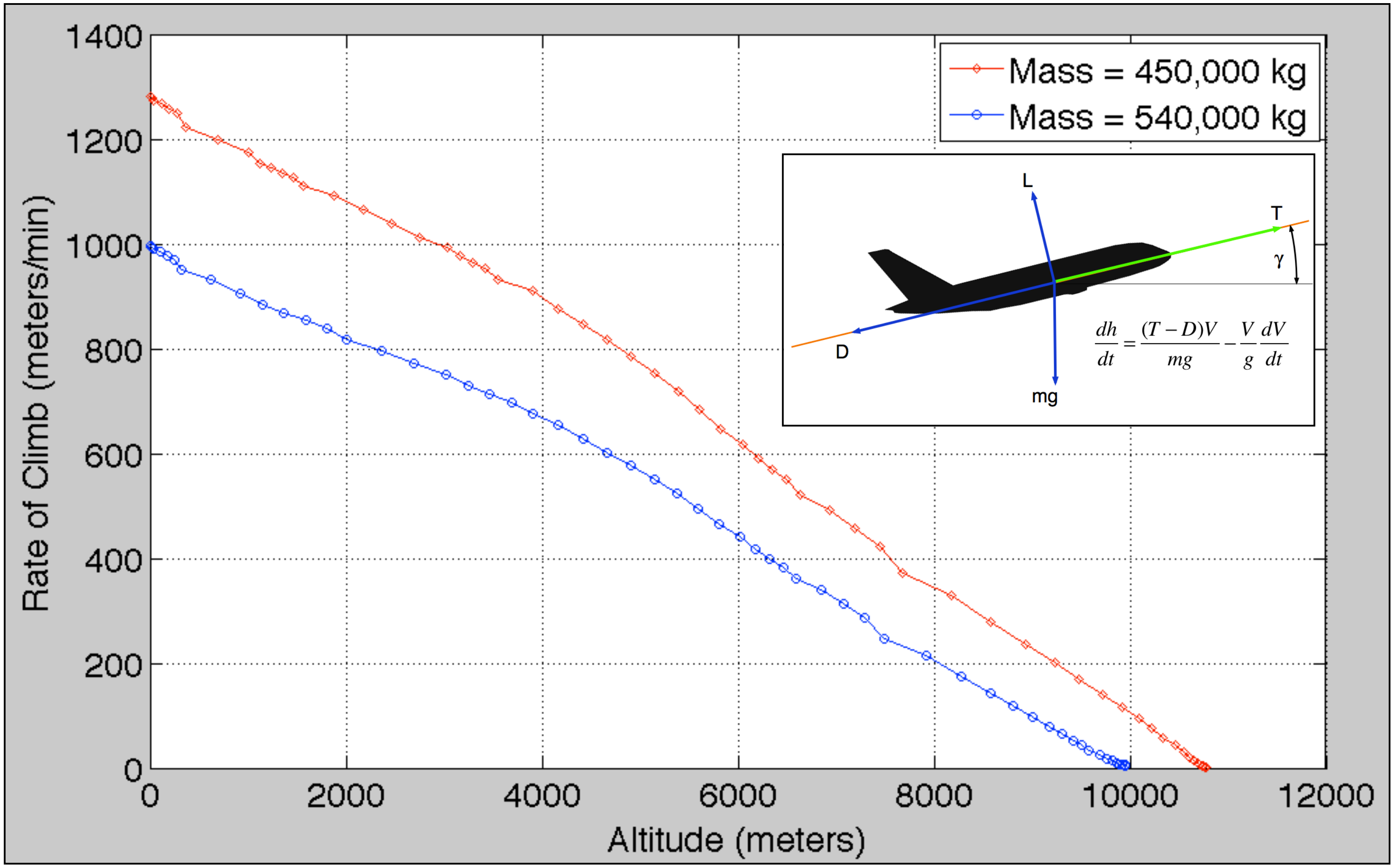


Observations

- Rate of climb is high at low altitudes (due to high thrust available and lower true airspeeds)
- The rate of climb decreases non-linearly with altitude (lower atmospheric density reduces engine thrust)
- The rate of climb is affected by other environmental and operational variables:
  - Aircraft weight
  - Temperature
  - Climb speed

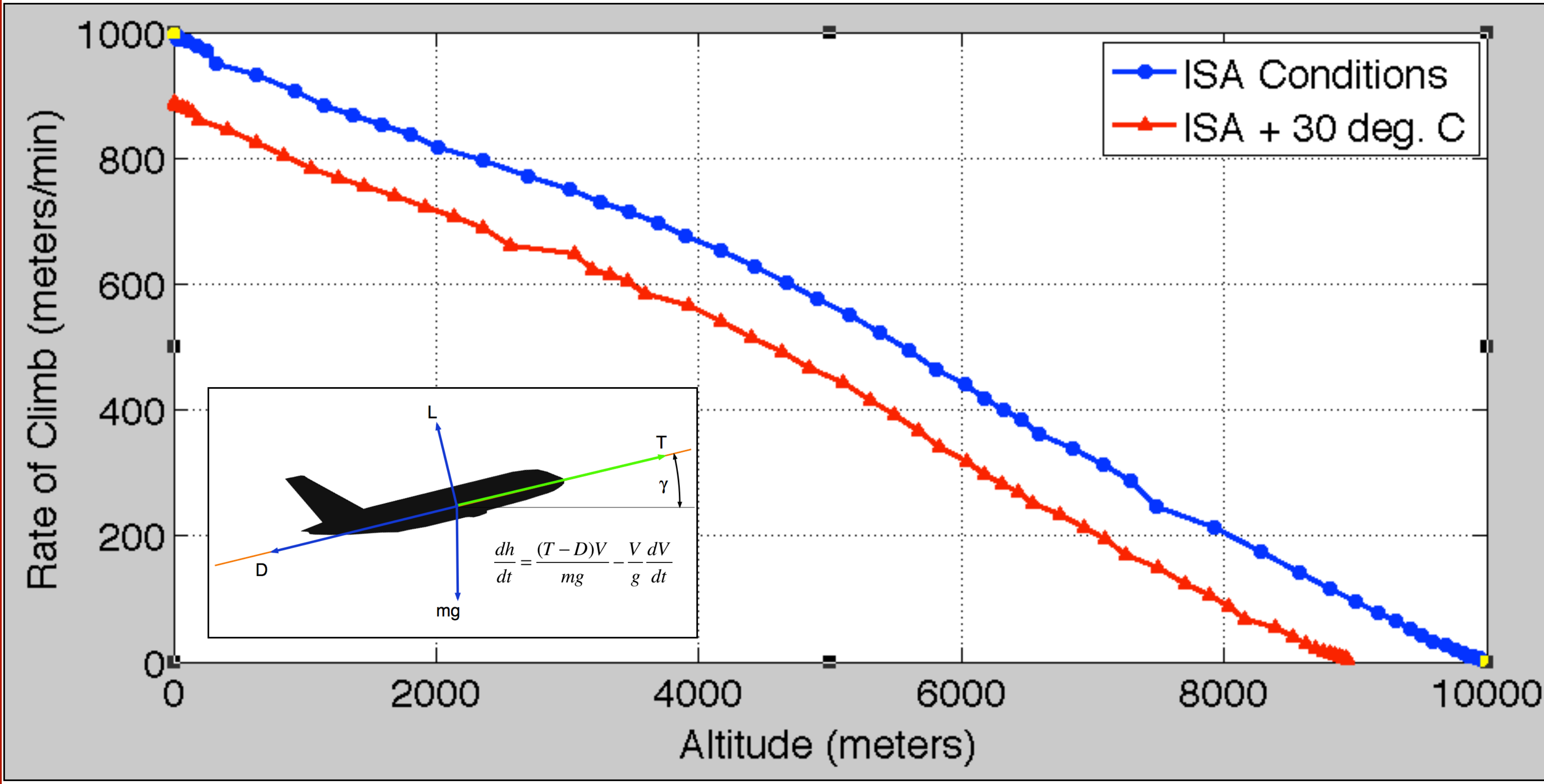
Sensitivity Analysis
Rate of Climb vs. Weight

- Varying the weight of the aircraft from 540,000 kg to 450,000 kg shows the effect on rate of climb



Sensitivity Analysis
Rate of Climb vs. Temperature

- Varying the temperature of the atmosphere from ISA to ISA + 30 degrees Celsius



Implications for Real-world Aviation Operations

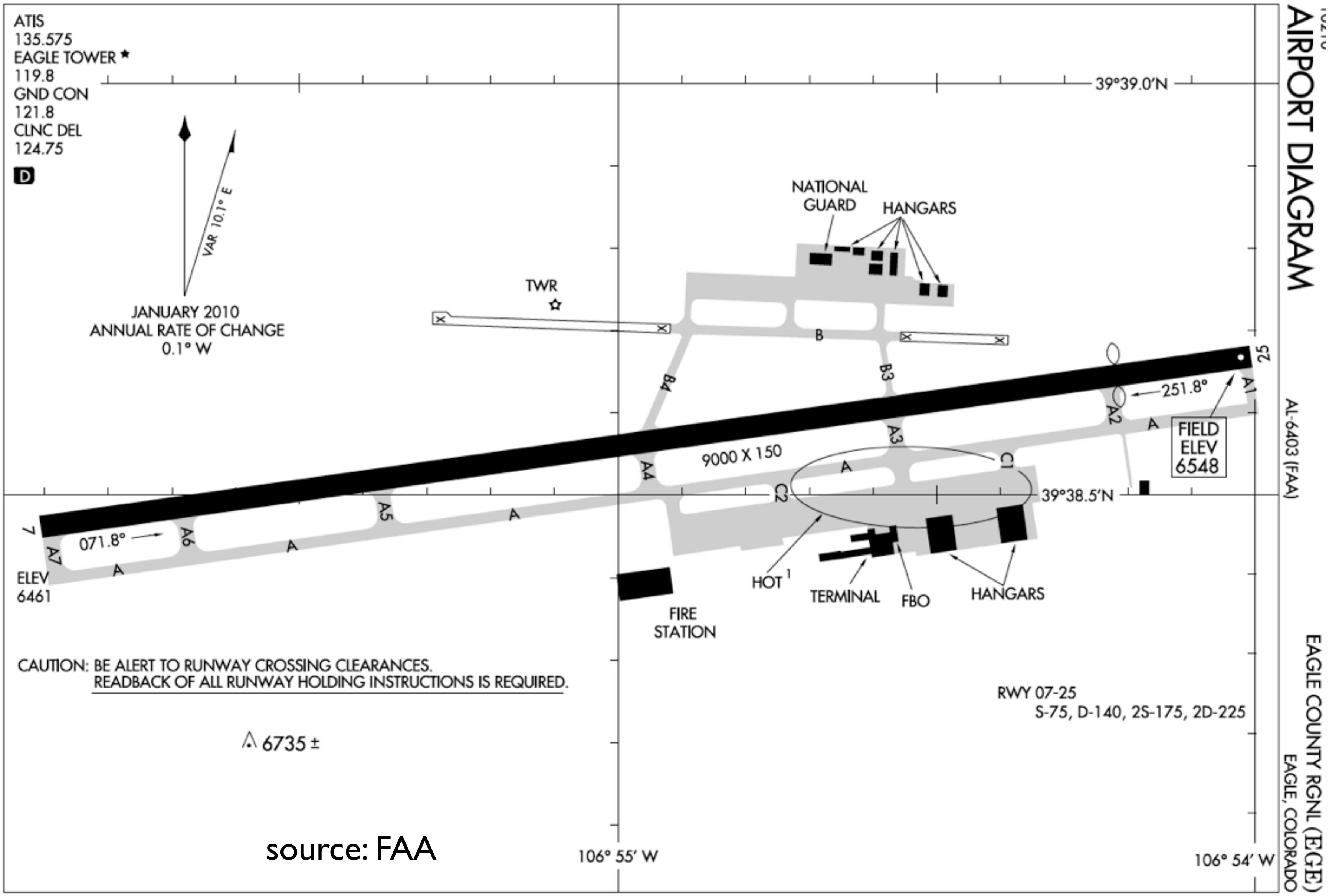
- The performance of the aircraft has profound effects in real-world flight planning applications
  - Obstacle accountability analysis
    - Obstacle clearance procedures in the terminal area (after takeoff)
    - Obstacle clearance in engine out conditions
  - Limits on carrying passengers and cargo from a challenging airport
  - Time to climb is affected and assigned cruise altitude

A Challenging Airport for Departure Performance

- Eagle County Airport in Colorado (EGE)
- 9000 x 150 foot runway
- Obstacles due to terrain on both approach and departure procedures
- Airport elevation is 6,535 feet above mean sea level
- Airport has commercial operations using high-performance twin-engine aircraft (Boeing 757-200)

Boeing 757-200 winglets departing LAX (A.A.Trani)

Eagle County Airport (EGE)



source: FAA

Use of Airport Approach and Departure Aeronautical Charts

- To illustrate some challenging airport operational procedures related to aircraft climb performance we use some approach and departure aeronautical charts

- These charts are used by pilots and ATC to plan and fly arrival and departure procedures (flight tracks) to the airport. These are called Standard Terminal Arrival Routes (STAR), Standard Instrument Departures (SID), and Instrument Approach Procedures (IAP)

- You can obtain airport STAR, SID and IAP charts at: http://flightaware.com/

- More information on how to read these charts can be found:

  - http://www.naco.faa.gov/index.asp?xml=naco/online/aero_guide

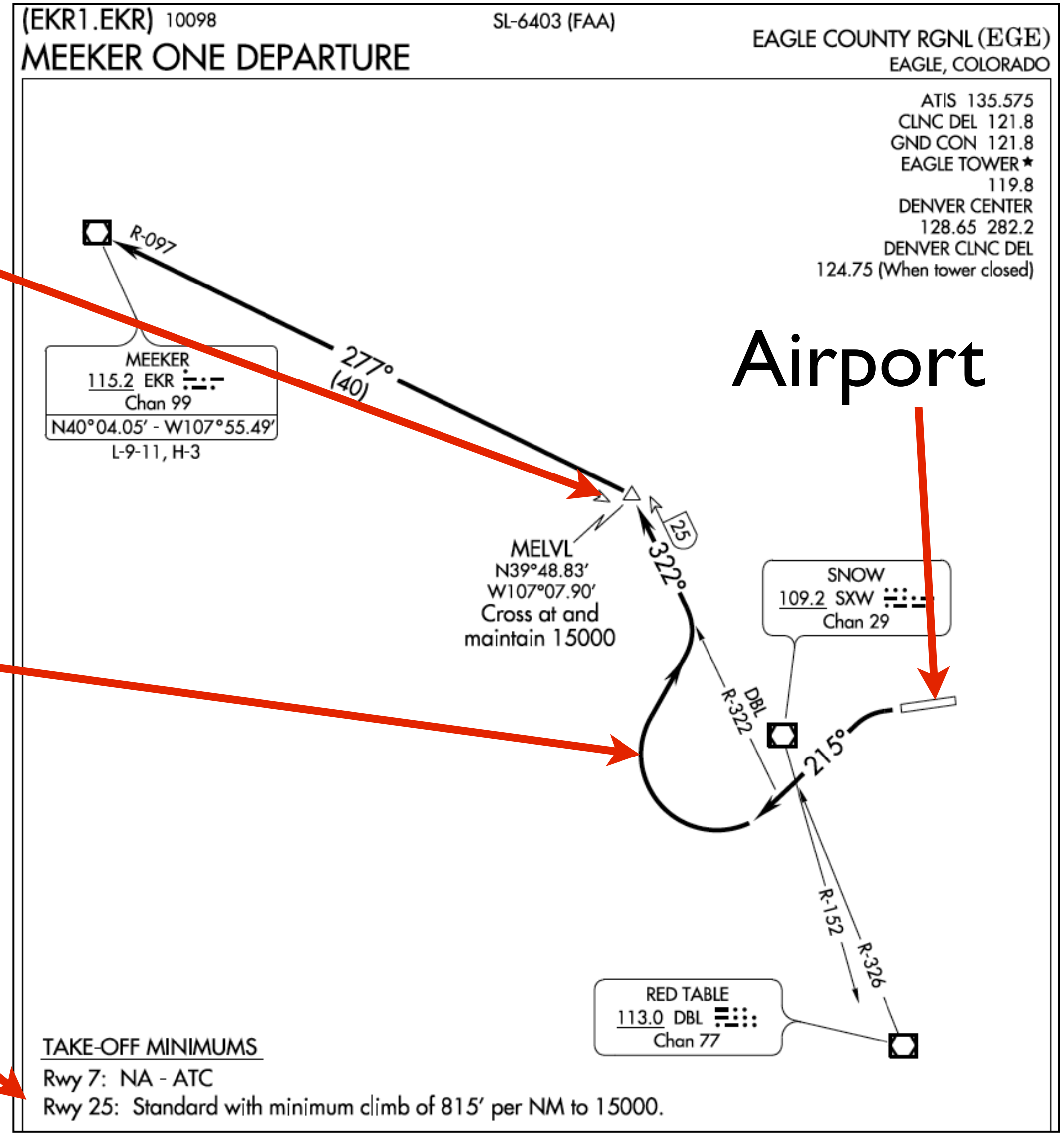
  - http://sunairexpress.com/images/How_to_Read_Approach_Plates.pdf

Departure Procedure from EGE (Runway 25)

Note crossing altitude restriction at MELVL

Climbing left turn heading 215 degrees to avoid natural terrain to the West of the airport

Requires 815 feet/nm of climb performance



Airport

source: FAA

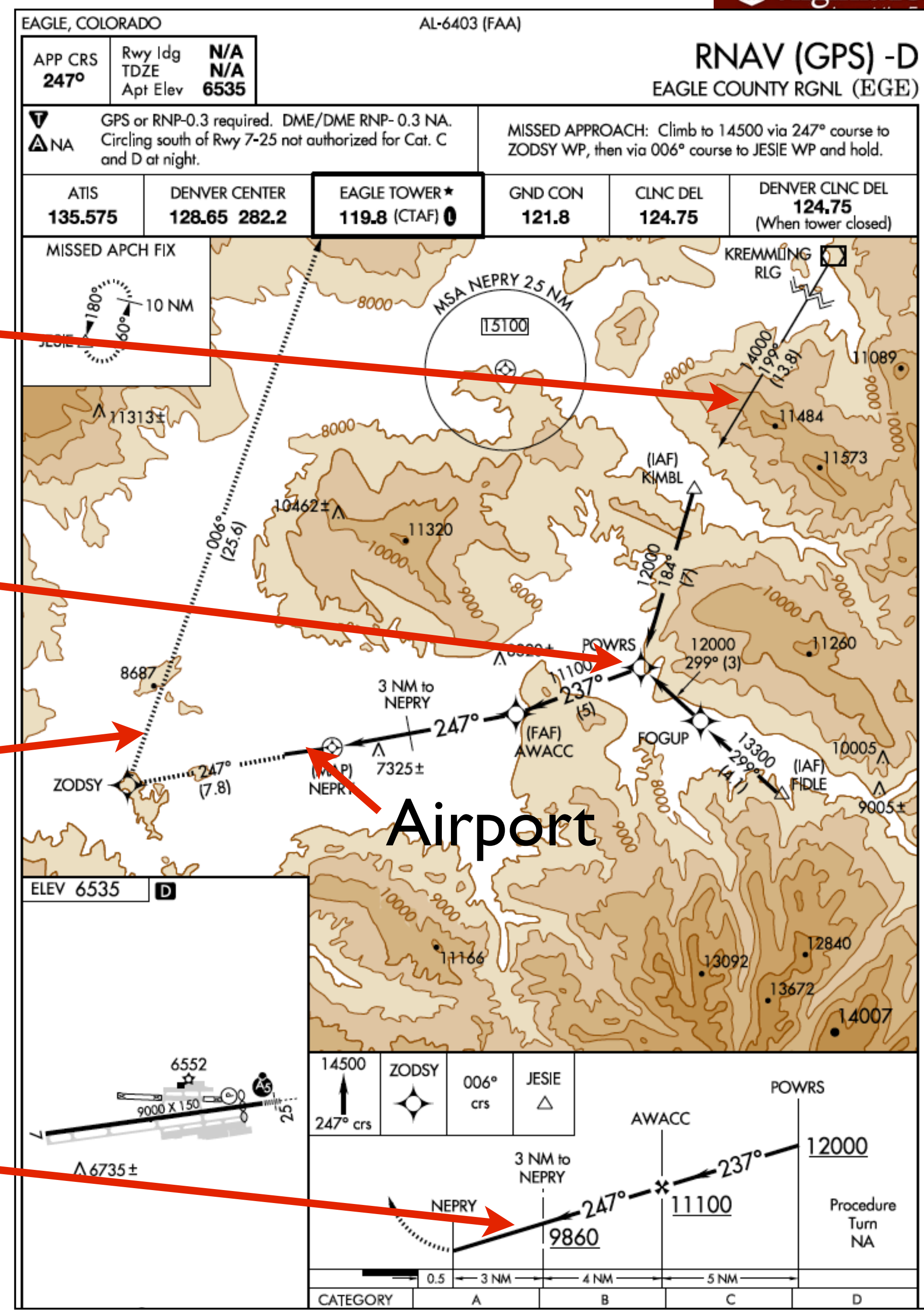
RNAV Approach Procedure to EGE (Runway 25)

Descent to the airport from NE avoids the highest terrain

RNAV waypoints provide a complex track

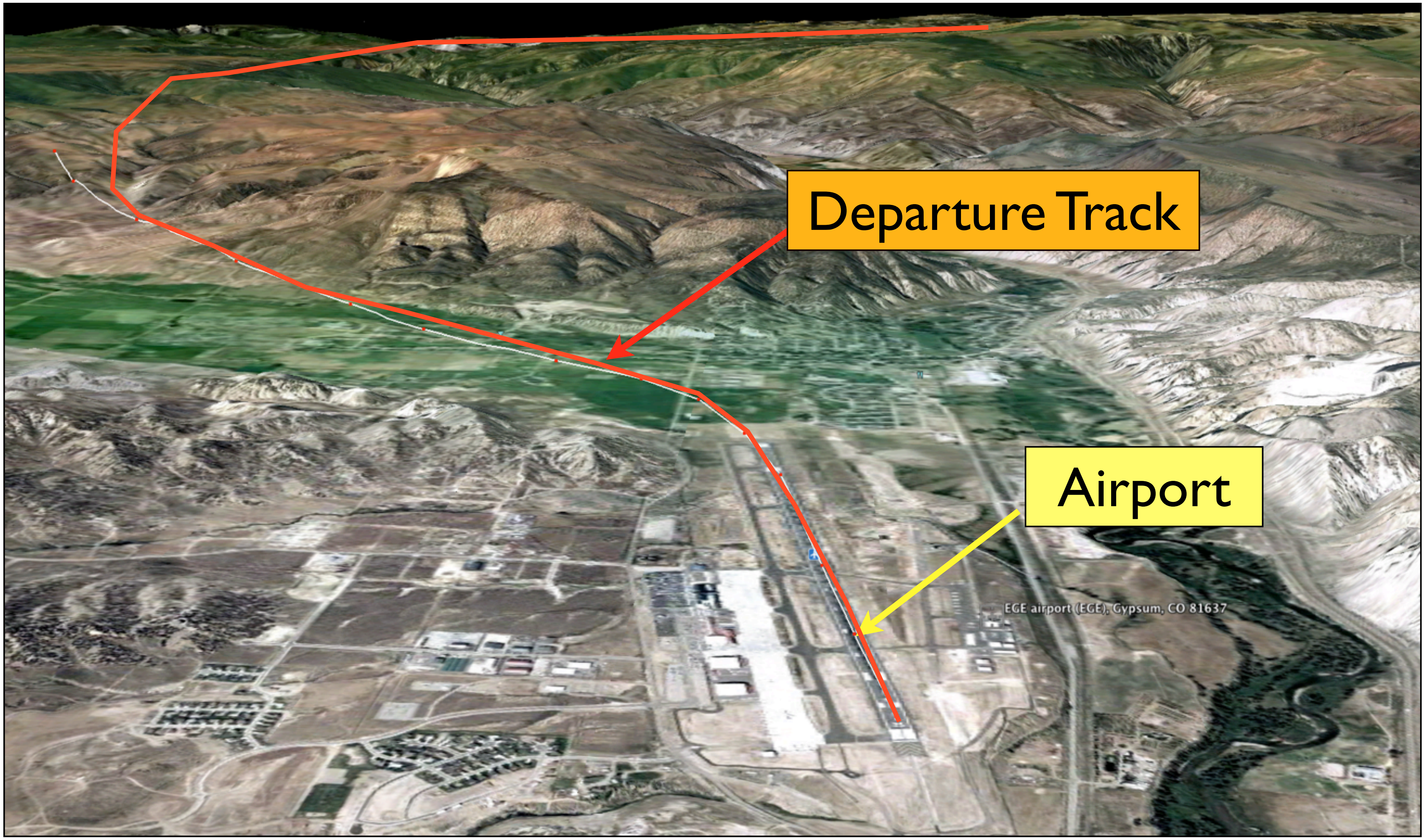
Missed approach track is a escape route through lower terrain

Vertical profile of approach track



source: FAA

Departure Procedure from EGE Airport (Runway 25)



source: Google Earth (2009)

Numerical Simulation of the Climb Equations of Motion

Using the Unrestricted Climb Profile Matlab Files

Purpose and Basic Explanations

- Explains how to use the **unrestricted climb profile files**
- Code simulates an aircraft climb profile up to the maximum altitude possible limited by thrust and atmospheric constraints
- Program calculates numerically, four aircraft state variables in the climb profile (altitude, mass, distance traveled along path and distance along flat earth)

Files are available at: http://128.173.204.63/courses/cee5614/matlab_files_cee5614.html

Input Files Needed Files

- Explains how to use the unrestricted climb profile files
- Code simulates an aircraft climb profile up to the maximum altitude possible limited by thrust and atmospheric constraints

Climb Performance Files

- UnrestrictedClimbAnalysis.m
  - fclimb_06.m
  - densityAltitudeOffISA.m
  - drag_03.m
  - thrust_calculationNoLoss.m
  - atmosphere

Plus one of the aircraft files from the list

Aircraft Files

- Very Light Jet (Eclipse 500 class)
- Large Twin Engine Transport (Boeing 777 class)
- Very Large Capacity Transport (Airbus A380 class)
- Regional Jet Transport (Bombardier CRJ200 class)
- Medium Size Jet Transport (Boeing 737-800 class)
- New Generation, Long-Range Transport (Boeing 787-800 class)

Files are available at: http://128.173.204.63/courses/cee5614/matlab_files_cee5614.html

File Organization

Climb Performance Files	

Climb Performance Files
• UnrestrictedClimbAnalysis.m
  ◦ fclimb_06.m
  ◦ densityAltitudeOffISA.m
  ◦ drag_03.m
  ◦ thrust_calculationNoLoss.m
  ◦ atmosphere
	UnrestrictedClimbAnalysis.m integrates the basic equations of motion of the aircraft as it climbs without restrictions to the maximum altitude the aircraft can reach. The state variables intgerated over time are: a) altitude, b) mass, c) distance traveled along the path, and d) distance traveled on the earth's surface.
	
	UnrestrictedClimbAnalysis.m calls **fclimb_06.m** which contains the rates of change of the state variables over time. The climb analysis also requires and aircraft file with the performance limits of the vehicle modeled.
	
	All files contained in this section are needed to successfully run **UnrestrictedClimbAnalysis.m** because drag and thrust computations are needed in the climb performance analysis.

UnrestrictedClimbAnalysis.m = Main Matlab script

Matlab functions
fclimb06.m (calculates the rates of change of state equations)
densityAltitudeOffISA.m (calculates atmospheric conditions)
drag03.m (calculates aircraft drag and lift characteristics)
thrust_calculationNoLoss.m (calculates engine thrust)
atmosphere.m (numerical values of ISA atmosphere)

Simulate an Aircraft

You specify the aircraft to be simulated in **UnrestrictedClimbAnalysis.m**

For example, Airbus380_class.m contains information similar to a very large capacity 4-engine aircraft

UnrestrictedClimbAnalysis.m

```
% Enter aircraft file desired - reads a file with aircraft characteristics
% The last three lines of information in the aircraft file provide the
% basis for the climb profile - Indicated airspeed vs. altitude

Airbus380_class

h_airport = 0;                  % airport altitude (m) - departing field elevation
rhos = 1.225;                   % sea level density (kg/m-m-m)
deltaTemp = 0;                  % ISA + deltaTemp conditions for analysis (deg. Kelvin)

Mass_init = mass*g;              % Initial aircraft weight (Newtons)
```

Aircraft file to be simulated

Aircraft weight

Controlling the Aircraft Climb Speed

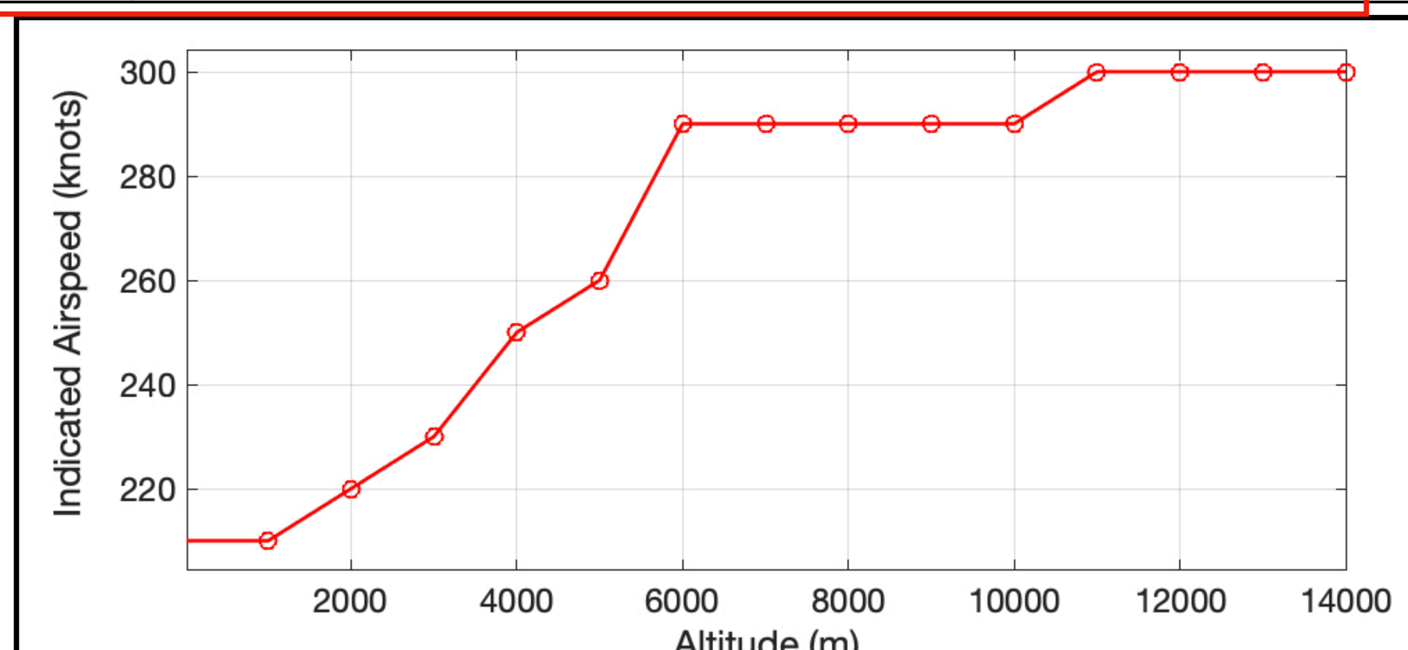
- You specify the aircraft climb speed inside the aircraft file (see lines 29 and 31)

- For example, Airbus380_class.m contains information similar to a very large capacity 4-engine aircraft

Airbus380_class.m

Aircraft climb speed profile to be simulated

```
26   % Computes the aircraft profile given altitude (Vcas given) – typical for
27   % four engine aircraft similar to the Airbus A380 aircraft
28
29 – Vclimb = [210 210  220 230 250  260  290  290  290  290  290  300 300  300  300];      % knots IAS
30 – Vdescent = [180 200  250 250 270  300  300  300  300  300  300  300 300  300  300];  % knots IAS
31 – altc = [ 0 1000  2000 3000 4000  5000  6000  7000  8000  9000  10000  11000 12000  13000  14000]; % altitude
```



Calculating Equations of Motion

- Program calculates numerically, four aircraft state variables in the climb profile (altitude, mass, distance traveled along path and distance along flat earth)

```
% Define the rate equations (4 rate equations for 4 state variables)
% y(1) - Aircraft altitude (m)
% y(2) - Aircraft weight (N)
% y(3) - Distance traveled along the path (m)
% y(4) - Distance traveled wrt to ground (m)

yprime(1) = vtas * (thrust - drag) / (y(2)) ;      % Rate of climb (m/s)
yprime(2) = - tsfc * thrust;                        % Mass flow rate (N/s)
yprime(3) = vtas ;                                  % Distance traveled along the flight path (m)


% Calculation of flight path angle (gamma)

flt_path_angle = asin(yprime(1)/ vtas);    % radians

% Calculate distance traveled across horizontal distance

yprime(4) = vtas * cos(flt_path_angle);    % distance traveled along horizontal (meters)

yprime = yprime'; % transposes the array going out
                  % MATLAB is very picky about this
                  % it likes the array in column format
```

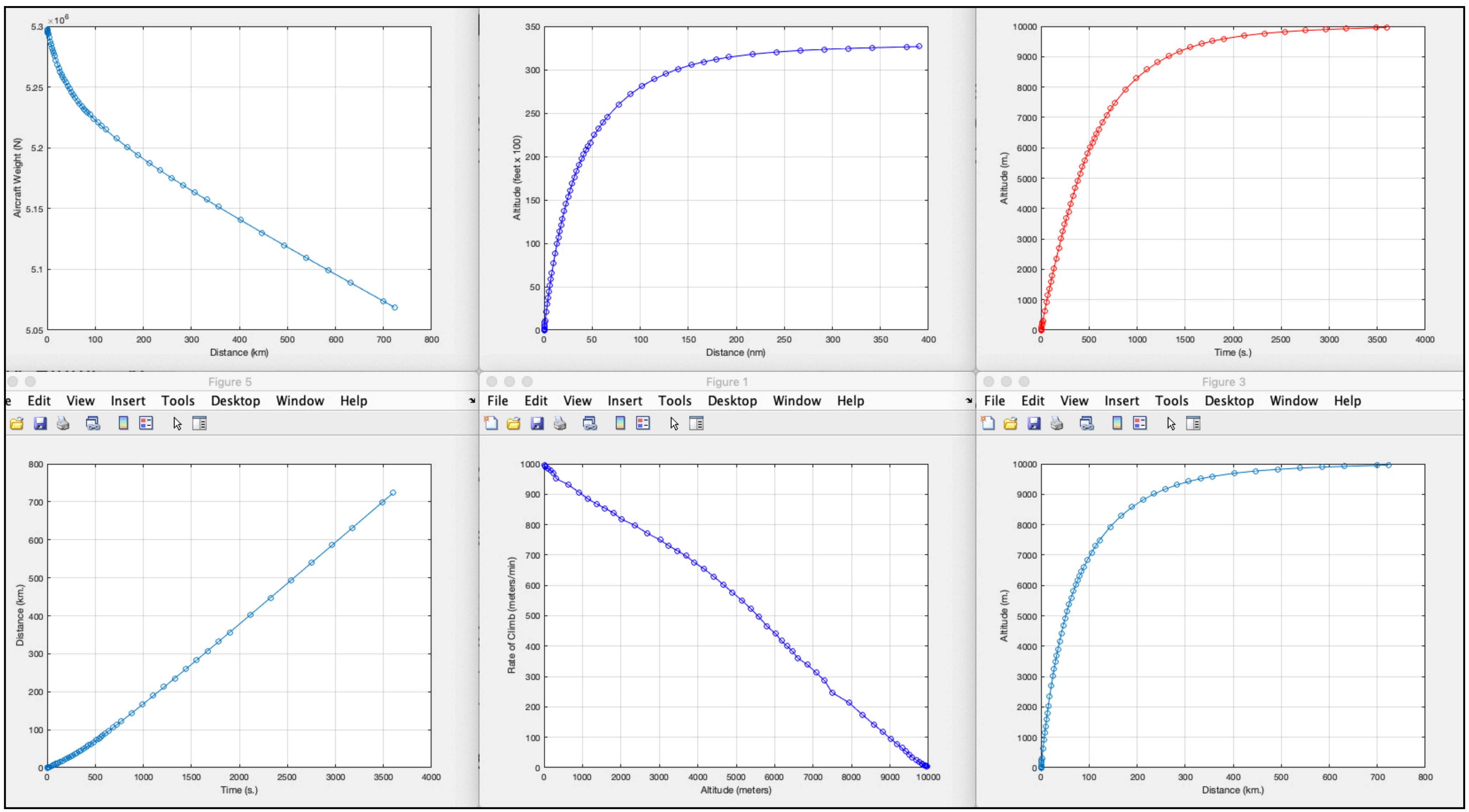
Function fclimb06.m

yprime(1) = dh/dt
yprime(2) = dW/dt
yprime(3) = dS/dt (along path)
yprime(4) = dS/dt (flat Earth)

State variables

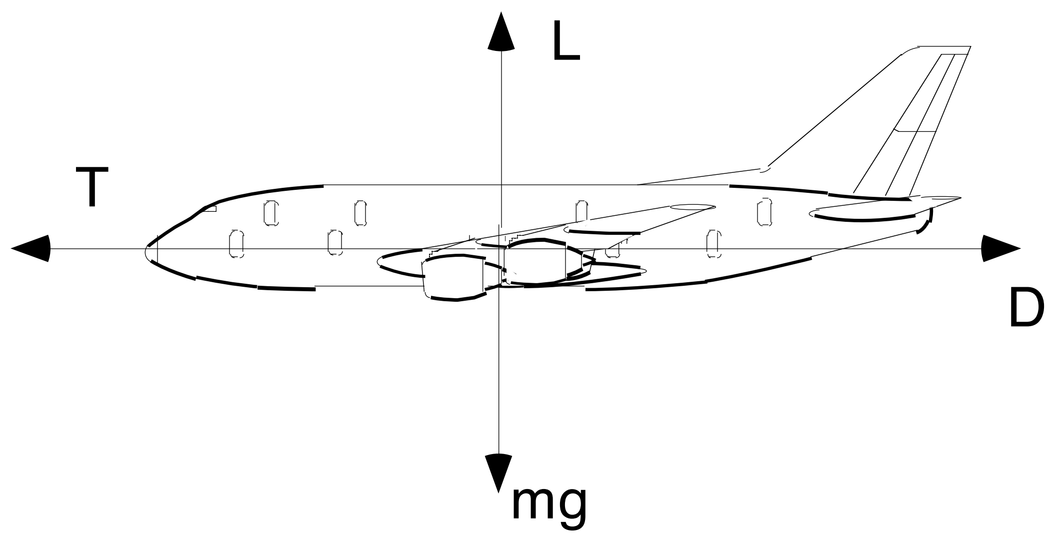
Output Produced by the Matlab Code

- Four aircraft state variables in the climb profile (altitude, mass, distance traveled along path and distance along flat earth)
- Plots of state variables vs. time
- Plots of state variables vs. distance flown and altitude



Cruise Analysis

The forces acting on the air vehicle during cruising flight are shown below. Note that drag generated by the aircraft and the thrust supplied by the engine are equal for steady and level flight. Similarly, the lift and weight are equal.



Cruise Analysis

Lift and drag can be computed according to the well known aerodynamic equations re-stated below.

$$L = \frac{1}{2}\rho S C_L V^2 \tag{15}$$

$$D = \frac{1}{2}\rho S C_D V^2 \tag{16}$$

$$C_D = C_{D0} + C_{Di} = C_{D0} + \frac{C_L^2}{\pi A R e} \tag{30}$$

$$C_L = \frac{2mg}{\rho S V^2} \tag{15B}$$

Cruise Analysis

For typical subsonic aircraft ($M < 0.8$) the drag rise beyond the so-called critical mach number (M_{crit}) is quite severe and this produces a well defined maximum speed capability dictated by the rapid rise in the C_{D0} term in Equation 30.

A drag divergence mach number exists for every aircraft. The drag divergence mach number is characterized by a fast rise in drag coefficient due to wave drag and parasite/friction drag effects at high speed.

Cruise Range Estimation

- An important consideration in assessing air vehicle performance is the range of the aircraft.
- Range is the maximum distance that an aircraft flies without refueling. Several range alternatives arise operationally for aircraft as will be shown in this section.
- The range represents a trade-off of how far and how much payload (i.e., the amount of passengers, cargo, or a combination of the two) an aircraft carries.

Range Estimation Methodology

The differential distance (or range), *dR*, traveled at speed V over a small interval of time *dt* is,

$$dR = Vdt \tag{34}$$

Since the aircraft only looses weight due to fuel expenditure we can define the rate of change of the weight over time as the product of the specific fuel consumption (TSFC) and the tractive force required to move the vehicle at speed V (T),

$$\frac{dW}{dt} = -(TSFC)T \tag{35}$$

This quantity is negative because the aircraft burns fuel
Weight decreases along the flight path

Range Estimation Methodology

Define the Specific Air Range - SAR - (a measure of the efficiency of the aircraft) as the ratio of the distance flown per unit of fuel consumed,

$$\frac{dR}{dW} = \frac{(V)}{(TSFC)T} = SAR \qquad (36)$$

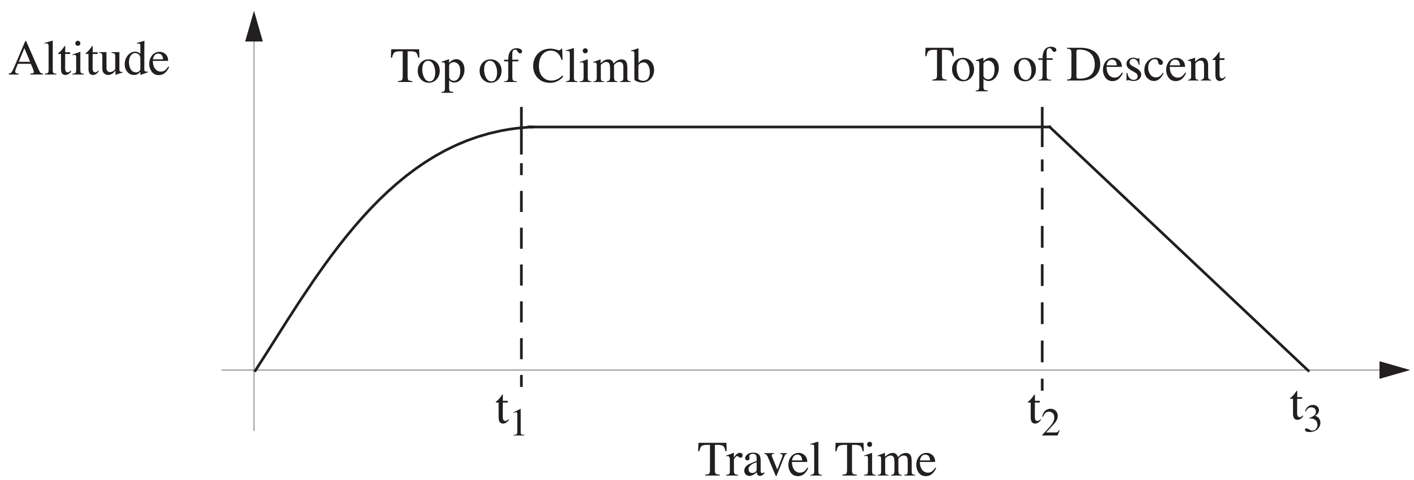
The typical units of TSFC are lb/hr/lbf (pounds per hour of fuel per pound of force produced) or kg/hr/kgf (kilograms per hour of fuel consumption per kilogram force produced). This parameter varies with altitude and speed.

Sample Use of TSFC

- The Pratt and Withney PW 4086 engine used in the Boeing 777 has a TSFC value of 0.6 lb/hr/lbf as the aircraft flies at mach 0.80 at 11,000 m. above mean sea level.
- If each engine produces 15,000 lb of thrust at that altitude to keep the aircraft flying straight and level then the average hourly fuel consumption would be (15,000 lb of thrust) (0.6 lb/hr/lbf) = 9,000 lb per hour (per engine).
- The solution to the so-called Breguet Range equation derived from SR is obtained if one separate variables and integrates over the weight expenditure of the vehicle from an initial weight, W_i to a final weight, W_f at the end of the cruising segment.

Cruise Range Analysis

In practical airline operations the initial and final cruising segment points are called Top of Climb (TOC) and Top of Descent (TOD), respectively.



Derivation of the Breguet Range Equation

Start with the basic equation of SAR (Equation 36),

$$\frac{dR}{dW} = \frac{(V)}{(TSFC)T} = SAR \qquad (36)$$

Multiplying the right hand side of the previous equation by L/W and rearranging terms,

$$\frac{dR}{dW} = \frac{(V)}{\mathrm{TSFC\ (T)}}\left(\frac{L}{W}\right) = \frac{(V)}{(TSFC)W}\left(\frac{L}{D}\right) \qquad (37)$$

Separating variables and integrating both sides,

$$\int_0^R dR = \int_{W_i}^{W_f} \frac{(V)}{(TSFC)}\left(\frac{L}{D}\right)\left(\frac{dW}{W}\right) \qquad (38)$$

In cruise T = D

$$R = \frac{(V)}{(TSFC)}\left(\frac{L}{D}\right)\ln\left(\frac{W_i}{W_f}\right) \tag{39}$$

where: R is the aircraft range, $TSFC$ is the thrust specific fuel consumption, V is the cruise true airspeed, L is the lift, D is the drag produced while moving at speed V, and W_i and W_f are the initial and final weights of the aircraft at the top of climb and top of descent, respectively.

Note that for constant altitude cruise the term L/D is not constant because as the aircraft depletes its fuel and gets lighter over time. Consequently, the amount of lift needed to keep it flying at the same altitude will vary over time. The derivation of an approximate range equation can, nevertheless treat the term L/D as constant to give a first order approximation of the expected aircraft range.

Modifications to Breguet-Range Equation

In the range equation (Eq. 39) the term L/D can be alternatively substituted by C_L/C_D. To avoid problems the range expression for very long range aircraft can be subdivided into various cruising segments and then integrated using corresponding values of C_L/C_D for each segment. One approach to estimate with more precision the range is to integrate numerically the Specific Air Range equation (Eq. 36) considering variations in C_L/C_D using standard numerical methods.

Example - Aircraft Range Performance

- Use the the vehicle characteristics for the very large capacity transport aircraft in the Matlab files for CEE 5614 to solve this problem (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)
- Estimate the rate of range for this aircraft for three altitudes and various Mach numbers:
  - Mach ranging from 0.74 to 0.86
  - Cruise altitudes from 8,000 to 12,000 meters
- Assume the International Standard Atmosphere applies

Example - Very Large Capacity Aircraft Data File

- Very large capacity transport aircraft (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)

```
% Aircraft file to support other computational modules
% Aircraft = Similar to Airbus A380 (heavy transport)

global A e S neng tsfc macht Cdoct mass thrust_table mach_table lapse_rate_factor Vclimb altc g

S = 858;            % wing area (square m)
A = 9.0;            % Wings aspect ratio
e = 0.84;           % Oswald's efficiency factor
g = 9.81;
neng = 4;               % Number of engines
tsfc = 1.6e-4;          % thrust specific fuel consumption (N/N/s)
mass = 540000;          % mass at operating point (kg)
```

geometric, mass and specific fuel consumption

```
% Drag characterictics - CDO function (zero lift drag function)

Cdoct   = [0.020   0.020   0.0204    0.022   0.037   0.038   0.040];
macht   = [0.0     0.75    0.80      0.85    0.90    0.95    1.00];
```

drag data

```
% Thrust parameters for Eclipse aircraft (at sea level)

thrust_table = [338000 180000];     % Thrust limits (sea level in Newtons)
mach_table = [0.0  0.9];                    % mach number limits to bound thrust
lapse_rate_factor = 0.96;                   % thrust lapse rate factor
```

engine thrust data

```
% Computes the aircraft profile given altitude (Vcas given) - typical for
% four engine aircraft similar to the Airbus A380 aircraft

Vclimb = [180 200  220 250 270  280  290  300  300  300  300  300 300  300  300];      % knots IAS
Vdescent = [ 300 300  300 300 300  300  300  300  300  300  300  300 300  300  300];  % knots IAS
altc = [ 0 1000  2000 3000 4000  5000  6000  7000  8000  9000  10000  11000 12000  13000  14000]; % altitude (meters)
```

speed profile data

Example - Very Large Capacity Aircraft Data File

- An aircraft similar in size and performance as the Airbus A380
- For range analysis assume mass at TOC = 530,000 kg
- Mass at Top of Descent (TOD) = 320,000 kg

Airbus A380 taxies to the gate at LAX (A.A.Trani)

Computations using the Breguet Range Equation

$$R = \frac{V}{TSFC}\frac{L}{D}\ln\left(\frac{W_i}{W_f}\right)$$

$$R = \frac{V}{TSFC}\frac{C_l}{C_d}\ln\left(\frac{W_{TOC}}{W_{TOD}}\right)$$

Steps:

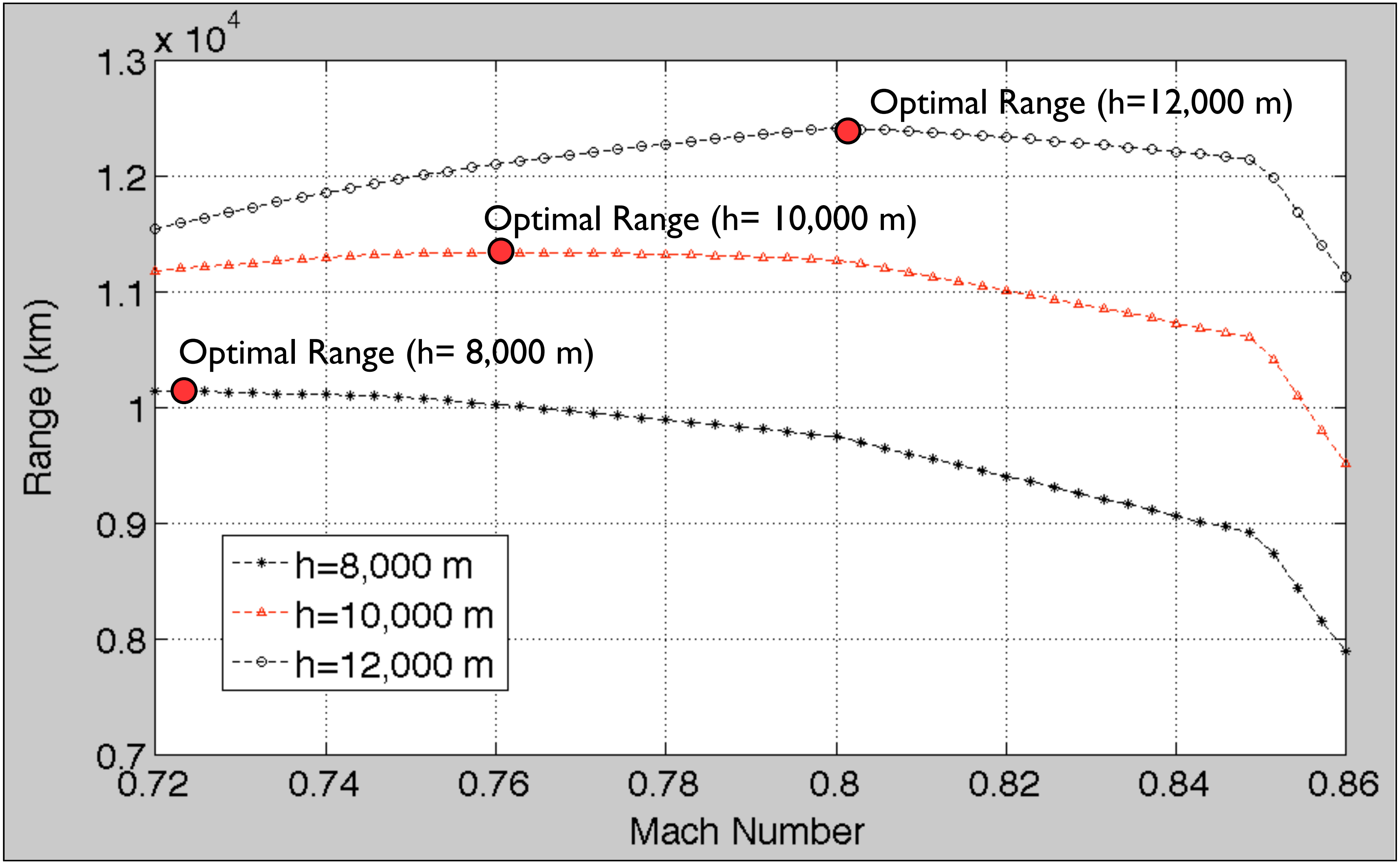
1) For a given cruise altitude and desired cruise Mach number calculate the true airspeed (*V*)

2) Calculate lift coefficient to maintain steady flight using the midpoint mass between TOC and TOD (C_l)

3) Calculate drag coefficient (C_d)

4) Estimate the range (*R*) assuming constant TSFC

Range Analysis Parametric Study



Observations

- The range varies with Mach number
- There is an optimum Mach number for long range cruise (0.80 for this aircraft at 12,000 m.)
- The optimal range occurs at different Mach numbers
- High-speed cruise in modern airliners like Boeing 777 and Airbus A380 is around Mach 0.83-0.85
- Maximum operating Mach number for a Boeing 747-400 is 0.92 (Mmo) - speed never used in practice
- Range penalties are associated with high cruise Mach numbers

CEE 5614
Analysis of Air Transportation
Cruise Analysis Calculations
Numerical Integration Example

Problem Description

- Use the large **four-engine** transport aircraft performance file provided in the Matlab files for CEE 5614 (http://128.173.204.63/courses/cee5614/matlab_files_cee5614.html to answer the following questions
- The aircraft cruises at FL 360 and Mach 0.83 over distance of 4,000 nm. The aircraft has a mass of 500,000 kg at the Top of Climb Point (TOC)
- Calculate the fuel burn in the cruise segment using a numerical integration procedure
- The procedure is to divide the cruise phase into smaller distance intervals
- In each interval we assume the fuel consumption is constant

Fuel Burn Calculation Formulas

$$\frac{dR}{dt} = V$$

$$T = D = \frac{1}{2}\rho V^2 S C_d$$

$$\frac{dW}{dt} = TSFC(D) = -TSFC(\frac{1}{2}\rho V^2 S C_d)$$

Negative sign because weight is decreasing with time

The system of two equations of motion can be solved using Matlab. Note that during the cruise phase, the speed of the aircraft does not change so the equation dR/dt is simple to solve.

The equation for dW/dt can be solved numerically in two ways:

1) Numerical integration with respect to time
2) Numerical integration over distance as parameter (inverse of SAR)

Fuel Burn Calculation Formulas

Numerical integration over time

$$\frac{dW}{dt} = -TSFC(T) = -TSFC(D) = -TSFC(\frac{1}{2}\rho V^2 S C_d)$$

$$W_{t+\Delta t} = W_t + \frac{dW}{dt}\Delta t = W_t - TSFC(\frac{1}{2}\rho V^2 S C_d)\Delta t$$

where:

$\frac{dW}{dt}$ =rate of change of aircraft weight per unit of time (i.e., N/s)

Δt = is a suitable time step size for the numerical integration

$TSFC$ = Thrust specific fuel consumption (N/s/N)

D = total drag (N)

T = thrust required to overcome drag (N)

Fuel Burn Calculation Formulas

Numerical integration over distance as parameter

$$\frac{dR}{dt} = V$$

$$dR = Vdt$$

$$\frac{dW}{dt} = TSFC(D) = -TSFC(\frac{1}{2}\rho V^2 SC_d)$$

$$\frac{dW}{dR} = \frac{1}{SAR} = \frac{-TSFC(D)}{V} = \frac{-TSFC(T)}{V}$$

where:

$$\frac{dW}{dR} = \text{rate of change of aircraft weight for a given distance (i.e., N/nm or N/m)}$$

The procedure to solve the differential equation numerically is illustrated in the following pages

Numerical Solution of dW/dR

$$\frac{dW}{dt} = TSFC(D) = -TSFC(\frac{1}{2}\rho V^2 S C_d)$$

$$\frac{dW}{dR} = \frac{1}{SAR} = \frac{-TSFC(D)}{V} = \frac{-TSFC(T)}{V}$$

where:

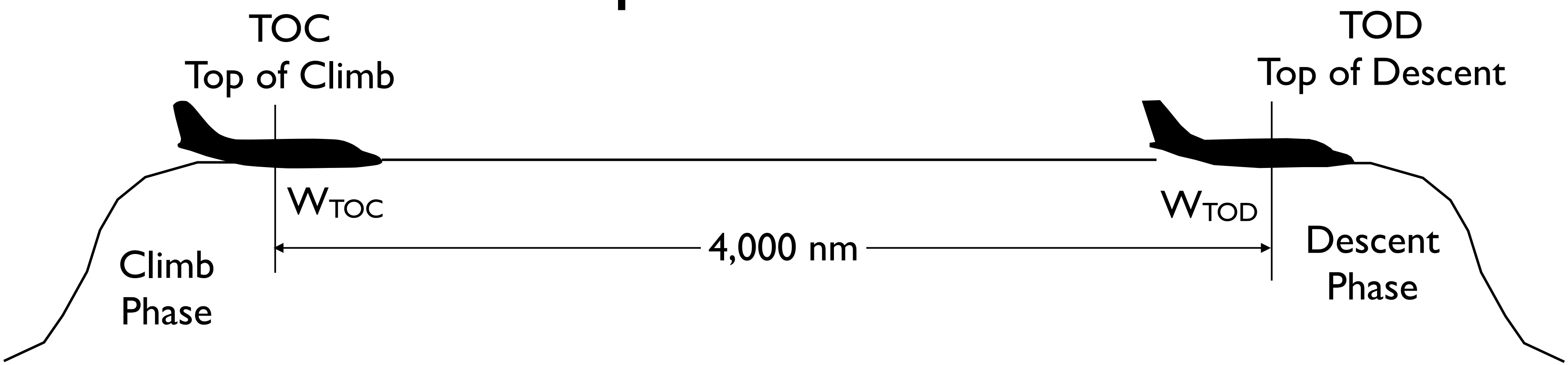
$$\frac{dW}{dR} = \text{rate of change of aircraft weight for a given distance (i.e., N/nm or N/m)}$$

The solution requires an evaluation of Drag and Speed at discrete points along the track as illustrated in the following page. TSFC is either known from engine data or estimated using a polynomial approximation.

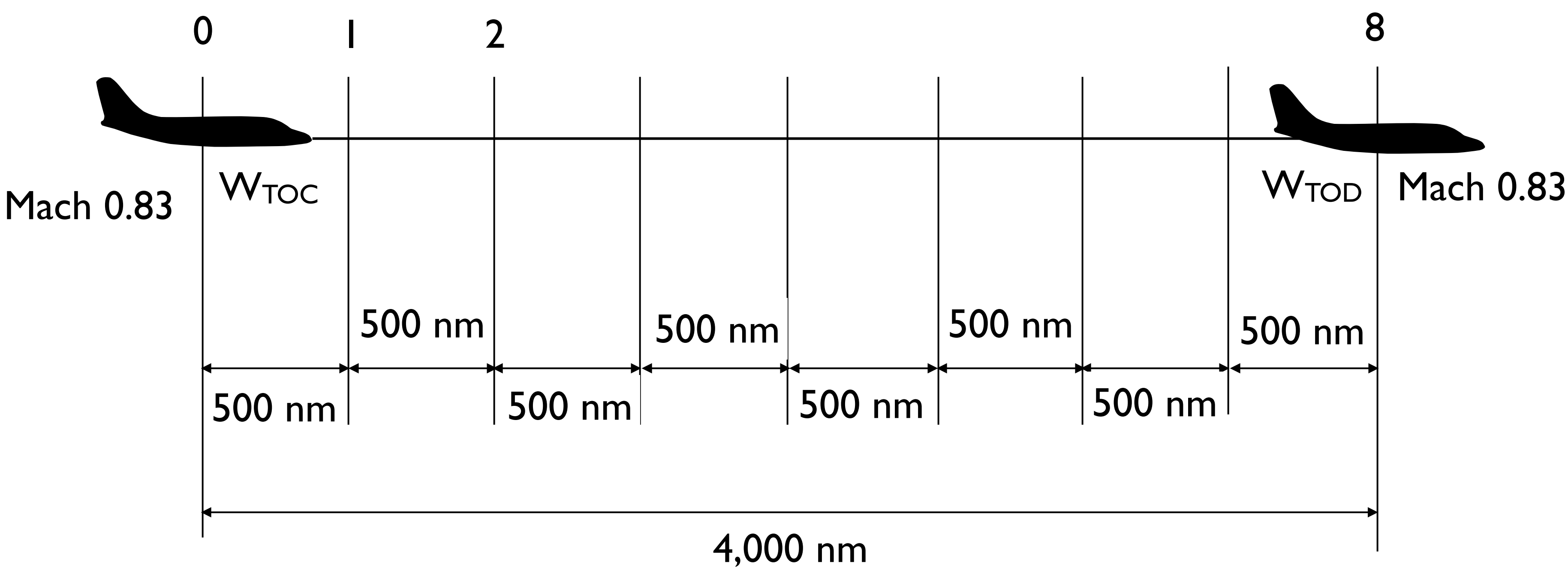
The aircraft cruises at Mach 0.83 at 36,000 feet. Assuming ISA conditions, the speed of sound (a) at 10,976 meters is 295 m/s. The true airspeed is:

$$V_{tas} = 295.1 \text{ m/s } (0.83) = 244.93 \text{ m/s}$$

Graphical Situation



Example: 8 intervals of 500 nm



Fuel Burn at TOC Point

$$m_{TOC} = 500{,}000 \text{ kg}$$

The drag at cruise at the Top of Climb point (point 0 in the diagram) would be:

$$V_{tas} = 244.93 \text{ m/s}$$

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2*(500{,}000)(9.81)}{(0.365)(244.94)^2(858)} \frac{(kg)(m/s^2)}{(kg/m^3)(m/s)(m^2)} = 0.5217$$

$$\mathrm{C}_l = 0.5217 \text{ (dim)}$$

$$\mathrm{C}_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi AR(e)} = 0.0211 + \frac{0.5217^2}{\pi(9.0)(0.84)} = 0.033 \text{ (dim)}$$

$$D = \frac{1}{2}\rho V^2 S C_d = \frac{1}{2}(0.365)(244.94)^2(858)(0.0330) = 310{,}070N$$

Fuel Burn at TOC Point

The fuel consumption at the TOC point is then,

$$\frac{dW}{dt} = -TSFC(D) = -TSFC(T) = -(1.6e-4 \text{ N/N/s}) * (310{,}070 \text{ N})$$

$$\frac{dW}{dt} = -49.61 \text{ N/s} = \text{-}5.06 \text{ kg/s}$$

The aircraft takes 3,780.2 seconds to travel 500 nm of the first interval of the 4,000 nm cruise phase.

In the process the aircraft burns 19,118 kg of fuel at 5.06 kg/s

At point (1), the aircraft mass is: 480,892 kg

Next Iteration (Segment 1-2)

- The process is repeated for all the remaining intervals. Here we show calculations for segment 1-2.

$$m_1 = 480{,}892 \text{ kg}$$

$$V_{tas} = 244.94 \text{ m/s}$$

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2*(480{,}892)(9.81)}{(0.365)(244.94)^2(858)} \frac{(kg)(m/s^2)}{(kg/m^3)(m/s)(m^2)} = 0.502$$

$$\mathrm{C}_l = 0.504 \text{ (dim)}$$

$$\mathrm{C}_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi AR(e)} = 0.0211 + \frac{0.502^2}{\pi(9.0)(0.84)} = 0.0321 \text{ (dim)}$$

$$D = \frac{1}{2}\rho V^2 S C_d = \frac{1}{2}(0.365)(244.94)^2(858)(0.0321) = 301{,}986\ N$$

Fuel Burn at Point (1) in Cruise

The fuel consumption at point 2 is then,

$$\frac{dW}{dt} = -TSFC(D) = -TSFC(T) = -(1.6e-4 \text{ N/N/s}) * (301{,}986 \text{ N})$$

$$\frac{dW}{dt} = -48.32 \text{ N/s} = -4.93 \text{ kg/s}$$

The aircraft takes 3,780.7 seconds to travel 500 nm of the second interval of the 4,000 nm cruise phase.

In the process the aircraft burns 18,618.6 kg of fuel at 4.93 kg/s

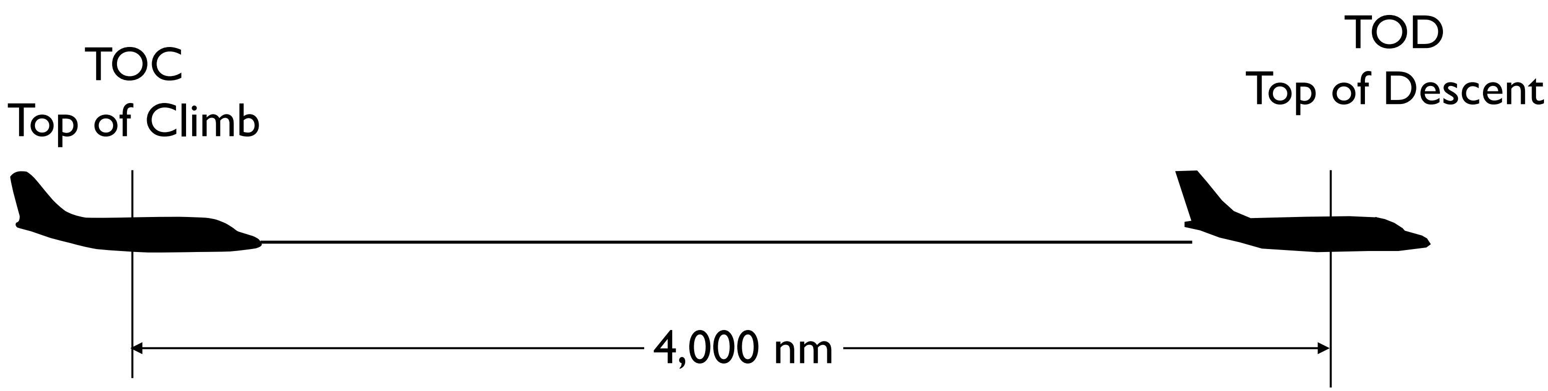
At point (2), the aircraft mass is: 462,273.3 kg

Complete the Numerical Analysis

The numerical procedure is repeated until the aircraft reaches the Top of Descent (TOD) - point (8)

$$fc_{cruise} = 140{,}611 \text{ kg}$$

$$m_{TOD} = 359{,}389 \text{ kg}$$



W_{TOC} = 500,000 kg

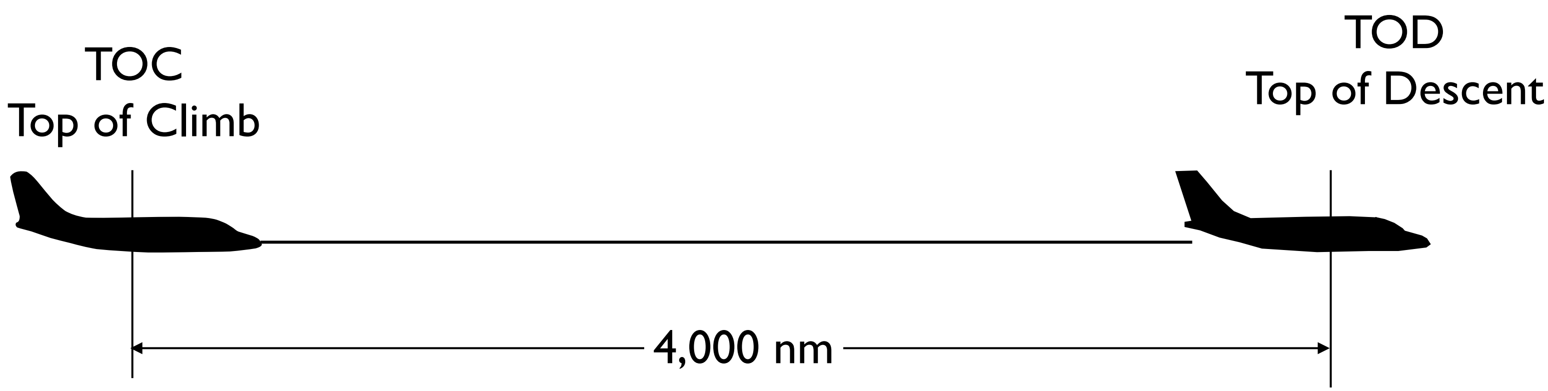
W_{TOD} = 359,389 kg

Numerical Analysis with 100 Intervals

The numerical procedure is now repeated using 100 intervals across the 4,000 nm segment to illustrate the improvement in the fuel burn calculation

$$fc_{cruise} = 139{,}248 \text{ kg}$$

$$m_{TOD} = 360{,}752 \text{ kg}$$



W_{TOC} = 500,000 kg

W_{TOD} = 360,752 kg

Comparison of Numerical Analyses

We compare the numerical procedure for various segments. The number of segments improves the accuracy of the solution (shown in the table below).

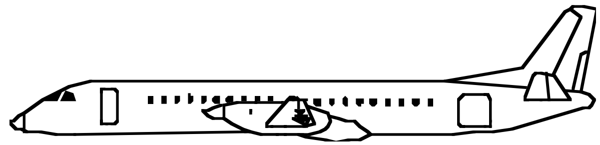
Number of Distance Segments	Cruise Fuel (kg)	Mass at TOD (kg)
1	152,935	347,065
8	140,611	359,389
100	139,248	360,762
500	139,154	360,850
5000	139,134	360,866

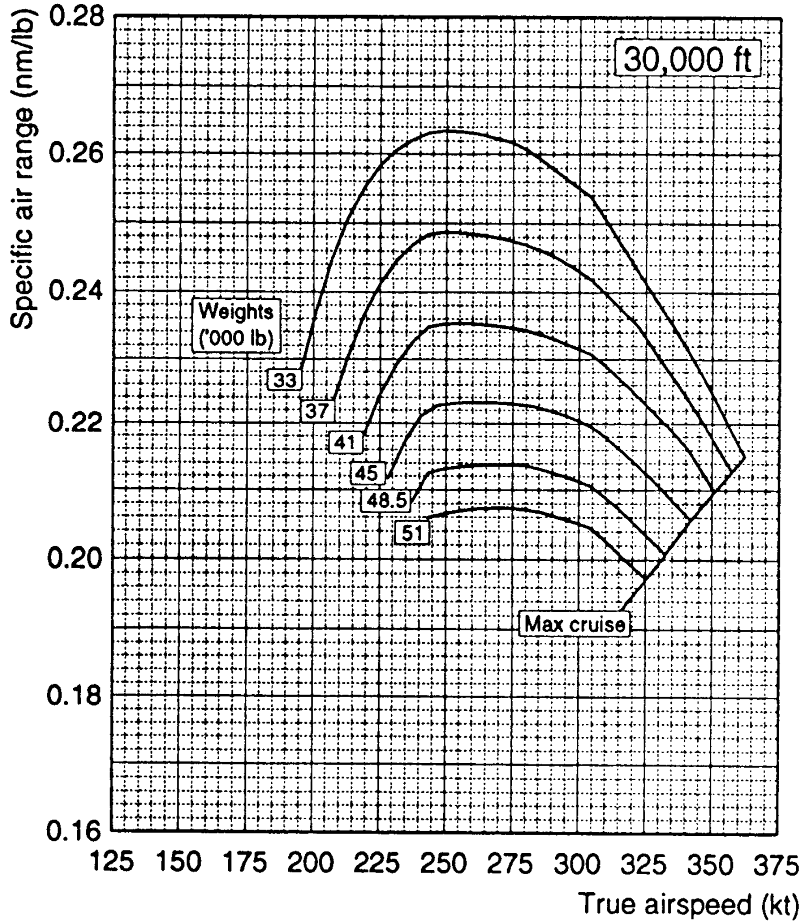
Conclusion

- The number of numerical steps improves the solution
- Beyond 500 steps, the solution cannot be improved significantly
- 500 distance steps in this problem is equivalent to calculating fuel burn every 8 nm (one minute at jet speeds)
- The method outlined can be employed in aviation simulation applications with good results

Presentation of Cruise Information

ISA + 15^0 C
950 Propeller RPM
Anti-Ice Off





source: Saab 2000 aircraft flight manual

Descent Flight Operations

- Usually, transport aircraft descent at a rate of 900 m/min. (3,000 ft./min.) during the early stages of the descent segment.
- Below 3,000 m. (10,000 ft.) aircraft enter a dense terminal area and are usually required to maneuver around other air vehicles to establish coordinated arrival flows to runways
- In the U.S is customary to limit the indicated airspeed to 250 knots or lower below 3,000 m to avoid accidents in the rare event of a bird strike.

Descent Profile Operations

- Below 3,000 m. the descent rate typically decreases to 500 m/min. (1,500 ft./min.) or less and the descent profile might follow a series of “steps” at designated altitudes in the final stages of flight.
- Aircraft manufacturers report typical fuel consumption vs. distance traveled curves similar to those shown in the figure below.

Manufacturers also include distance vs. altitude curves for the descent phase of aircraft operations.

Descent Profile Operations

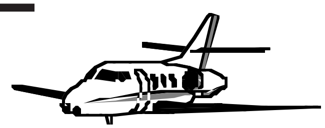
Decent operations can be studied using the same principles used to model climb operations

The vehicle is now placed into a shallow dive with engines running at lo power

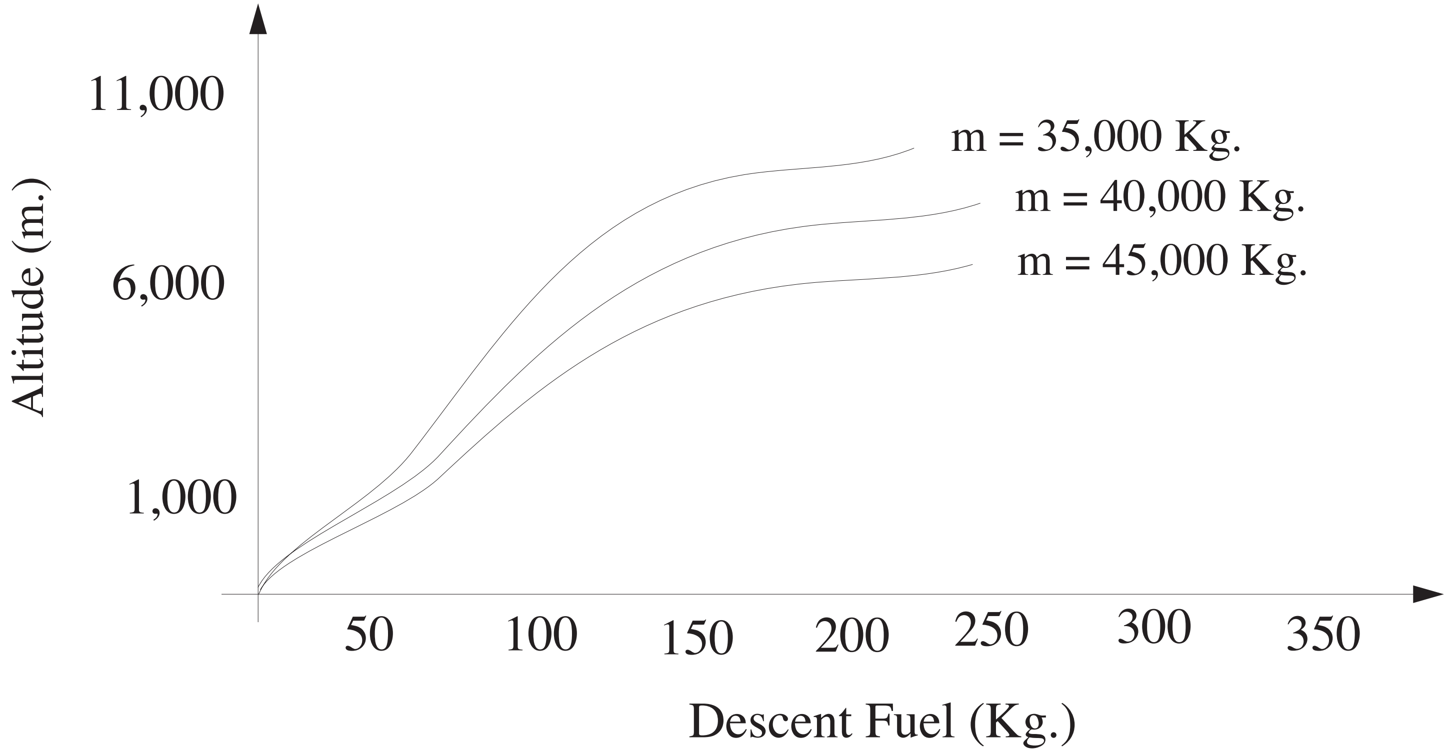
The flight path angle is negative and since thrust is limited, the vehicle glides at a specific speed starting from the Top of Descent Point (TOD) into the terminal area

The analysis of the descent flight is presented in detail with an example in the following pages.

Presentation of Descent Profile Information



Sample fuel descent charts for Fokker F-100 aircraft.



Adapted from Fokker 100 aircraft flight manual

Aircraft Performance Calculations: Descent Analysis

CEE 5614
Analysis of Air Transportation Systems

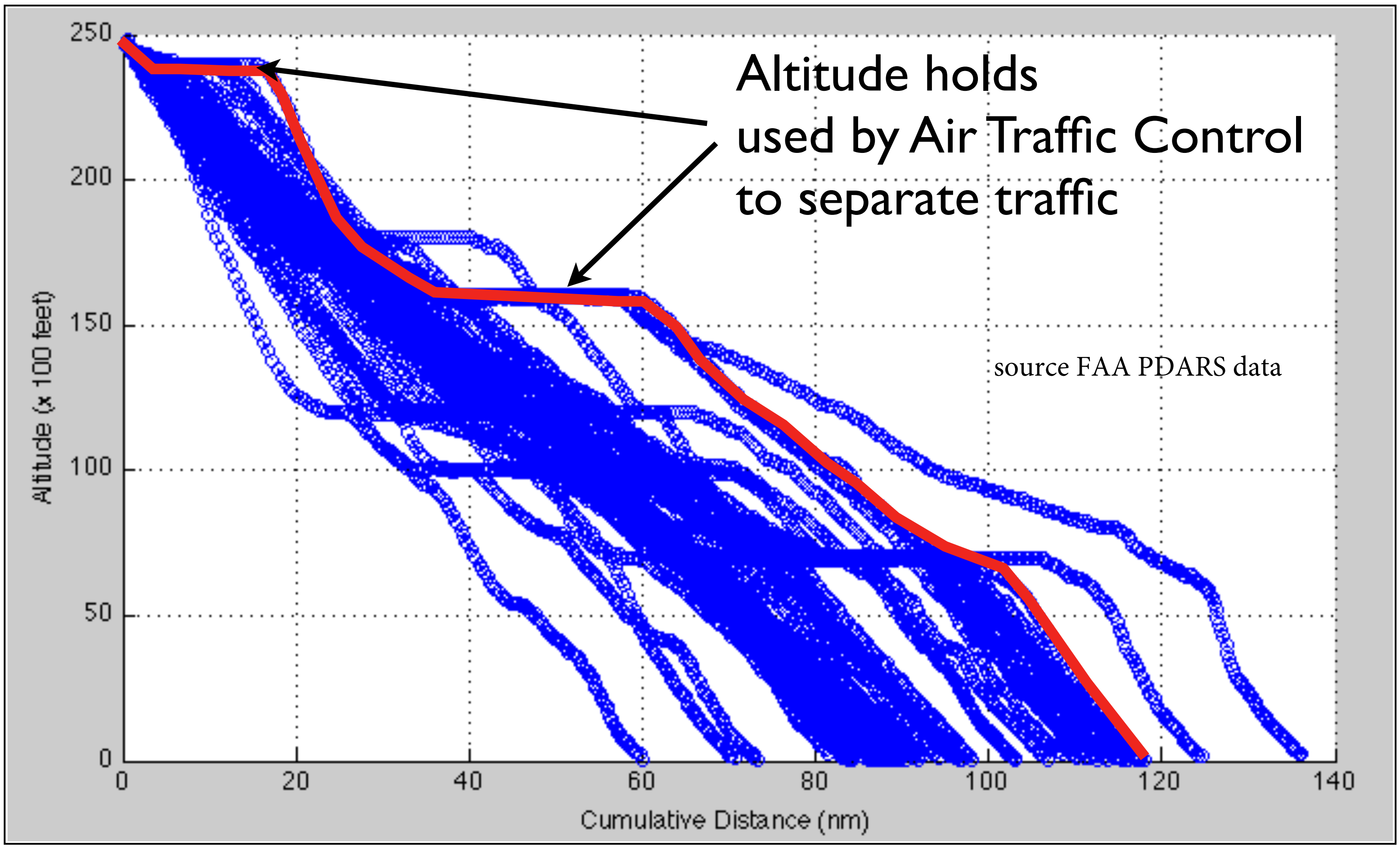
Dr. Antonio A. Trani
Professor

Aircraft Descent Performance

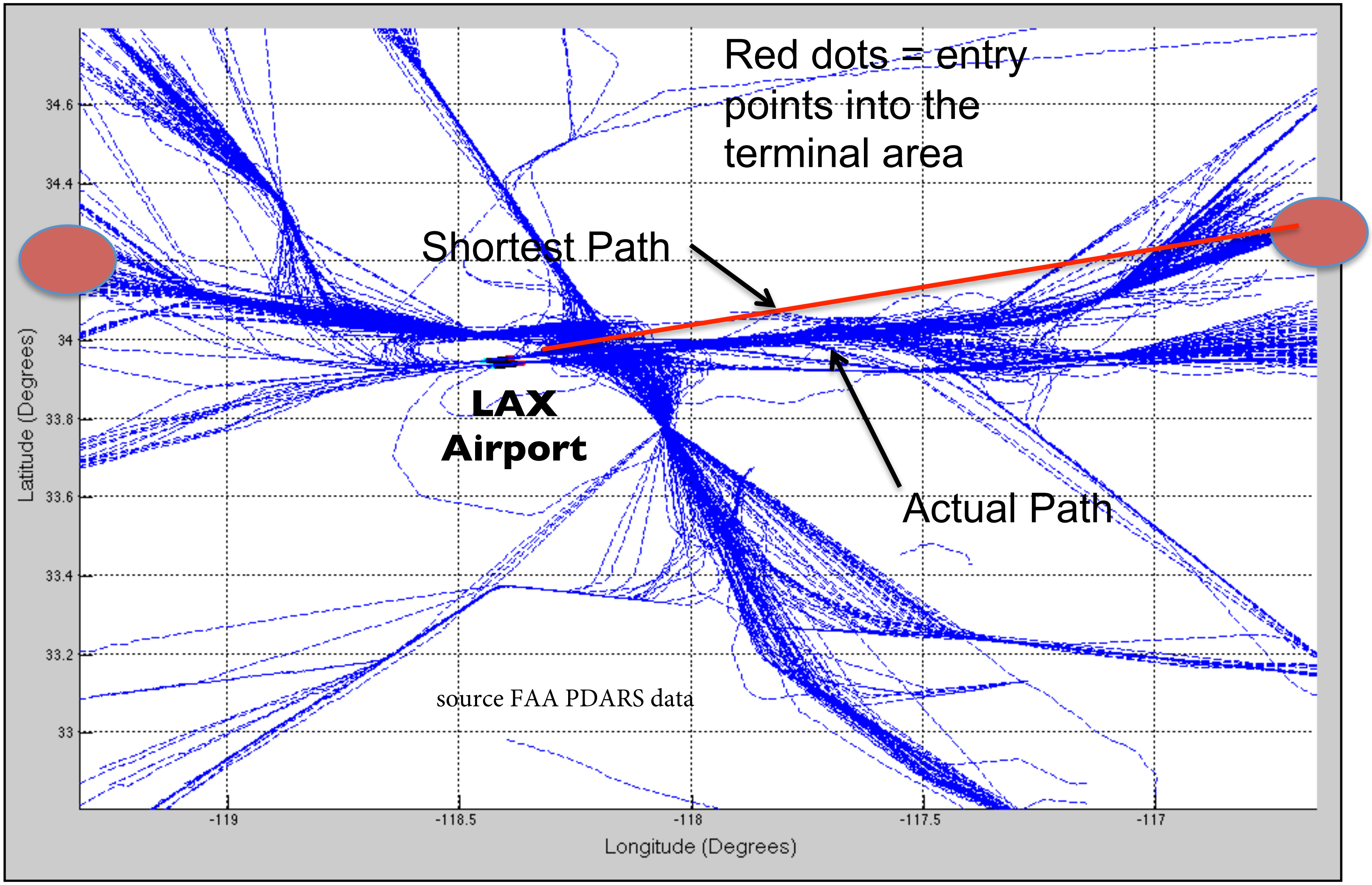
- The top of descent point typically starts 80-120 miles away from the destination airport (depending upon the cruise altitude assigned)
- A descent on commercial transport aircraft is initiated by setting the engine thrust to a very low power condition (i.e.,idle thrust)
- The analysis done for climb is now reversed
- Once in the airport terminal area, thrust adjustments are necessary to compensate for altitude holds or flap configuration changes as needed

Sample Descent Profile (LAX Data)

- Shown are sample descent profiles for Boeing 767-300 flying into LAX International airport
- Clearly, not all aircraft fly the same descent profiles



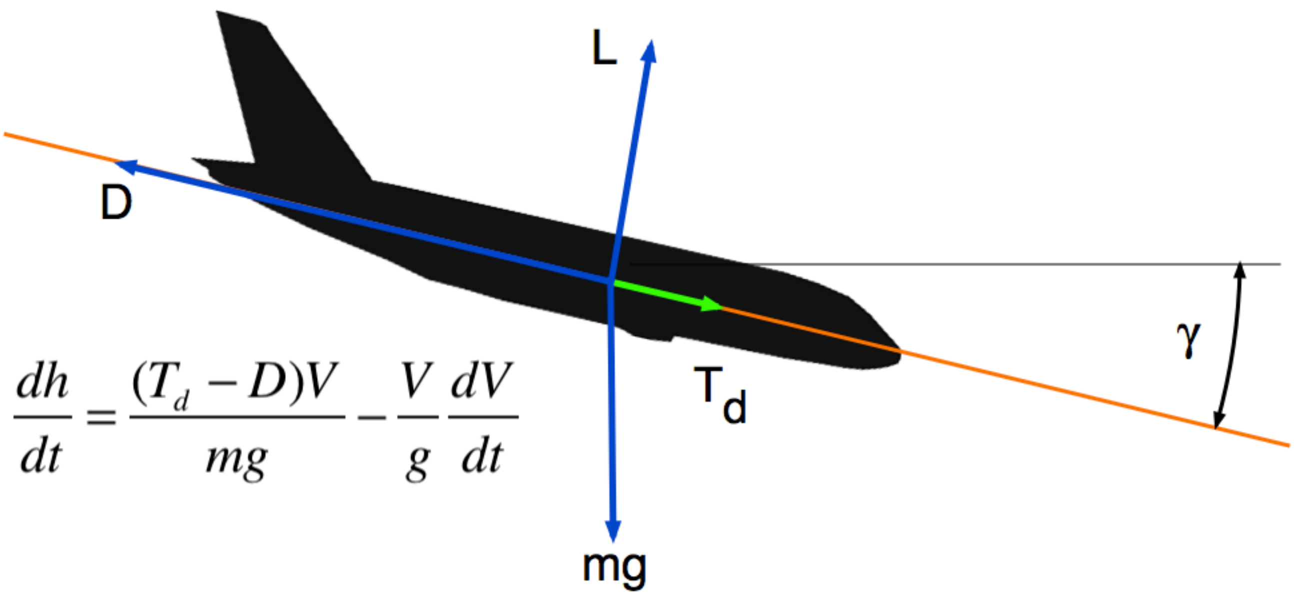
Arrival Flight Profiles into the LAX Airport Terminal Area



7

Aircraft Descent Performance Analysis

- The pilot reduces thrust to near idle conditions
- If we let the reduced thrust be T_d, then the analysis done for the climb procedure applies to the descent
- The most economical descent would be a continuous descent flown at idle conditions until a point where flaps and landing gear are deployed. At such point adjustments in thrust are required to maintain a safe rate of descent in the final approach



$$\frac{dh}{dt} = \frac{(T_d - D)V}{mg} - \frac{V}{g}\frac{dV}{dt}$$

Example - Aircraft Descent Performance Controlling the Speed Profile

- Very large capacity transport aircraft (http://128.173.204.63/courses/cee5614/cee5614_pub/AirbusA380_class.m)
- The aircraft descent speed is controlled by the last **two lines of data** in the aircraft data file
- Vdescent is the vector of descent speed for altitudes (altc)
- altc is a vector of altitudes to complete the table function of speed vs altitude
- For example: The aircraft below descends at 300 knots indicated at the top of descent. However below 3000 meters, the aircraft slows down to 250 knots or below.

```
% Computes the aircraft profile given altitude (Vcas given) – typical for
% four engine aircraft similar to the Airbus A380 aircraft

Vclimb = [210 210  220 230 250  260  290  290  290  290  290  300 300  300  300];      % knots IAS
Vdescent = [ 180 200  250 250 270  300  300  300  300  300  300  300 300  300  300];  % knots IAS
altc = [ 0 1000  2000 3000 4000  5000  6000  7000  8000  9000  10000  11000 12000  13000  14000]; % altitude (m
```

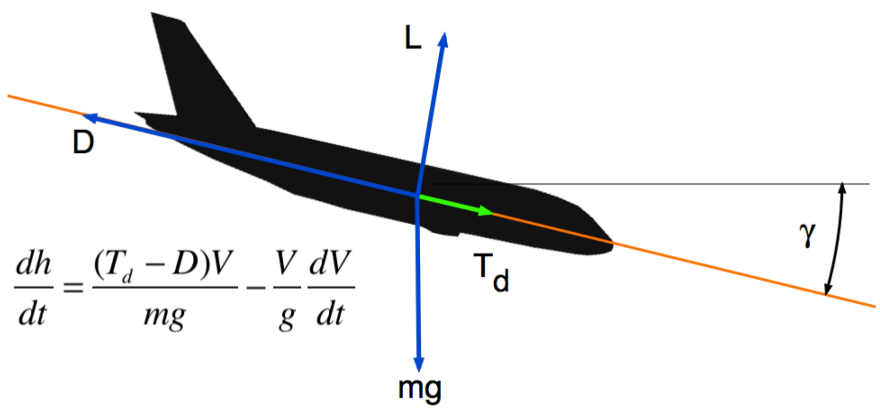
Example - Very Large Capacity Aircraft Data File

- An aircraft similar in size and performance as the Airbus A380
- Four turbofan engines each developing 34,400 kg (338,000 N) at sea level. **Assume idle thrust produces 1/10 of the full continuous thrust**
- **Top of Descent (TOD) mass is 400,000 kg.**

Airbus A380 taxies to the gate at LAX (A.A. Trani)

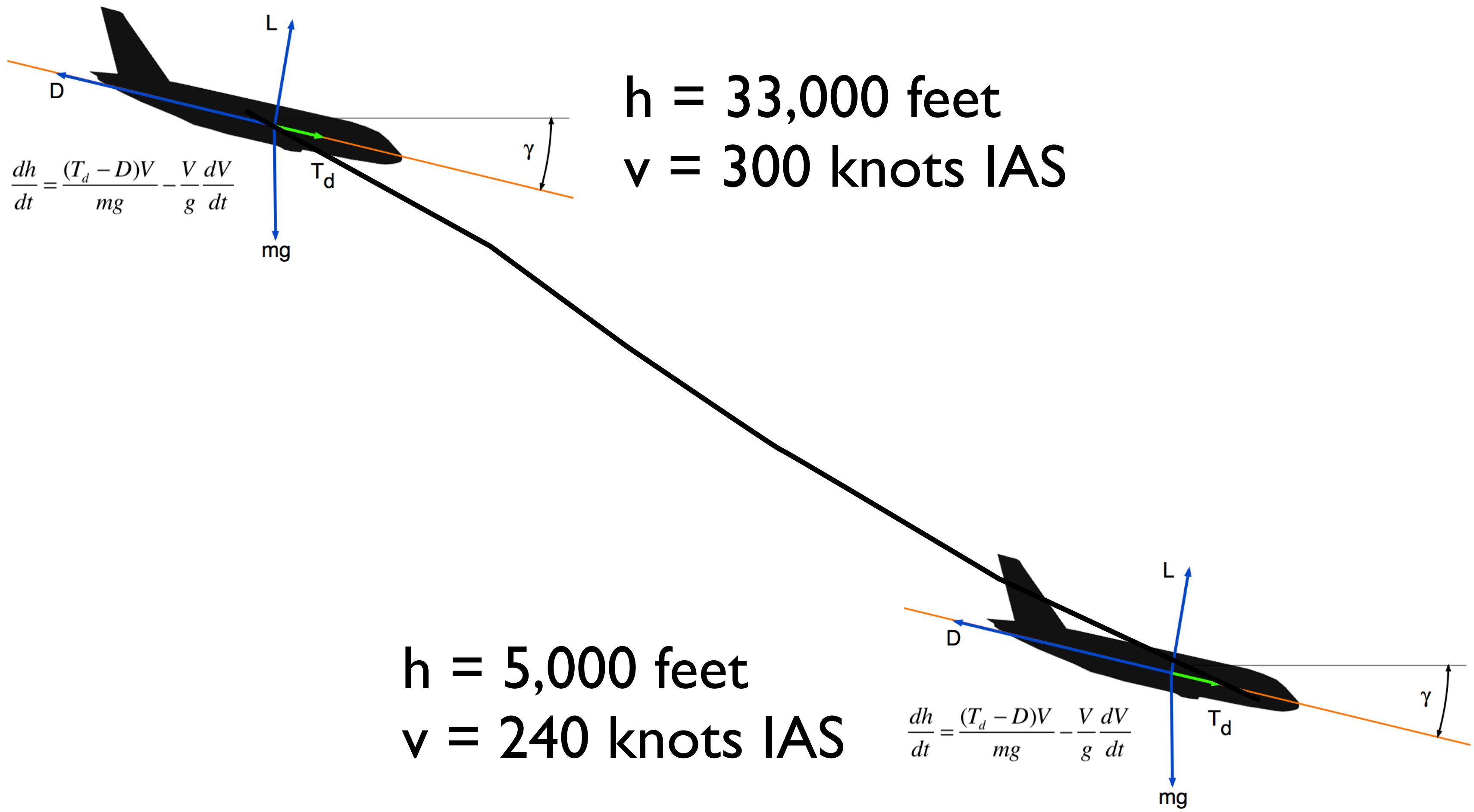
Descent Analysis Calculation Procedure

- Step 1: Estimate true airspeed using the atmospheric model
- Step 2: Estimate the lift coefficient needed to sustain flight using the basic lift equation
- Step 3: Estimate drag coefficient
- Step 4: Estimate total drag (D)
- Step 5: Estimate the reduced thrust produced by the engines at altitude (T_d)
- Step 6: Find the rate of descent (dh/dt)



$$\frac{dh}{dt} = \frac{(T_d - D)V}{mg} - \frac{V}{g}\frac{dV}{dt}$$

Sample Calculations for Two Aircraft States



Aircraft Descent Performance
33,000 feet (10,061 m) and 300 knots IAS

- Using the standard expression to estimate the true mach number of the aircraft at altitude,

$$M_{true} = \sqrt{5\left[\left\{\frac{\rho_0}{\rho}\left(\left[1+0.2\left(\frac{V_{IAS}}{661.5}\right)^2\right]^{3.5}-1\right)+1\right\}^{0.286}-1\right]}$$

- The true mach number is 0.751, the speed of sound at 10,061 meters is 299.2 m/s and the density of air is 0.41 kg/cu. m.
- The true airspeed (TAS) is 224.65 m/s or 437 knots
- Use the fundamental lift equation to estimate the lift coefficient under the known flight condition

$$L = mg = \frac{1}{2}\rho V^2 S C_l \longrightarrow C_l = \frac{2mg}{\rho V^2 S}$$

Aircraft Descent Performance
33,000 feet and 300 knots IAS

- The lift coefficient needed to maintain steady descent is,

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2*(400,000)(9.81)}{(0.41)(224.65)^2(858)} \frac{(kg)(m/s^2)}{(kg/m^3)(m/s)(m^2)} = 0.4421$$

- The lift coefficient is non-dimensional

- The drag coefficient can be calculated using the standard parabolic drag polar model

$$C_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi A \mathrm{Re}} = 0.020 + \frac{0.4421^2}{\pi(9.0)(0.84)} = 0.0282$$

- Note that the value of C_{do} is found by interpolation in the table function relating C_{do} and Mach number (C_d is non-dimensional)

Aircraft Climb Performance
33,000 feet and 300 knots IAS

- The total drag is,

$$D = \frac{1}{2}\rho V^2 S C_d = \frac{1}{2}(0.410)(224.65)^2(858)(0.0282) = 255{,}660\,N$$

- The residual thrust developed is assumed to be 1/10 of the thrust produced at altitude for the given Mach number and altitude

- The calculation of thrust is done in the same way as before. However, the solution is multiplied by 1/10 (assumed idle residual thrust) as shown in the next page

Aircraft Descent Performance
35,000 feet and 300 knots IAS

$$T_{0,M} = T_{0,M=0} - \lambda M_{true}$$

$$T_{0,M} = 338,000 - 175,560 M_{true}$$

$$T_{0,M} = 338,000 - 175,560(0.751)$$

$$T_{0,M} = 206,150 \text{ Newtons}$$

$$T_{h,M} = T_{0,M}\left(\frac{\rho_h}{\rho_0}\right)^m$$

$$T_{h,M} = 206,150\left(\frac{0.410}{1.225}\right)^{0.96}$$

$$T_{h,M} = 72,087 \text{ Newtons}$$

But thrust is just 1/10 of that produced by the engine, therefore,

$$\mathrm{T}_{produced} = \frac{1}{10} T_{h,M} = 7,209 \text{ Newtons}$$

For four engines,

$$\mathrm{T}_{total} = nT_{produced} = (4)(7,209) = 28,835 \text{ Newtons}$$

Aircraft Descent Performance
33,000 feet and 300 knots IAS

- The rate of descent of the aircraft can be calculated,

$$\frac{dh}{dt} = \frac{(T_d - D)V}{mg} =$$

$$\frac{dh}{dt} = \frac{(28,835 - 255,660)(224.65)}{400,000(9.81)} \frac{(N - N)}{kg(m/s^2)}$$

$$\frac{dh}{dt} = -12.98 \ m/s$$

- This is equivalent to 779 meters per minute or 2,556 feet per minute
- This descent rate is typical of transport aircraft at the TOD point
- The process is now repeated for state 2

Aircraft Descent Performance
5,000 feet and 240 knots IAS

- The true mach number is 0.390, the speed of sound is 334.3 m/s and the density of air is 1.056 kg/cu. m.
- The true airspeed (TAS) is 130.4 m/s or 253.4 knots
- The lift coefficient needed to maintain flight at 130.4 m/s is,

$$C_l = \frac{2mg}{\rho V^2 S} = \frac{2*(400,000)(9.81)}{(1.056)(130.4)^2(858)} \frac{(kg)(m/s^2)}{(kg/m^3)(m/s)(m^2)} = 0.5094$$

- The drag coefficient at 5,000 feet and 240 knots (IAS) can be calculated using the standard parabolic drag polar model

$$C_d = C_{do} + C_{di} = C_{do} + \frac{C_l^2}{\pi A \mathrm{Re}} = 0.020 + \frac{0.5094^2}{\pi(9.0)(0.84)} = 0.0309$$

- Note that the value of C_{do} at Mach 0.390 is 0.020

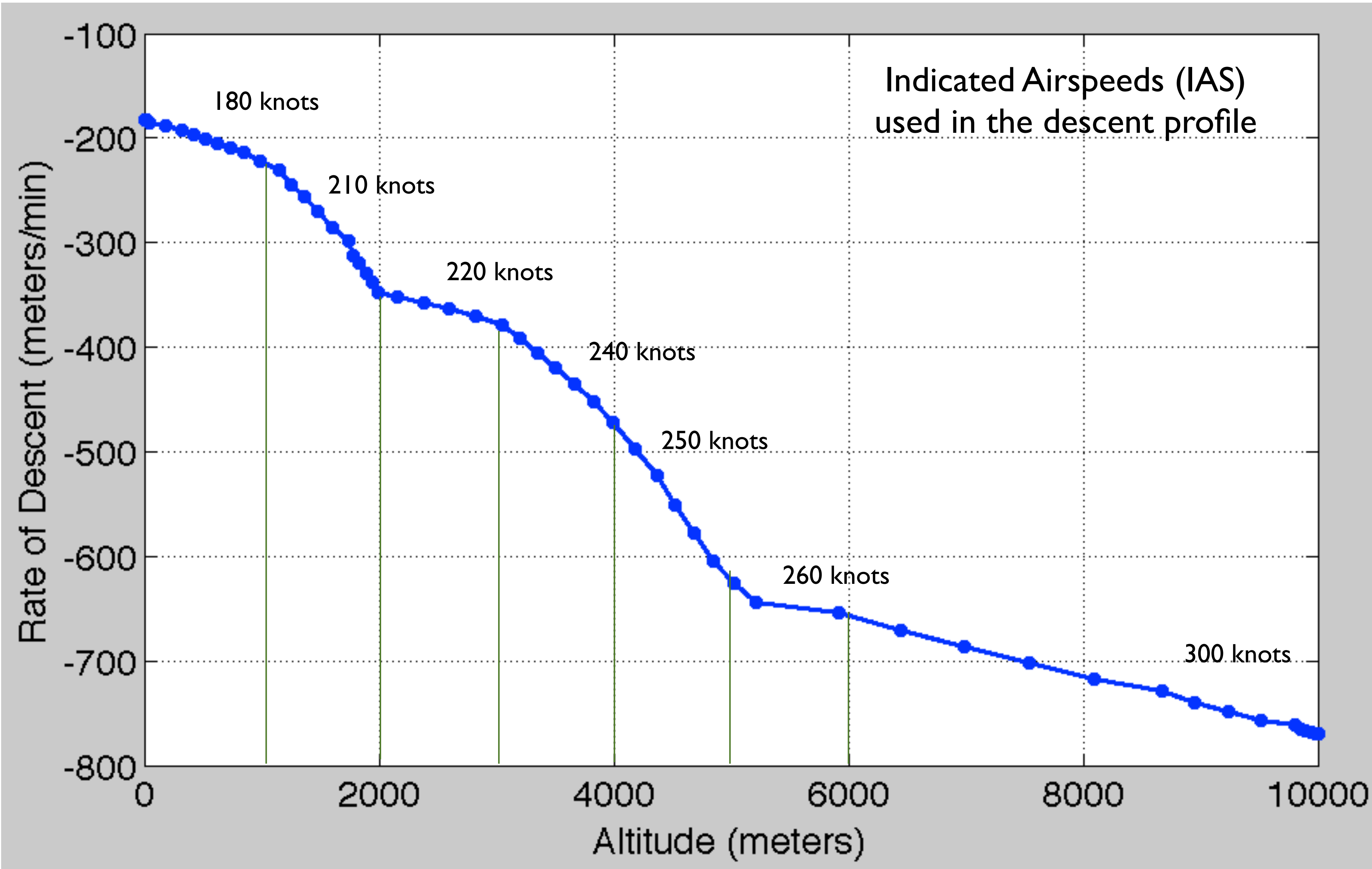
Aircraft Descent Performance
5,000 feet and 240 knots IAS

- The total drag is,

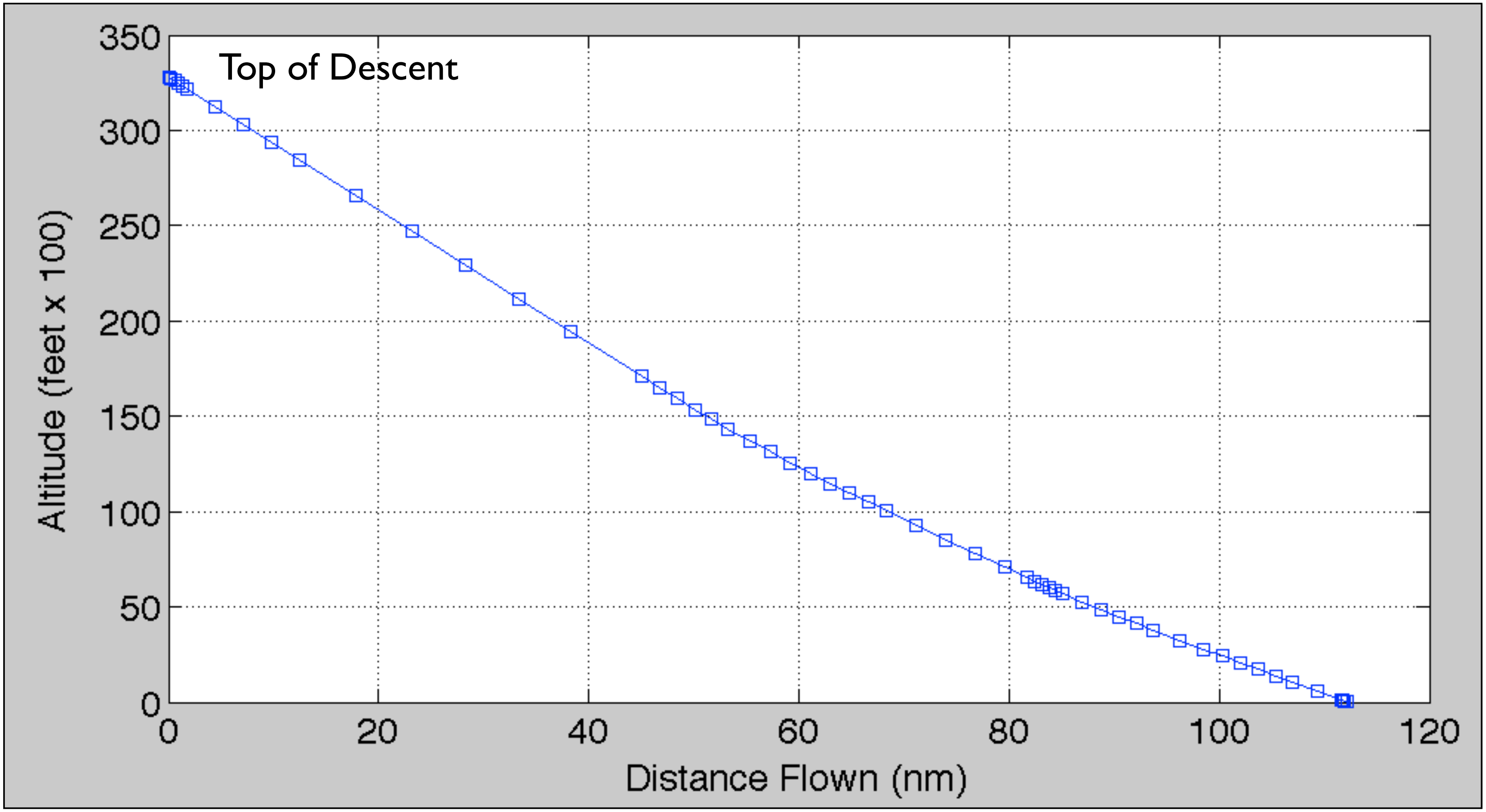
$$D = \frac{1}{2}\rho V^2 S C_d = \frac{1}{2}(1.056)(130.4)^2(858)(0.0309) = 238{,}230\,N$$

- The rest of the process can be easily computed
- Repeating the same steps outlined here we can derive a rate of descent equation for various altitudes
- The analysis presented in the following pages includes variations in aircraft weight as the the aircraft descents from teh Top of Descent (TOD) point to the airport elevation

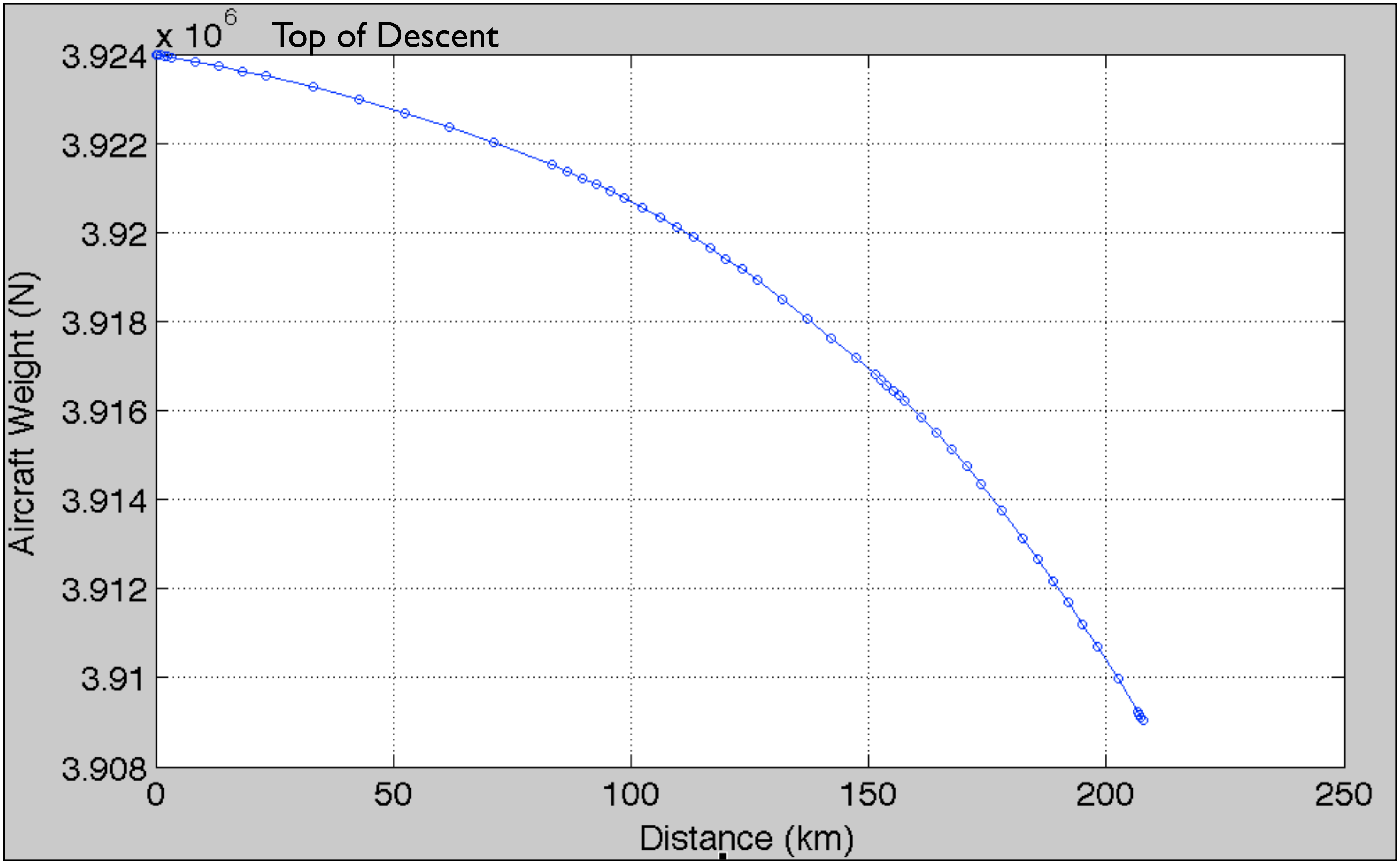
Rate Of Descent Analysis



Descent Profile for Very Large Capacity Aircraft



Descent Profile for Very Large Capacity Aircraft



Observations

- Rate of descent is controlled by the speed profile and the assumed residual thrust
- Typical rates of descent vary from 2600 ft/min (at TOD) to 700 feet per minute (at lower altitudes)
  - The final approach phase is not well represented in this analysis because flaps and landing gear are usually deployed below 3,000 feet and change the character of the drag coefficient (i.e., higher drag coefficient)
  - The aircraft mass changes by 1,526 kg in the descent. This a relatively small amount of fuel for a vehicle that could carry 182,000 kg of fuel at takeoff
- The aircraft performs a continuous descent from TOD to the airport elevation and travels 112 nautical miles

Implications for Real-world Aviation Operations

- The performance of the aircraft has profound effects in real-world flight planning applications
  - Obstacle accountability analysis
    - Obstacle clearance procedures in the terminal area (before landing)
- Current terminal operations do not support continuous descent approaches but for a few, isolated flights
- Continuous descent profiles are expected to save fuel and time once NextGen technologies are implemented

Use of Matlab Code

- The previous analysis has been done using the **UnrestrictedDescentAnalysis.m** program
- This main file integrates numerically the equations of motion of the aircraft
- Four state variables:
  - Altitude (y(1))
  - Aircraft weight (y(2))
  - Distance traveled along path (y(3))
  - Distance traveled along the plane of the earth (y(4))
- The initial conditions of the states are specified in the file under line 54
- `yN = [h_TOD  Mass_init  0 0];   % Vector of initial values of state variables`

UnrestrictedDescentAnalysis.m

- Main program to execute the descent analysis
- Employs Matlab Ordinary Differential Equation solver (ODE15s)
- Function Calls:
    - **fdescent_06.m** - function that contains the equations of motion of the aircraft in the descent phase
    - **densityAltitudeoffISA.m** - function to estimate the atmospheric conditions for both ISA and non-ISA conditions
    - **drag03.m** - function to estimate the aircraft drag at any altitude (h) and Mach number (M)
    - **thrust_calculation.m** - fundtion to estimate the thrust produced by the engine for any Mach number and altitude (h) condition

UnrestrictedDescentAnalysis.m

- **Inputs to the Program**
- Aircraft file to be used in analysis (line 36)

```
33      % Enter aircraft file desired - reads a file with aircraft characteristics
34      % eclipse500New_class
35      % regionalJetDescent
36 -    AirbusA380_class                 % aircraft file used
```

- Speed profile. Descent speed profile specified as a table function in the aircraft file (lines 30 and 31 in aircraft file)

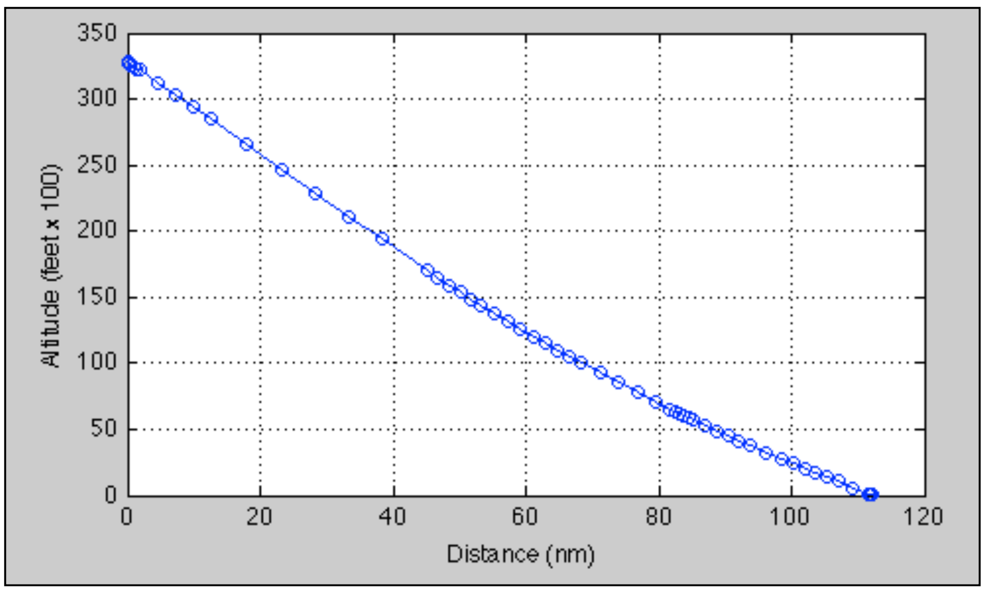
```
29 -    Vclimb = [210 210  220 230 250  260  290  290  290  290  290  300 300  300  300];      % knots IAS
30 -    Vdescent = [180 200  250 250 270  300  300  300  300  300  300  300 300  300  300];   % knots IAS
31 -    altc = [ 0 1000  2000 3000 4000  5000  6000  7000  8000  9000  10000  11000 12000  13000  14000];
```

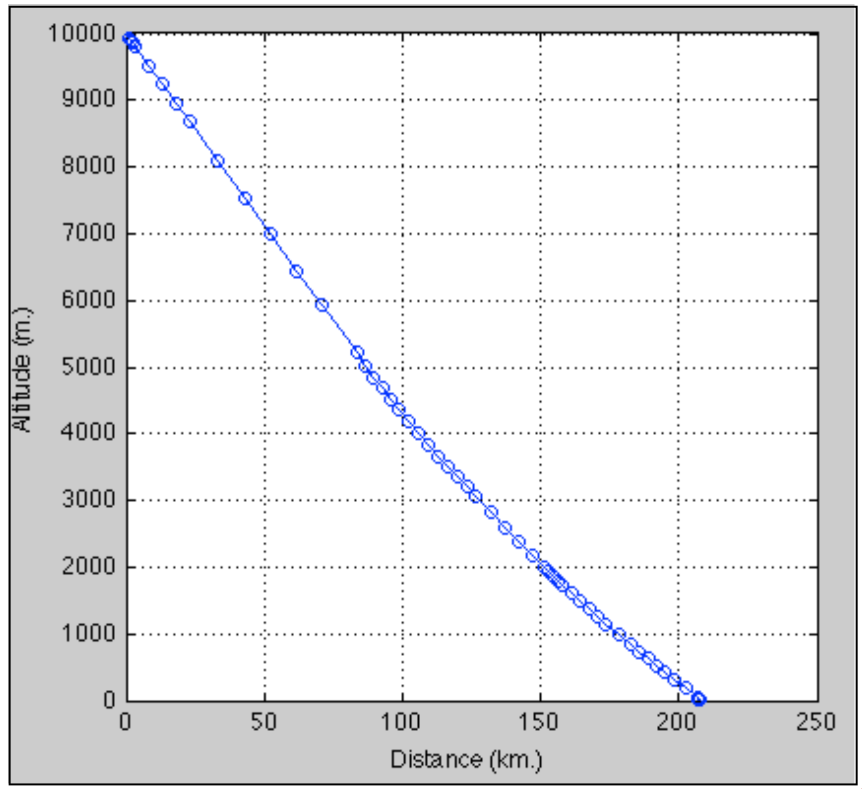
- Initial aircraft states (lines 38-41 in main program)
    - altitude, mass, distance traveled along path and distance traveled along a flat earth

```
38 -    h_TOD = 10000;              % initial altitude (m) - at Top of Descent Point
39 -    mass_TOD = 400000;          % mass at TOD point (kg)
40 -    rhos = 1.225;               % sea level density (kg/m-m-m)
41 -    deltaTemp = 0;              % ISA + deltaTemp conditions for analysis (de
```

UnrestrictedDescentAnalysis.m

- **Outputs of the Program**
  - Results of the four aircraft state variables in the climb profile (altitude, mass, distance traveled along path and distance along flat earth)
  - Plots of state variables vs. time
  - Plot of state variables vs. distance

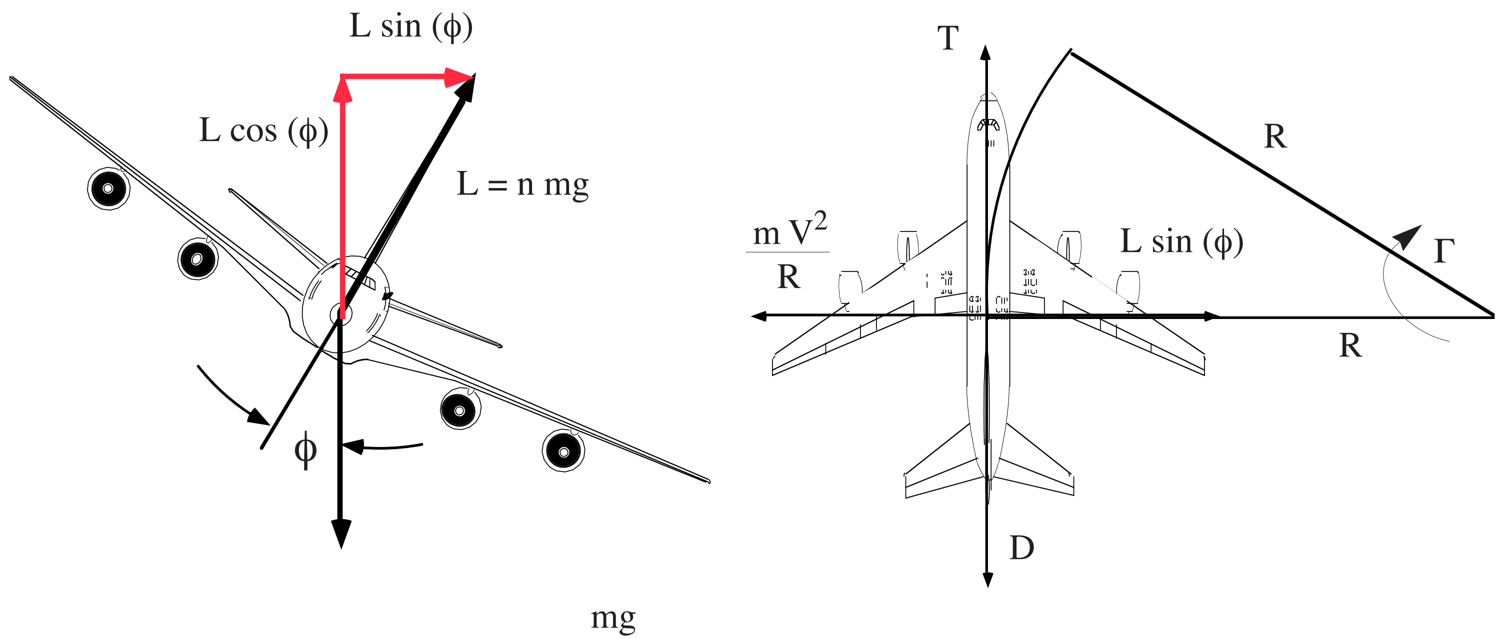




Basic Turning Performance

- An important consideration in air transportation systems analysis (i.e., terminal areas operations and climb out procedures)
- Turning and climbing are the two most common maneuvers executed in the terminal area while an aircraft transitions from enroute airspace to the terminal and airport areas.

Basic Turning Performance Diagram



Turning Performance Analysis

- The basic forces acting on a turning aircraft that executes a steady level turn. It must realized that in many instances aircraft are instructed (or commanded by the pilot) to turn while climbing and descending. The equations of motion can be modified to include these three dimensional effects.

Balance of forces along the vertical axis (z-axis in aeronautical terms) yields,

$$L \cos\phi = mg \tag{40}$$

Turning Performance Basics

Similarly, balancing forces perpendicular to the turning motion,

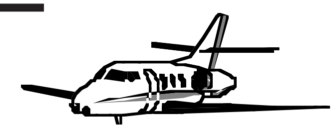
$$L \sin\phi = \frac{mV^2}{R} \tag{41}$$

Note that along the flight path (x axis of the aircraft) thrust and drag are the same if the aircraft is in unaccelerated flight.

$$T = D = \frac{1}{2}\rho V^2 S C_D \tag{42}$$

Using the previous equations we can derive the radius of the turn, R for a given bank angle (ϕ) and airspeed (V), and the resulting turn rate (Γ).

Turning Radius and Rate of Turn Analysis



$$R = \frac{V^2}{g\tan\phi} \tag{43}$$

and,

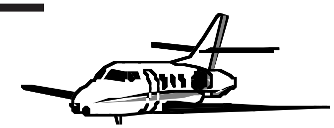
$$\Gamma = \frac{V}{R} = \frac{g\tan\phi}{V} \tag{44}$$

In many instances you will find that pilots and engineers define the so-called load factor, n, as follows:

$$n = \frac{L}{mg} = \frac{1}{\cos\phi} \tag{45}$$

This parameter tells us how large the lift vector has to be to overcome the weight of the aircraft while turning.

Turning Performance



Note that as the bank angle (ϕ) increases so does L to maintain a coordinated, level turn.

Substituting n into equations 43 and 44 yields,

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} \tag{46}$$

and,

$$\Gamma = \frac{g\sqrt{n^2 - 1}}{V} \tag{47}$$

In practical airspace operations commercial aircraft seldom bank more than 30 degrees to keep passengers in comfort (this implies a load factor of 1.16 or less).

Standard Turn

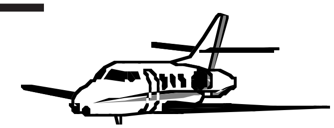
- It is also interesting to note that from the ATC perspective a standard turning maneuver is usually assumed in the design of terminal area flight paths using a three-degree per second turn rate.
- This implies that in a standard turn the aircraft takes one minute to complete a 180^0 maneuver.

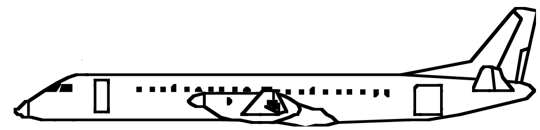
Example of Terminal Area Maneuvering

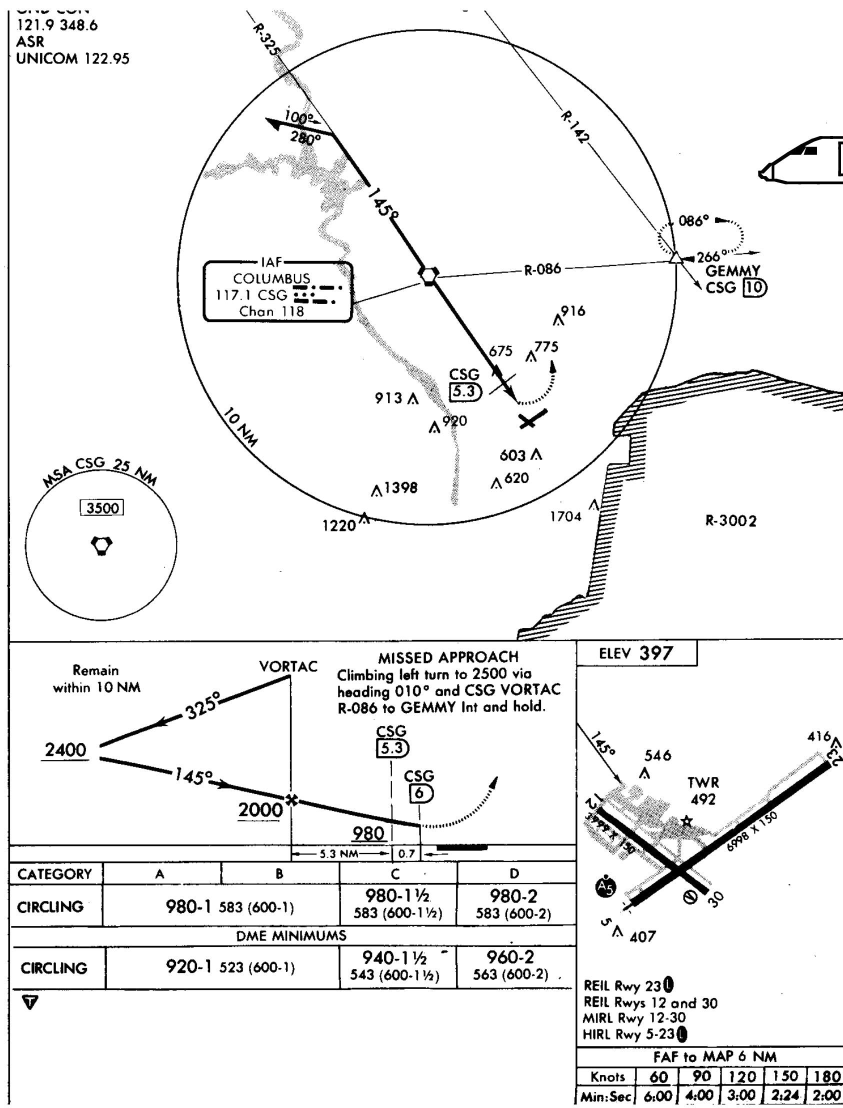
Suppose a Saab 2000 commuter aircraft approaches an airport and executes a VOR non-precision approach to Columbus, Georgia runway 12. This approach requires a flight outbound from the VORTAC and execute a procedure turn (225^0) before landing (see Figure).

If the pilot maneuvers the aircraft down to 150 knots (indicated airspeed) while executing the procedure turn. Find the bank angle, the load factor imposed on cargo and passengers, the turning time (t_t) and the radius of turn in the maneuver.

Approach Plate to Columbus, Georgia





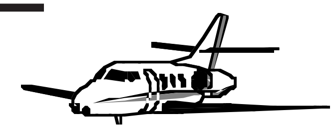


Solution

- Solving for the load factor in Equation 35 and substituting the corresponding values for g, V and Γ yields a bank angle of 9.35 degrees and a load factor of 1.0315.
- The resulting turn radius of the maneuver is 5,305 meters (2.86 nm). Note that the approach plate calls for the aircraft to stay within 10 nautical miles of the Columbus VOR (called initial approach fix).
- Note also that this aircraft is expected to complete the procedure turn while at 2,400 ft. above mean sea level (the airport is at 407 ft. MSL) and then descend to 2,000 ft. MSL at the VORTAC (see insert in left hand corner of

the approach plate) and then continue towards the airport.

Solution (cont.)



- The aircraft continues Missed Approach Point (MAP) altitude of 980 ft. MSL (the Saab 2000 is classified as a TERP B aircraft for ATC procedures).
- According to the approach chart this aircraft would take more than 2 minutes if flown at 180 knots from the Final Approach Fix (FAF) to the MAP. A more reasonable speed in final would be 120 knots for this aircraft.
- In the computation of turning radii true airspeeds should always be used. This is because the design of ATC procedures always looks at the topographical obstructions surrounding the airport facility to avoid collisions with terrain.

Final Note on Turning Performance Example

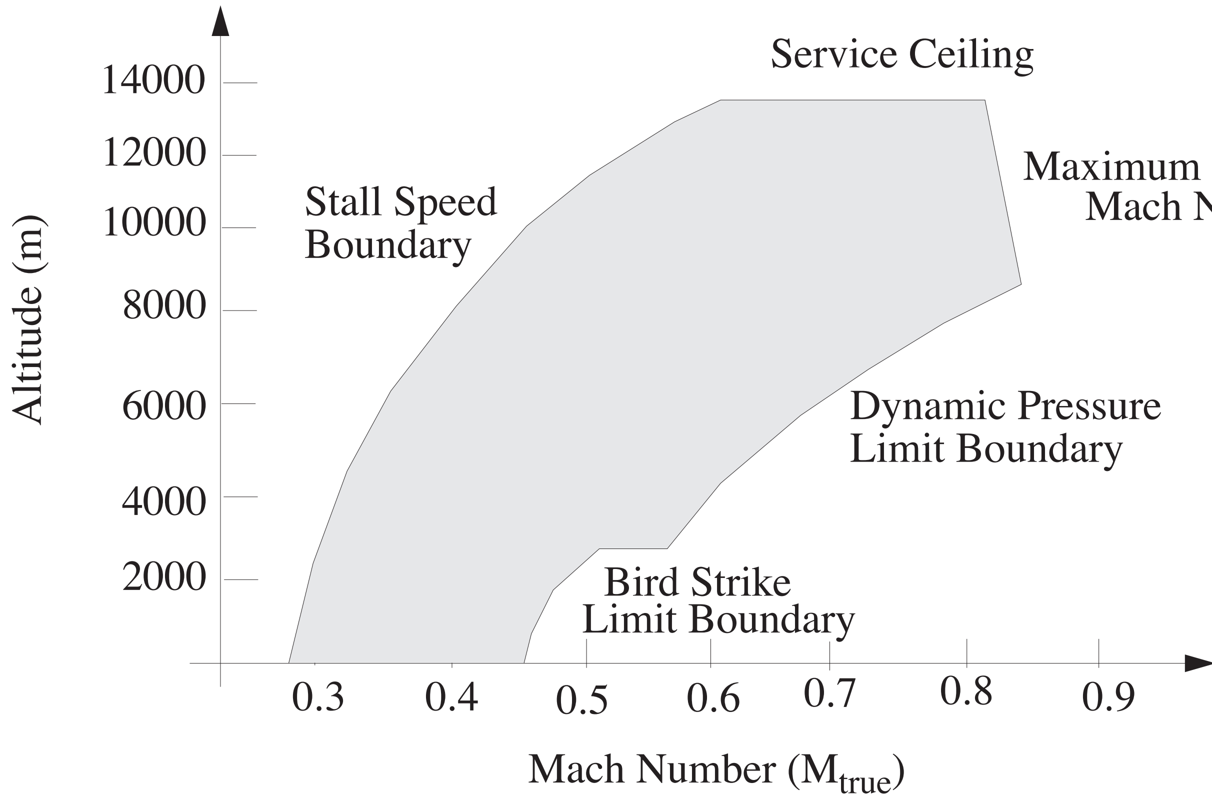
- The implication of using true airspeed is that TAS increases dramatically with altitude resulting in very large turning radii at moderate and high altitudes.
- For example an aircraft flying at an altitude of 5.0 km and 250 knots IAS - 300 knots true airspeed - would require a turning radius of 5.0 nautical miles while executing a standard turn.

Aircraft Flight Envelope Characteristics

- Once the analysis of climb, cruise and descent trajectories has been made we are in the position to draw the typical boundaries that restrict the operation of an aircraft in flight.
- The following figure illustrates a typical flight envelope for a turbofan-powered, subsonic aircraft.

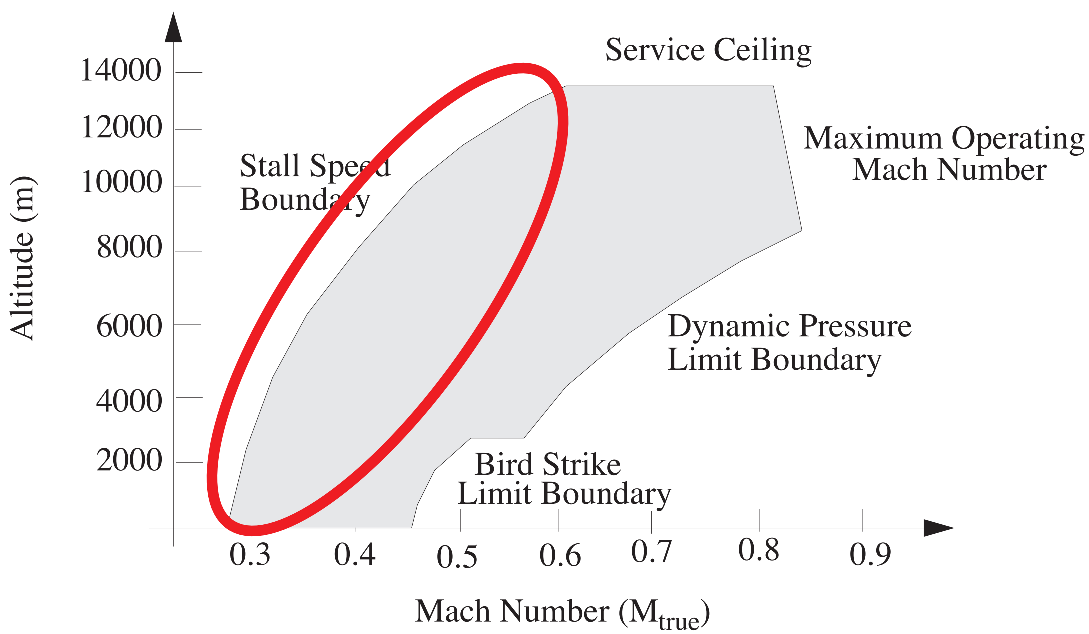
Aircraft Envelope Analysis

Typical subsonic aircraft envelope

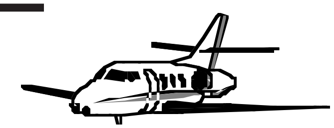


Low Speed Aircraft Envelope Boundary

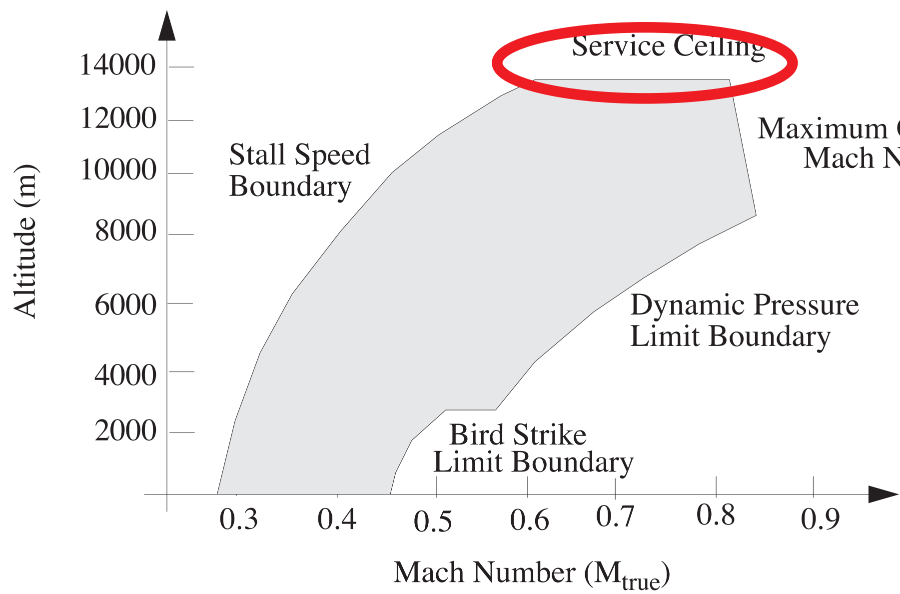
- Low speed boundary indicating that the aircraft wing can only produce enough lift for a given speed at various altitudes.
- Stalling speed (in terms of true airspeed) increases with altitude



Service Ceiling Aircraft Envelope Boundary

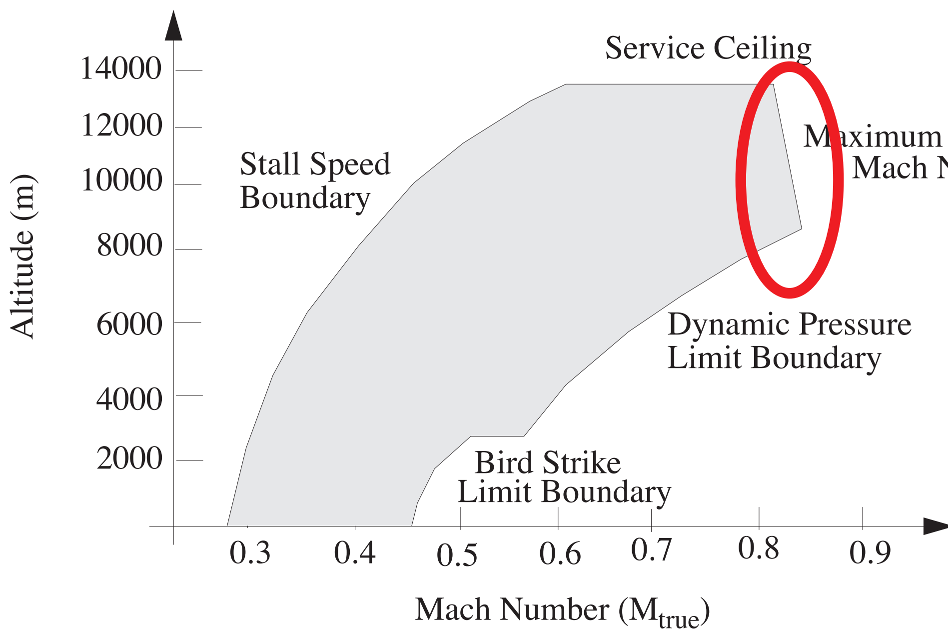


- Is the maximum altitude that the aircraft attains while climbing at a very small climb rate (typically 100 ft./min. according to FAR Part 25 regulations). Modern transport aircraft such as the Boeing 757-200 and the Boeing 777 have been certified to fly up to 13,720 m (45,000 ft.) at moderate to light weights.



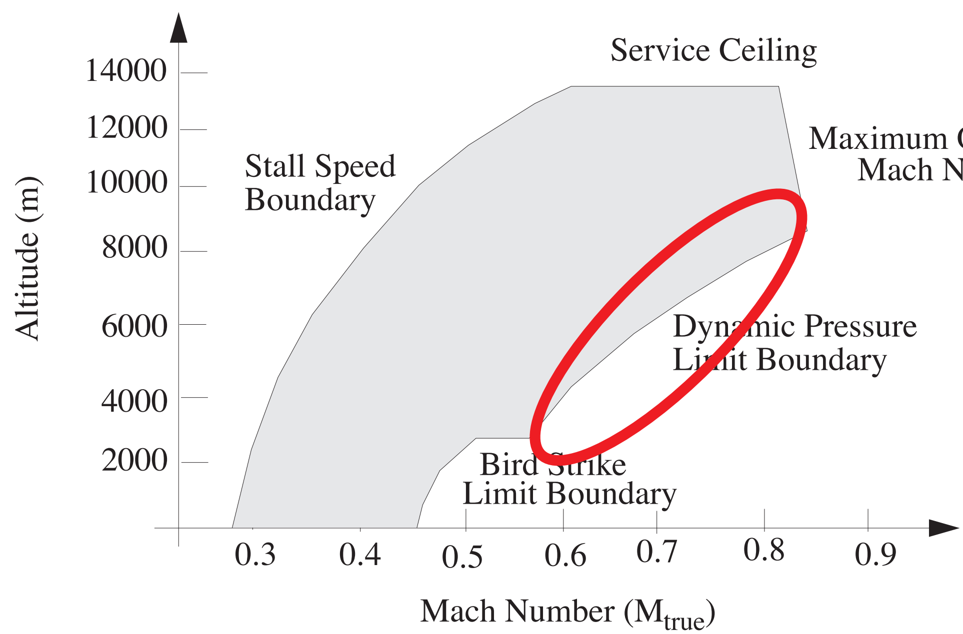
Maximum Speed Aircraft Envelope Boundary

- The aircraft reaches a region of flight where the drag produced increases sharply (i.e., drag divergence Mach number boundary) and thus the aircraft engines are incapable of producing enough thrust to accelerate the aircraft to faster speeds.



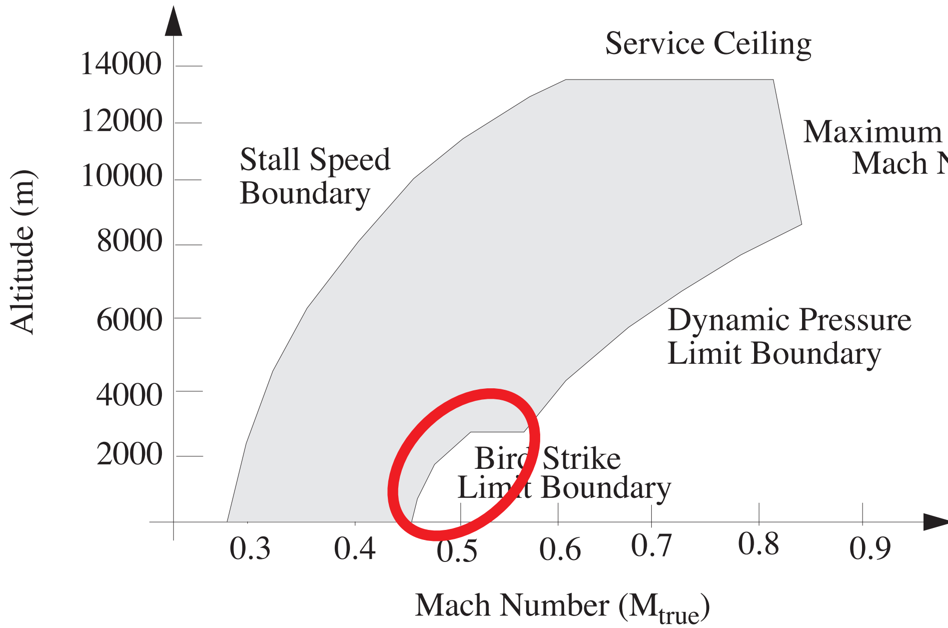
Dynamic Pressure Aircraft Envelope Boundary

- The design of all aircraft structures carries an assumption about the maximum loads that can be tolerated in flight. For our hypothetical aircraft a maximum dynamic pressure limit of 25,490 kg/m^2 has been used.
-



Bird Strike Aircraft Envelope Boundary

- Common sense and certification of flight deck windshields dictated a natural boundary below 10,000 ft. Traditionally this boundary has been set at 250 knots.

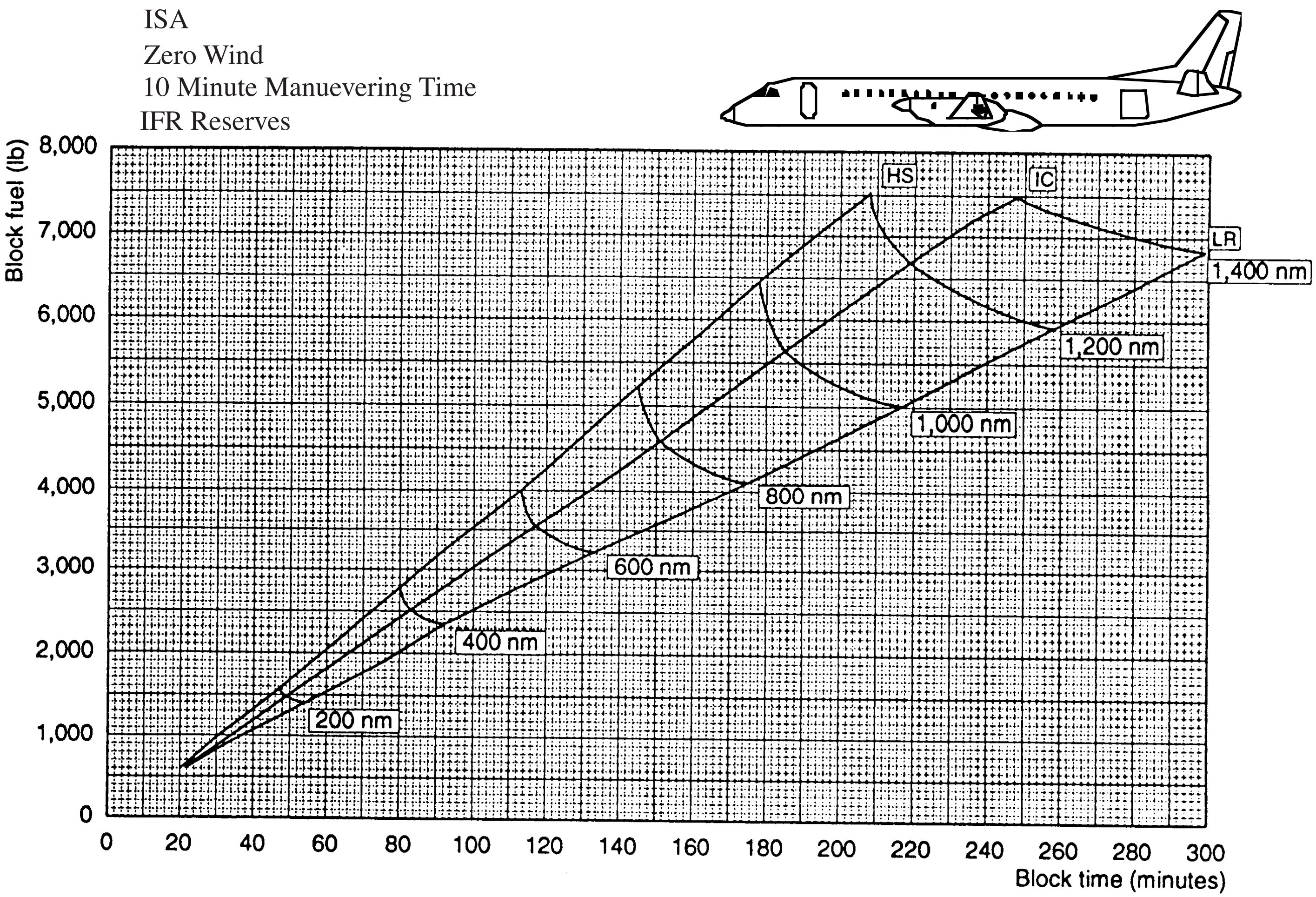


Fuel and Block Time Diagrams

The complete understanding of a flight trajectory allows us to estimate block time and block fuel for an entire trip.

- Block time is defines as the time it takes an aircraft to complete its trip from gate to gate. This may include taxiing times and departure delay times that are common today in NAS operations.
- The figure illustrates a typical presentation of block fuel and block times for the Saab 2000 commuter aircraft. In this figure we identify three operating speed regimes: 1) high speed (HS), 2) typical cruise (TC) and 3) long-range (LR).

Sample Fuel and Block Time Diagram



Interpretation Fuel and Block Time Diagrams

- It is evident from this diagram that a clear trade-off exists between block speed and block fuel.
- For example, if the airline operator wants to fly this aircraft between two cities 800 nm apart it could use a long range speed profile taking 175 minutes and consuming 4,100 lb. of fuel.
- The same operator could use a high-speed profile using 5,250 lb. of fuel and taking 145 minutes for the same trip.
- One question that perhaps we should ask ourselves is whether or not a 30 minute block time savings is significant or not and at what operating cost.

Interpretation of Fuel and Block Time Diagram

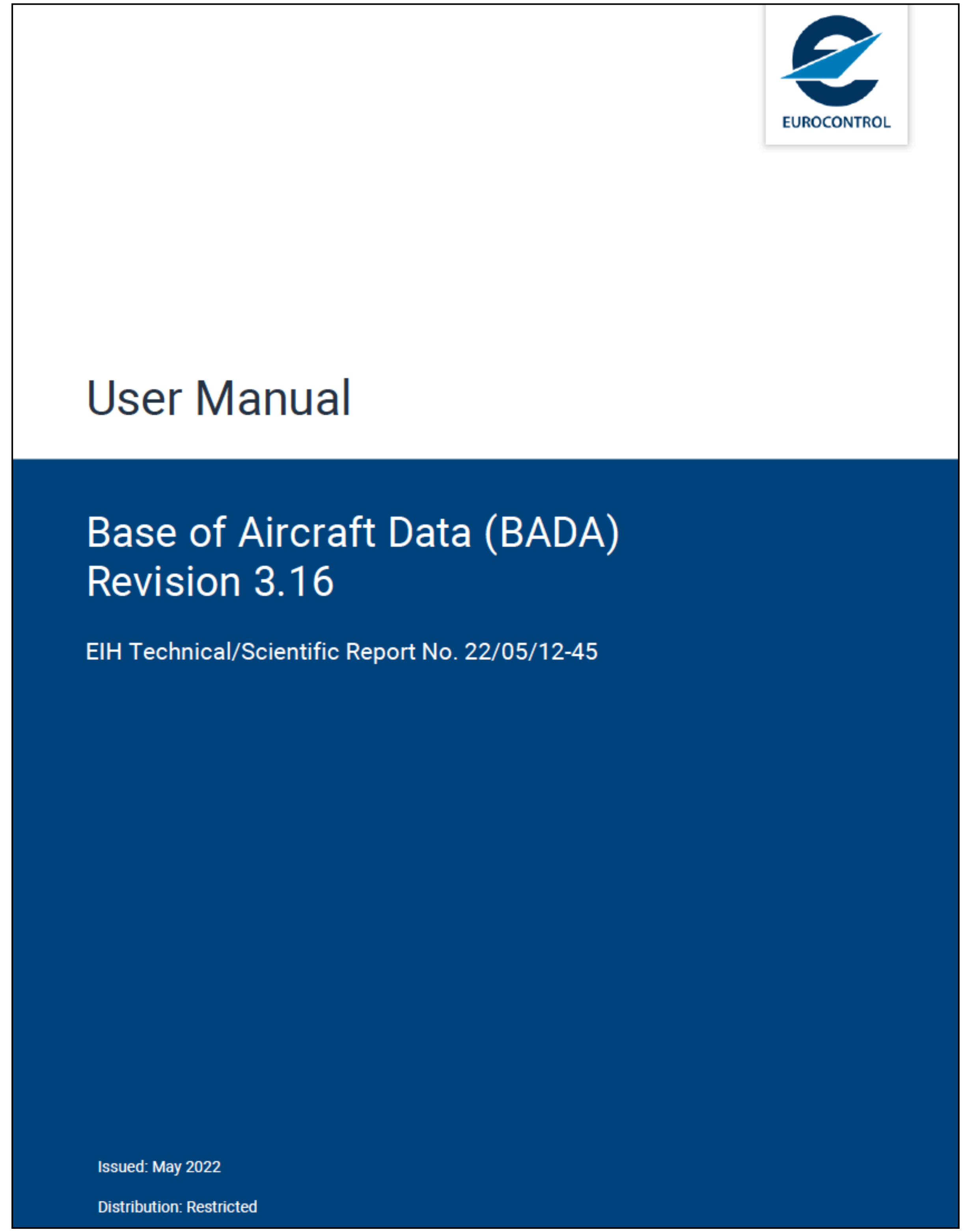
- Saving 1,150 lb. of fuel could be significant considering that an aircraft of this type makes three to four trips per day. This could easily translate into several hundreds of thousand pounds of fuel saved in a year
- It is interesting to note that few airlines operate their aircraft at the most economical speed regime (i.e., long range) because in long trips the resulting block times could be quite high thus reducing the number total trips that a single aircraft completes in one day
- The operational cost is directly linked to the productivity of the aircraft in terms of the number of seat-miles offered. Therefore, faster block times could make a difference in the profit of the operator.

Sample Flight Performance Models

- BADA - Eurocontrol
  - + Trajectory and fuel burn
- OPGEN - FAA and CSSI
  - + Optimal trajectory and fuel burn
- ASAC - NAS and LMI
  - + Optimal trajectory and fuel burn
- VPI Neural Network Fuel Burn and FTM Model
  - + Fuel burn and flight trajectory (using BADA data)
- Most airspace and airport simulators have their own fuel burn models (Old SIMMOD fuel consumption model and TAAM fuel consumption table functions)

The BADA Performance Model

- Developed by Eurocontrol Experimental Centre (ECC) to model various Air Traffic Management (ATM)
- 313 aircraft modeled directly
- Approximated aircraft drag polars are included in the model
- Current version 3.16
- BADA 4 has more detail but is restricted to a few aircraft


EUROCONTROL

User Manual

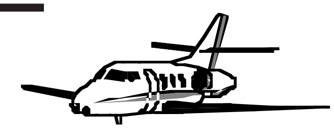
Base of Aircraft Data (BADA)
Revision 3.16

EIH Technical/Scientific Report No. 22/05/12-45

Issued: May 2022

Distribution: Restricted

BADA Data Organization



BADA Model

Operations Performance File (OPF)

1) Aero coefficients
2) Mass coefficients
3) Flight envelope restrictions
4) Engine thrust
5) Fuel consumption

Airline Procedures File (APF)

Recommended speed procedures: climb, cruise and descent

Performance Table File (PTF)

Typical speed procedures for climb, cruise and descent at ISA conditions

Airbus A320 OPF File

Aircraft mass Values

Drag Polars

$$Cd = C_{do} + C_{d2} * k$$

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC A320__.OPF CCCCCCCCCCCCCCCCCCCCCCCCCCCC/
CC                                                                    /
CC                AIRCRAFT PERFORMANCE OPERATIONAL FILE               /
CC                                                                    /
CC                                                                    /
CC      File_name: A320__.OPF                                         /
CC                                                                    /
CC      Creation_date: Apr 30 2002                                    /
CC                                                                    /
CC      Modification_date: Apr 08 2019                                /
CC                                                                    /
CC                                                                    /
CC====== Actype ======================================================/
CD    A320__         2 engines    Jet                         M       /
CC   A320-231 with V2500-A1 engines                          wake     /
CC                                                                    /
CC====== Mass (t) ====================================================/
CC     reference      minimum      maximum     max payload  mass grad  /
CD      .64000E+02   .39000E+02   .77000E+02   .21500E+02   .43250E+00 /
CC====== Flight envelope =============================================/
CC      VMO(KCAS)      MMO         Max.Alt       Hmax      temp grad  /
CD      .35000E+03   .82000E+00   .41000E+05   .33295E+05   -.3136E+03 /
CC====== Aerodynamics ================================================/
CC Wing Area and Buffet coefficients (SIM)                            /
CCndrst Surf(m2)     Clbo(M=0)      k          CM16                   /
CD 5   .12260E+03   .14041E+01   .79242E+00   .00000E+00              /
CC    Configuration characteristics                                   /
CC n Phase  Name     Vstall(KCAS)    CD0          CD2        unused   /
CD 1 CR    CLEAN     .14050E+03   .26659E-01   .38726E-01   .00000E+00 /
CD 2 IC    1         .11800E+03   .23000E-01   .44000E-01   .00000E+00 /
CD 3 TO    1+F       .11210E+03   .33000E-01   .41000E-01   .00000E+00 /
CD 4 AP    2         .10510E+03   .38000E-01   .41900E-01   .00000E+00 /
CD 5 LD    FULL      .10130E+03   .96000E-01   .37100E-01   .00000E+00 /
```

Sample BADA 3.0 APF File

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC B767__.APF CCCCCCCCCCCCCCCCCCCCCCCCCCCC
CC
CC                 AIRLINES PROCEDURES FILE
CC
CC    File Name        Current Revision            Last Modification
CC                     revision    date            revision     date
CC    B767__.APF           3.0   98/03/12          2.4.1.2    96/09/05
CC
CC   BADA Revision:
CD   Rev  3.0
CC
CC   LO= 90.00 to ---.--  /  AV= ---.-- to ---.--  /  HI= ---.-- to 181.40
CC
CC==================================================================================
CC  COM CO    Company name ------climb-------  --cruise--  -----descent------  --app
CC                 mass  lo  hi                  lo  hi         hi  lo           (ur
CC    version engines  ma  cas cas mc xxxx xx  cas cas mc  mc cas cas xxxx xx  xxx x
CC===:=======:=======:===:====:===:===:====:===:====:===:===:===:===:===:====:===:====:
CD  *** **    Default Company
CD    300ER   PW4060   LO  290 290 78          310 310 80  78 290 290             0
CD    300ER   PW4060   AV  290 290 78          310 310 80  78 290 290             0
CD    300ER   PW4060   HI  290 290 78          310 310 80  78 290 290             0
CC===:=======:=======:===:====:===:===:====:===:====:===:===:===:===:===:====:===:====:
CC/////////////////////////////////// THE END  ///////////////////////////////////
```

Airbus A320 PTF File

BADA PERFORMANCE FILE May 06 2019

AC/Type: A320__

Source OPF File: Apr 08 2019
Source APF file: Mar 31 2011

Speeds:	CAS(LO/HI)	Mach	Mass Levels [kg]	
climb	250/310	0.78	low	46800
cruise	250/310	0.78	nominal	64000
descent	250/300	0.79	high	77000

Temperature: ISA

Max Alt. [ft]: 41000

FL	CRUISE TAS [kts]	CRUISE fuel [kg/min] lo	CRUISE fuel [kg/min] nom	CRUISE fuel [kg/min] hi	CLIMB TAS [kts]	CLIMB ROCD [fpm] lo	CLIMB ROCD [fpm] nom	CLIMB ROCD [fpm] hi	CLIMB fuel [kg/min] nom	DESCENT TAS [kts]	DESCENT ROCD [fpm] nom	DESCENT fuel [kg/min] nom
0					151	2687	2164	1860	113.5	137	707	44.7
5					152	2672	2147	1842	112.5	138	723	44.3
10					153	2656	2130	1825	111.4	144	788	44.0
15					159	2765	2210	1891	110.5	155	770	17.4
20					160	2748	2193	1873	109.5	187	824	17.4
30	230	26.9	32.7	38.3	184	3166	2501	2130	108.1	230	935	8.7
40	233	26.9	32.7	38.3	218	3671	2857	2418	107.1	233	956	8.6
60	272	31.5	36.1	40.6	272	4186	3078	2512	104.5	272	1266	8.4
80	280	31.5	36.2	40.6	280	4027	2946	2392	100.4	280	1316	8.2
100	289	31.5	36.2	40.7	357	3667	2710	2223	98.4	345	2072	8.0
120	297	31.5	36.2	40.7	367	3441	2527	2060	94.3	356	2131	7.8
140	378	44.2	47.4	50.5	378	3209	2340	1892	90.2	366	2190	7.6
160	389	44.1	47.3	50.4	389	2972	2148	1719	86.2	377	2248	7.4
180	401	44.0	47.3	50.4	401	2728	1951	1543	82.2	388	2306	7.2
200	413	43.9	47.2	50.3	413	2480	1750	1362	78.3	400	2363	7.0
220	425	43.7	47.0	50.2	425	2227	1545	1178	74.4	412	2418	6.8

Airbus A320 PTF File

Example:
True airspeed in Cruise at FL 370 Is 447 knots (Mach 0.78)

Fuel burn is 36.8 kg/min At 64,000 kgs (nominal mass)

```
BADA PERFORMANCE FILE                                  May 06 2019

AC/Type: A320__
                        Source OPF File:               Apr 08 2019
                        Source APF file:               Mar 31 2011

 Speeds:    CAS(LO/HI)  Mach   Mass Levels [kg]      Temperature:  ISA
 climb    - 250/310     0.78   low      -  46800
 cruise   - 250/310     0.78   nominal  -  64000     Max Alt. [ft]:  41000
 descent  - 250/300     0.79   high     -  77000
```

FL	CRUISE TAS [kts]	CRUISE fuel [kg/min] lo	CRUISE fuel [kg/min] nom	CRUISE fuel [kg/min] hi	CLIMB TAS [kts]	CLIMB ROCD [fpm] lo	CLIMB ROCD [fpm] nom	CLIMB ROCD [fpm] hi	CLIMB fuel [kg/min] nom	DESCENT TAS [kts]	DESCENT ROCD [fpm] nom	DESCENT fuel [kg/min] nom
220	425	43.7	47.0	50.2	425	2227	1545	1178	74.4	412	2418	6.8
240	438	43.6	46.9	50.2	438	1969	1337	991	70.5	425	2472	6.7
260	452	43.4	46.8	50.1	452	1708	1125	801	66.7	438	2524	6.5
280	464	42.8	46.3	49.7	464	2055	1300	870	62.9	452	2574	6.3
290	462	41.3	44.9	48.4	462	1998	1242	808	60.8	459	2598	6.2
310	458	38.4	42.4	46.2	458	1865	1108	667	56.8	464	3438	6.0
330	454	35.8	40.2	44.4	454	1940	1005	505	52.8	459	3241	5.8
350	450	33.5	38.3	42.9	450	1738	820	322	48.9	455	3077	5.6
370	447	31.5	36.8	41.9	447	1395	567	109	45.0	453	2711	5.4
390	447	29.8	35.6	41.2	447	1173	359	0	41.3	453	2643	5.2
410	447	28.3	34.7	40.9	447	926	128	0	37.6	453	2603	5.0

The BADA Performance Model

BADA uses a total energy model to derive aircraft performance.

$$mg\frac{dh}{dt} + mV\frac{dV}{dt} = V[T - D] \qquad (48)$$

where:

$\frac{dh}{dt}$ is the rate of climb (m/s)

$\frac{dV}{dt}$ is the acceleration along the flight path (m/s^2)

h is the aircraft altitude (m)

m is the aircraft mass (kg)

V is the aircraft true airspeed (m/s)

g is the gravitational acceleration (9.81 m/s^2)

T is the aircraft thrust (N) and

D is the aircraft drag (N)

Computation of Aircraft Cruise Performance Parameters

Drag Coefficient:

$$C_D = C_{D0-CR} + C_{D2-CR} C_L^2 \qquad (49)$$

where: C_D is the total aircraft drag coefficient (dim)

C_{D0-CR} is the zero lift drag coefficient in the cruise configuration (dim)

C_{D2-CR} is a lift-dependent coefficient (dim)

C_L is the aircraft lift coefficient (dim)

Estimation of Aircraft Lift Coefficient

The estimation of the aircraft drag coefficient requires knowledge of C_L. This non-dimensional coefficient is calculated assuming small flight path angles,

$$C_L = \frac{2mg}{\rho S V^2 \cos(\phi)} \qquad (50)$$

where: S is the aircraft wing reference area (m^2)

ρ is the air density (kg/m^3)

m is the aircraft mass (kg)

$\cos(\phi)$ is the cosine of the bank angle (dim) and all other parameters as previously defined.

The total aircraft drag is then,

$$D = \frac{1}{2}\rho V^2 S C_D$$

where:

D is the total aircraft drag (N)

S is the aircraft wing reference area (m^2)

BADA 3.0 Fuel Consumption

The aircraft thrust specific fuel consumption (η) is estimated as follows:

$$\eta = C_{f1}(1 + V / C_{f2}) \qquad (51)$$

where:

η is the aircraft thrust specific fuel consumption (kg/min./kN)

V is the aircraft true airspeed (knots)

C_{f1} and C_{f2} are model coefficients

BADA 3.0 Fuel Consumption

$$f_{nom} = \eta T \qquad (52)$$

where:

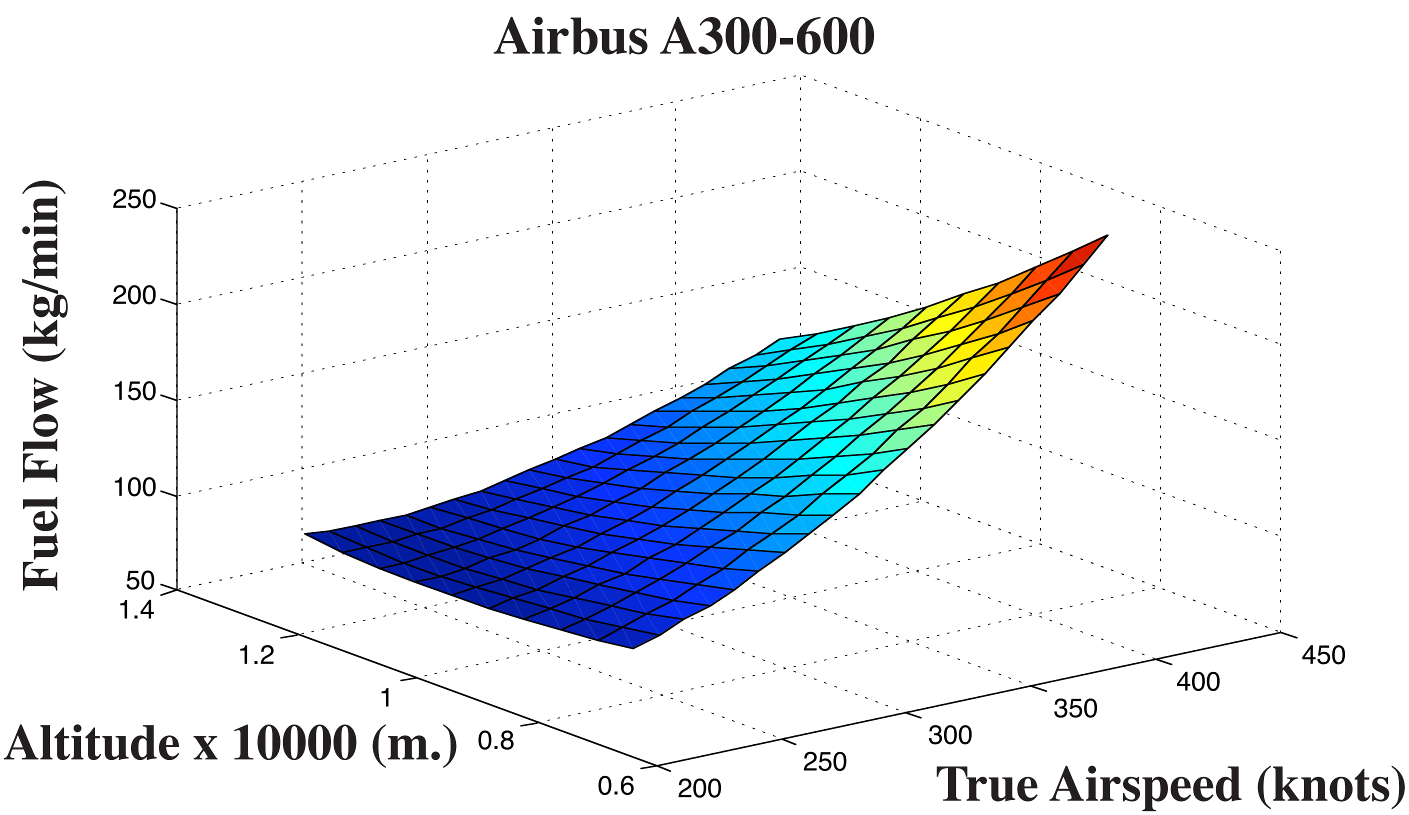
f_{nom} is the nominal aircraft fuel consumption (kg/min.)

The Specific Air Range (SAR) a measure of aircraft fuel efficiency is,

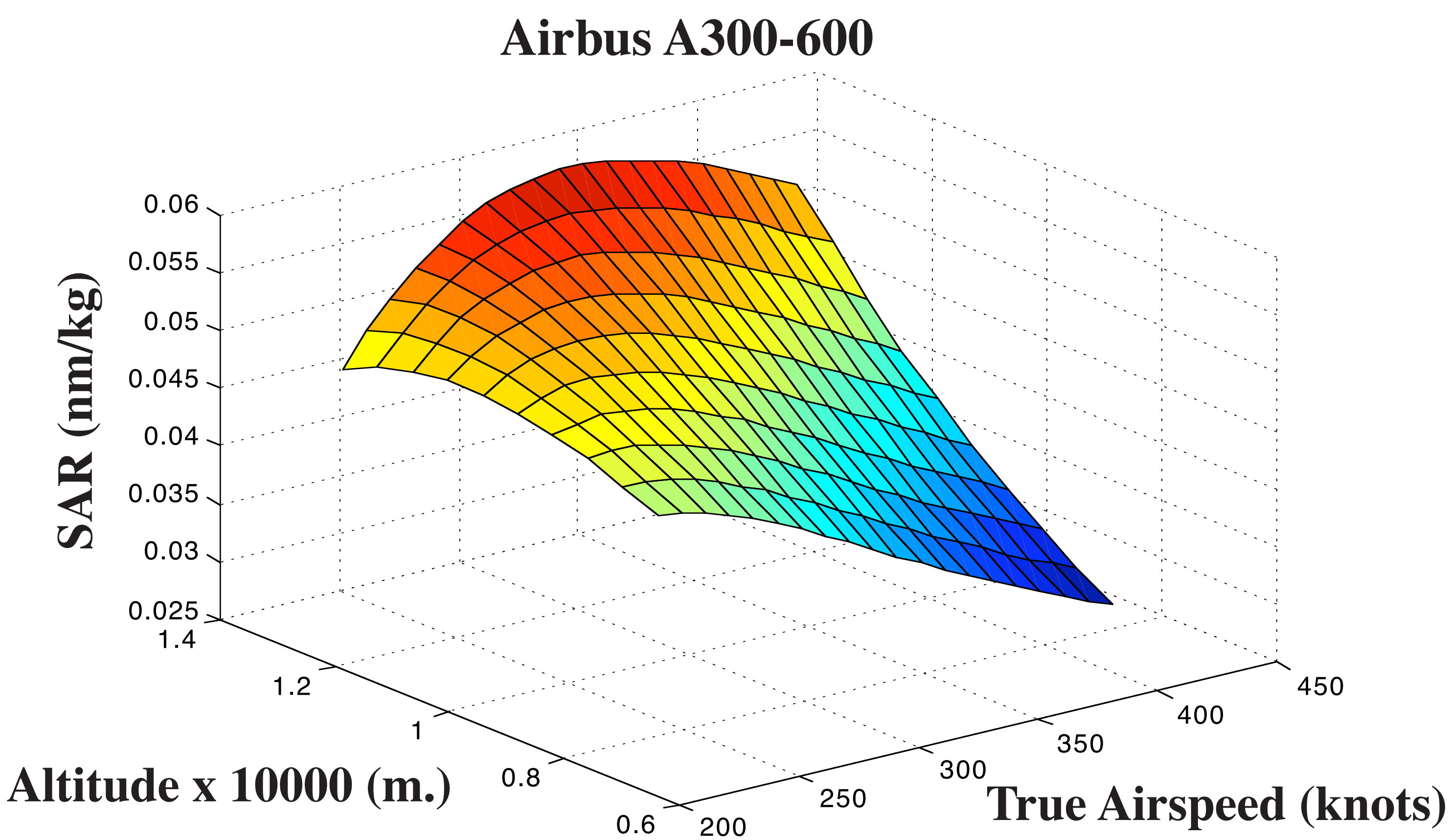
$$SAR = \frac{\Delta d}{\Delta f} \qquad (53)$$

where: Δd is the change in position over time t and Δf is the fuel consumed traveling distance Δd.

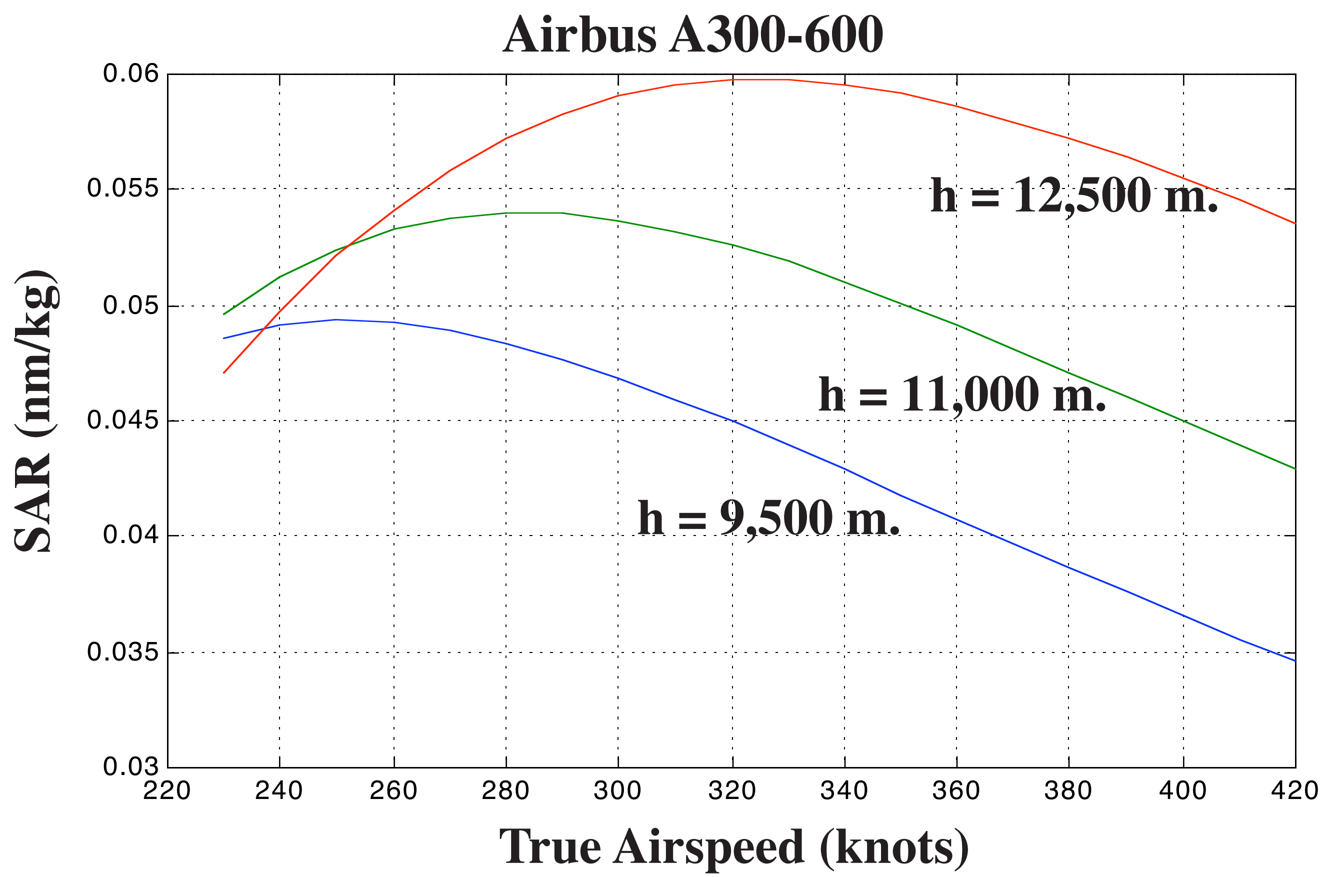
BADA 3.0 Results (Fuel Consumption)



BADA 3.0 Results (Specific Air Range - SAR)



BADA 3.0 Results (Specific Air Range)

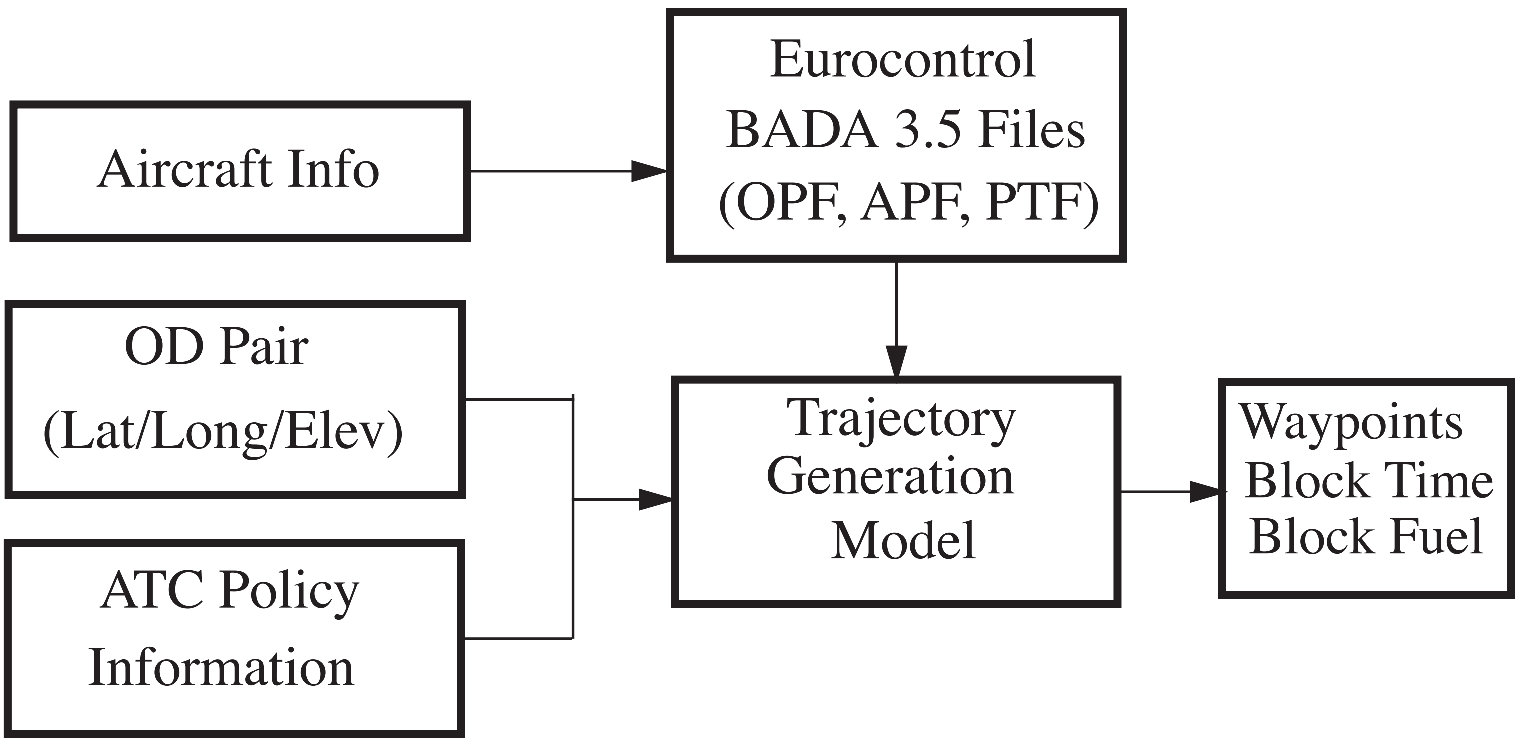


Interpretation of SAR - Specific Air Range

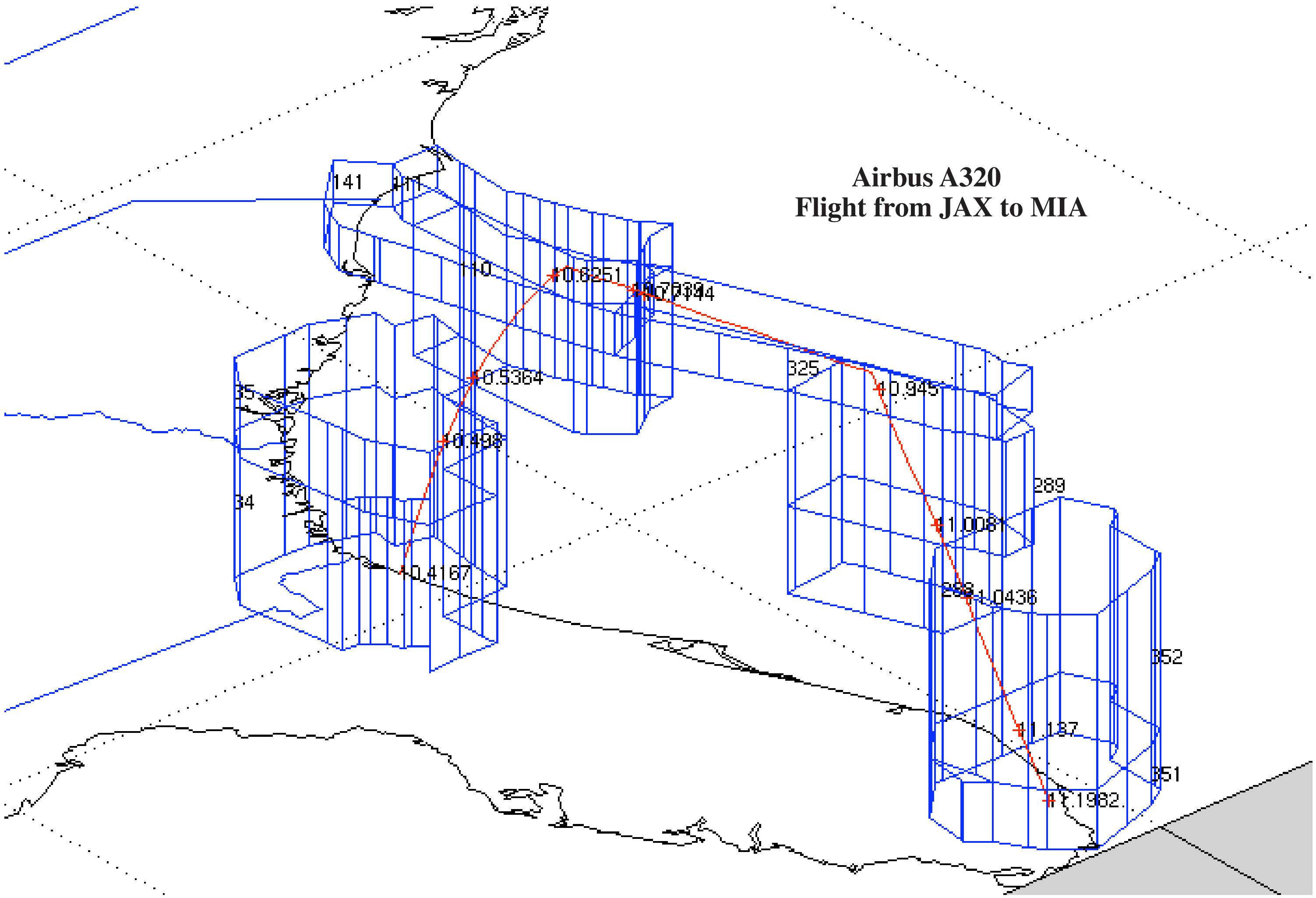
- SAR represents a measure of aircraft efficiency
- the higher the SAR parameter, the more fuel efficient the aircraft is
- For example: in the figure of page 20, the A300 has a maximum SAR value near 0.06. This implies that the aircraft covers 0.06 nautical miles per kg of fuel used.
- The aircraft is more fuel efficient when flying higher (at 12,500 m. instead of 9,500 m.).

Potential Implementation Scheme

- Virginia Tech flight trajectory and fuel consumption implementation



Sample Implementation (VT Flight Trajectory Model)



BADA Model Calculations

BADA Model General Notes

- BADA model consists of:
- Aircraft performance specification contained in a text file YYYY__.OPF (YYYY is the aircraft name)
  - Example: B744__.OPF is the operational procedure file for the Boeing 747-400 aircraft
  - OPF file contains aircraft model coefficients
- BADA also provides "rules" on how to fly the aircraft
  - Derived from airline data gathered in Europe
- BADA provides "canned" solutions about the performance of each aircraft modeled

BADA File Structure

BADA Model Files

BADA Performance File Results (PTD)

Pre-computed performance tables (ISA and Off-ISA)

Operations Performance File (OPF)

1) Aero coefficients
2) Mass coefficients
3) Flight envelope restrictions
4) Engine thrust
5) Fuel consumption

Airline Procedures File (APF)

Recommended speed procedures: climb, cruise and descent

Performance Table File (PTF)

Typical speed procedures for climb, cruise and descent at ISA conditions

Directory Structure

B744__.APF
B744__.OPF
B744__.PTD
B744__.PTF
B748__.APF
B748__.OPF
B748__.PTD
B748__.PTF
B752__.APF
B752__.OPF
B752__.PTD
B752__.PTF
B753__.APF
B753__.OPF
B753__.PTD
B753__.PTF

Sample BADA OPF File

Boeing 747-8

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC B748__.OPF CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC/
CC                                                                       /
CC              AIRCRAFT PERFORMANCE OPERATIONAL FILE                    /
CC                                                                       /
CC                                                                       /
CC      File_name: B748__.OPF                                            /
CC                                                                       /
CC      Creation_date: Jan 30 2012                                       /
CC                                                                       /
CC      Modification_date: Apr 10 2012                                   /
CC                                                                       /
CC                                                                       /
CC====== Actype ============================================================/
CD    B748__           4 engines     Jet                       H           /
CC   B747-8F with GENX-2B67 engines                           wake         /
CC                                                                         /
CC====== Mass (t) ==========================================================/
CC     reference       minimum       maximum      max payload   mass grad  /
CD      .36633E+03    .19105E+03    .44225E+03    .13390E+03   .62663E-01  /
CC====== Flight envelope ===================================================/
CC      VMO(KCAS)        MMO          Max.Alt        Hmax       temp grad  /
CD      .36500E+03    .90000E+00    .42100E+05    .32973E+05   -.1492E+03  /
CC====== Aerodynamics ======================================================/
CC Wing Area and Buffet coefficients (SIM)                                 /
CCndrst Surf(m2)     Clbo(M=0)         k           CM16                    /
CD 5    .51097E+03    .10815E+01    .30373E+00    .00000E+00               /
CC    Configuration characteristics                                        /
CC n Phase  Name      Vstall(KCAS)     CD0          CD2         unused     /
CD 1 CR    CLEAN      .18400E+03    .25669E-01    .39082E-01   .00000E+00  /
CD 2 IC    F1         .14600E+03    .19021E-01    .60437E-01   .00000E+00  /
CD 3 TO    F5         .14200E+03    .22667E-01    .56399E-01   .00000E+00  /
CD 4 AP    F20        .13100E+03    .48043E-01    .47344E-01   .00000E+00  /
CD 5 LD    F30        .12400E+03    .10733E+00    .43809E-01   .00000E+00  /
```

```
CC    Spoiler                                                            /
CD 1        RET                                                          /
CD 2        EXT                                  .00000E+00   .00000E+00 /
CC    Gear                                                               /
CD 1        UP                                                           /
CD 2        DOWN                    .24000E-01   .00000E+00   .00000E+00 /
CC    Brakes                                                             /
CD 1        OFF                                                          /
CD 2        ON                                   .00000E+00   .00000E+00 /
CC====== Engine Thrust ==================================================/
CC        Max climb thrust coefficients (SIM)                            /
CD     .75979E+06   .52423E+05   .40968E-10   .10356E+02   .77315E-02    /
CC     Desc(low)    Desc(high)   Desc level   Desc(app)    Desc(ld)      /
CD     .11581E+00   -.2525E-01   .38370E+05   .24054E+00   .39605E+00    /
CC     Desc CAS     Desc Mach    unused       unused       unused        /
CD     .33500E+03   .86000E+00   .00000E+00   .00000E+00   .00000E+00    /
CC====== Fuel Consumption ===============================================/
CC    Thrust Specific Fuel Consumption Coefficients                      /
CD     .58259E+00   .12839E+04                                           /
CC    Descent Fuel Flow Coefficients                                     /
CD     .45380E+02   .72929E+05                                           /
CC    Cruise Corr.   unused       unused       unused       unused       /
CD     .89625E+00   .00000E+00   .00000E+00   .00000E+00   .00000E+00    /
CC====== Ground =========================================================/
CC        TOL          LDL          span         length       unused     /
CD     .32300E+04   .23770E+04   .68400E+02   .76250E+02   .00000E+00    /
CC=======================================================================/
```

Source: Eurocontrol BADA Model

Sample BADA APF File


```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC B748__.APF CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC/
CC                                                                                                 /
CC                  AIRLINES PROCEDURES FILE                                                       /
CC                                                                                                 /
CC       File_name: B748__.APF                                                                     /
CC                                                                                                 /
CC       Creation_date: Jan 30 2012                                                                /
CC                                                                                                 /
CC       Modification_date: Jan 30 2012                                                           /
CC                                                                                                 /
CC                                                                                                 /
CC                                                                                                 /
CC    LO= 191.05 to ---.--  /  AV= ---.-- to ---.--  /  HI= ---.-- to 442.25                       /
CC                                                                                                 /
CC=================================================================================================/
CC   COM CO     Company name ------climb-------  --cruise--  -----descent------  --approach-  model- /
CC                           mass   lo  hi            lo  hi        hi   lo       (unused)          /
CC     version engines  ma   cas cas mc xxxx xx  cas cas mc  mc cas cas xxxx xx  xxx xxx xxx  opf___ /
CC===:=======:=======::==::===:===:==:====:==::===:===:==::==:===:===:====:==::===:===:===::======:/
CD   *** **     Default Company                                                                    /
CD     8F                LO  335 335 86          320 320 84  86 335 335            0   0   0  B748__ /
CD     8F                AV  335 335 86          320 320 84  86 335 335            0   0   0  B748__ /
CD     8F                HI  335 335 86          320 320 84  86 335 335            0   0   0  B748__ /
CC===:=======:=======::==::===:===:==:====:==::===:===:==::==:===:===:====:==::===:===:===::======:/
CC//////////////////////////////////////////////THE END///////////////////////////////////////////
```

Sample PTF File (ISA Conditions)

```
BADA PERFORMANCE FILE                                     Apr 10 2012

AC/Type: B748__
                              Source OPF File:              Apr 10 2012
                              Source APF file:              Jan 30 2012

 Speeds:   CAS(LO/HI)  Mach   Mass Levels [kg]          Temperature:  ISA
 climb   - 250/335     0.86   low      -  229260
 cruise  - 250/320     0.84   nominal  -  366330        Max Alt. [ft]:  42100
 descent - 250/335     0.86   high     -  442250
==========================================================================================
 FL |          CRUISE            |               CLIMB                |      DESCENT
    |  TAS          fuel         |  TAS          ROCD          fuel   |  TAS  ROCD    fuel
    | [kts]       [kg/min]       | [kts]         [fpm]      [kg/min]  | [kts] [fpm] [kg/min]
    |          lo    nom     hi  |          lo    nom    hi     nom   |        nom     nom
==========================================================================================
  0 |                            |  190    3501  2508  2129   508.0   |  166   1117  198.0
    |                            |                                    |
  5 |                            |  191    3482  2489  2108   503.6   |  167   1137  196.3
    |                            |                                    |
 10 |                            |  192    3463  2469  2088   499.3   |  174   1209  195.2
    |                            |                                    |
 15 |                            |  199    3572  2530  2135   496.6   |  185    889  118.4
```

Sample PTF File (ISA Conditions)

```
BADA PERFORMANCE FILE                                   Apr 10 2012

AC/Type: B748__
                                Source OPF File:          Apr 10 2012
                                Source APF file:          Jan 30 2012

 Speeds:   CAS(LO/HI)  Mach   Mass Levels [kg]        Temperature:  ISA
 climb   - 250/335     0.86   low     -  229260
 cruise  - 250/320     0.84   nominal -  366330       Max Alt. [ft]:  42100
 descent - 250/335     0.86   high    -  442250
```

FL	CRUISE TAS [kts]	CRUISE fuel [kg/min] lo	CRUISE fuel [kg/min] nom	CRUISE fuel [kg/min] hi	CLIMB TAS [kts]	CLIMB ROCD [fpm] lo	CLIMB ROCD [fpm] nom	CLIMB ROCD [fpm] hi	CLIMB fuel [kg/min] nom	DESCENT TAS [kts]	DESCENT ROCD [fpm] nom	DESCENT fuel [kg/min] nom
0					190	3501	2508	2129	508.0	166	1117	198.0
5					191	3482	2489	2108	503.6	167	1137	196.3
10					192	3463	2469	2088	499.3	174	1209	195.2
15					199	3572	2530	2135	496.6	185	889	118.4
20					200	3552	2509	2113	492.3	217	965	119.8
30	230	93.6	140.8	176.4	224	3973	2755	2309	490.3	230	1046	118.4
40	233	93.7	141.2	176.8	259	4495	3040	2425	491.7	233	1080	116.3
310	493	154.2	184.8	207.8	505	2624	1245	634	[illegible]	505	3504	26.1
330	489	143.9	177.3	202.5	[illegible]	2377	1012	407	255.3	500	3350	24.8
350	484	[illegible]	171.3	198.9	496	2413	756	157	234.7	496	3227	23.6
370	482	126.7	167.0	197.3	493	1879	431	0	214.6	493	2841	22.4
390	482	120.1	164.5	197.8	493	1562	158	0	195.1	493	3290	21.1
410	482	114.6	163.4	200.2	493	1216	0	0	175.7	493	3247	19.9
421	482	112.0	163.5	202.2	493	1014	0	0	165.1	493	3236	19.2

Provides "ready-to-use" point performance solutions for each aircraft

Example: Boeing 747-8 flies at 484 knots in cruise at FL 350 (35,000 ft)

Sample PTF File (ISA Conditions)

Boeing 747-8
flying at 484 knots (TAS)
at FL 350 (35,000 ft)
burns 171.3 kg/min

TAS = true airspeed

BADA PERFORMANCE FILE Apr 10 2012

AC/Type: B748__

Source OPF File: Apr 10 2012
Source APF file: Jan 30 2012

Speeds:	CAS(LO/HI)	Mach	Mass Levels [kg]	
climb	250/335	0.86	low	229260
cruise	250/320	0.84	nominal	366330
descent	250/335	0.86	high	442250

Temperature: ISA

Max Alt. [ft]: 42100

FL	CRUISE TAS [kts]	CRUISE fuel [kg/min] lo	CRUISE fuel [kg/min] nom	CRUISE fuel [kg/min] hi	CLIMB TAS [kts]	CLIMB ROCD [fpm] lo	CLIMB ROCD [fpm] nom	CLIMB ROCD [fpm] hi	CLIMB fuel [kg/min] nom	DESCENT TAS [kts]	DESCENT ROCD [fpm] nom	DESCENT fuel [kg/min] nom
310	493	154.2	184.8	207.8	505	262[illegible]	1245	[illegible]34	276.3	505	3504	26.1
330	480	143.9	[illegible]	202.5	500	237[illegible]	1012	[illegible]07	[illegible]	500	3350	24.8
350	484	134.7	171.3	198.9	496	241[illegible]	756	[illegible]57	234.7	496	3227	23.6
370	482	126.7	167.0	197.3	493	187[illegible]	431	0	214.6	493	2841	22.4
390	482	120.1	164.5	197.8	493	156[illegible]	158	0	195.1	493	3290	21.1
410	482	114.6	163.4	200.2	493	121[illegible]	0	0	175.7	493	3247	19.9
421	482	112.0	163.5	202.2	493	101[illegible]	0	0	165.1	493	3236	19.2

Boeing 747-8
has a rate of climb
of 756 ft/minute
while passing 35,000 ft
at a nominal weight
of 366,330 kg

VirginiaTech
Invent the Future

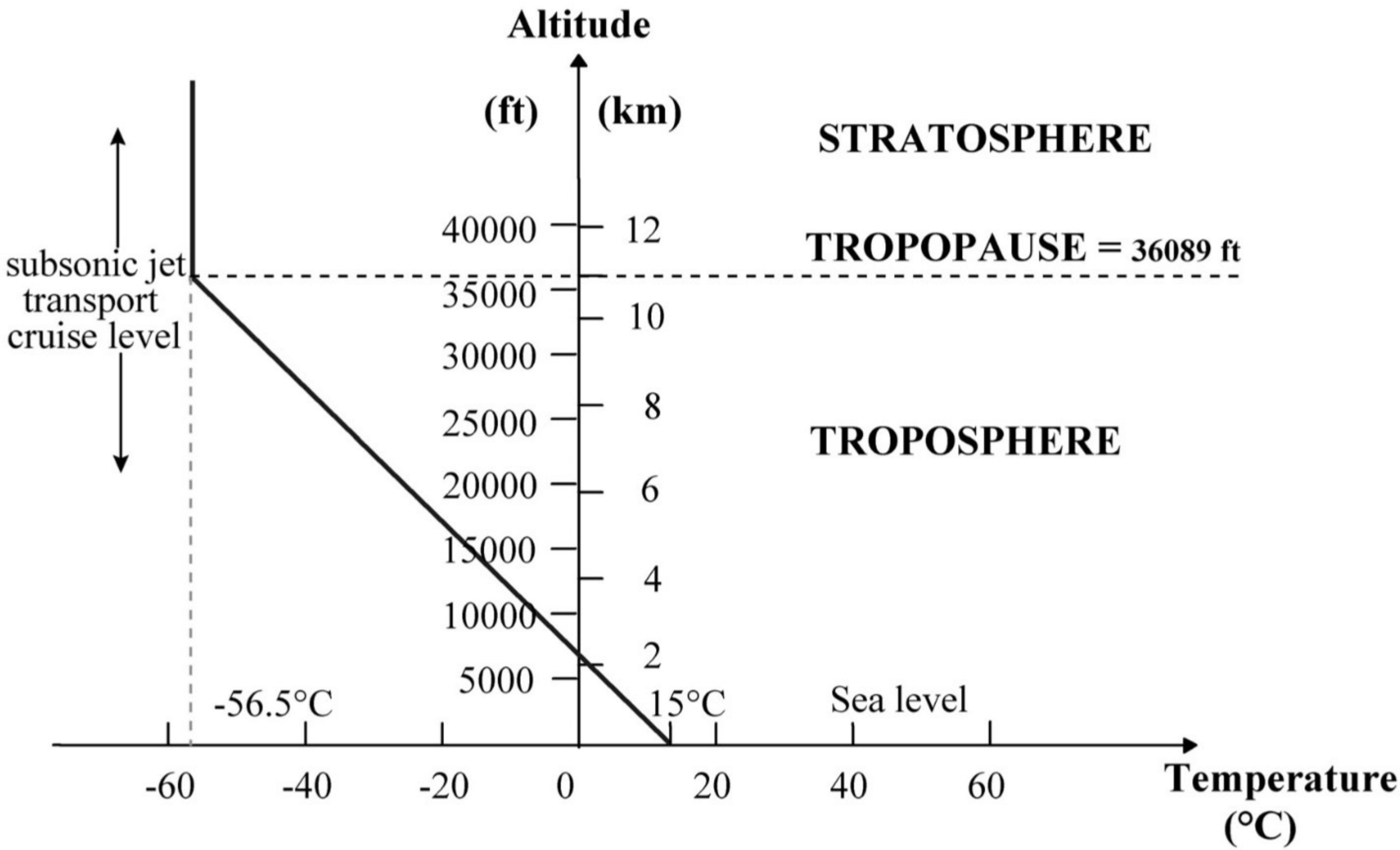
Sample PTD File (ISA Conditions)

Note: ISA density and temperature conditions

High mass CLIMBS
================

FL[-]	T[K]	p[Pa]	rho[kg/m3]	a[m/s]	TAS[kt]	CAS[kt]	M[-]	mass[kg]	Thrust[N]	Drag[N]	Fuel[kgm]	ESF[-]	ROC[fpm]	TDC[N]	PWC[-]
0	288	101325	1.225	340	207.83	207.83	0.31	442250	759790	297310	514.3	0.95	2129	462480	1.00
5	287	99508	1.207	340	209.31	207.83	0.32	442250	752551	297360	509.9	0.95	2108	455191	1.00
10	286	97717	1.190	339	210.81	207.83	0.32	442250	745328	297411	505.5	0.95	2088	447917	1.00
15	285	95952	1.172	339	217.42	212.83	0.33	442250	738120	292381	502.8	0.94	2135	445739	1.00
20	284	94213	1.155	338	218.99	212.83	0.33	442250	730928	292430	498.5	0.94	2113	438498	1.00
30	282	90812	1.121	337	242.97	232.83	0.37	442250	716590	279144	496.5	0.93	2309	437446	1.00
40	280	87511	1.088	336	264.57	250.00	0.41	442250	702314	274938	493.5	0.92	2425	427377	1.00
60	276	81200	1.024	333	272.30	250.00	0.42	442250	673950	274971	475.9	0.91	2317	398979	1.00
80	272	75262	0.963	331	280.34	250.00	0.44	442250	645835	275010	458.4	0.91	2203	370824	1.00
100	268	69682	0.905	328	384.74	335.00	0.60	442250	617968	313605	467.9	0.84	2307	304363	1.00
120	264	64441	0.849	326	395.81	335.00	0.62	442250	590351	312612	450.0	0.84	2144	277739	1.00
140	260	59524	0.796	324	407.27	335.00	0.65	442250	562983	311553	432.0	0.83	1975	251429	1.00
160	256	54915	0.746	321	419.15	335.00	0.67	442250	535863	310425	414.1	0.82	1802	225438	1.00
180	252	50600	0.698	319	431.46	335.00	0.70	442250	508993	309224	396.2	0.81	1624	199769	1.00
200	249	46563	0.653	316	444.21	335.00	0.72	442250	482372	307947	378.3	0.80	1441	174425	1.00
220	245	42791	0.610	314	457.41	335.00	0.75	442250	456000	306593	360.3	0.79	1255	149407	1.00
240	241	39271	0.569	311	471.07	335.00	0.78	442250	429876	305159	342.3	0.78	1064	124717	1.00
260	237	35989	0.530	308	485.20	335.00	0.81	442250	404002	303646	324.3	0.76	869	100357	1.00
280	233	32932	0.493	306	499.82	335.00	0.84	442250	378377	302052	306.3	0.75	670	76326	1.00
290	231	31485	0.475	304	507.31	335.00	0.86	442250	365658	301225	297.2	0.75	570	64433	1.00
310	227	28745	0.442	302	504.62	322.12	0.86	442250	340406	291863	276.3	1.11	634	48544	1.00
330	223	26201	0.410	299	500.19	308.29	0.86	442250	315404	283975	255.3	1.11	407	31429	1.00
350	219	23842	0.380	297	495.72	294.77	0.86	442250	290650	278451	234.7	1.11	157	12199	1.00
370	217	21663	0.348	295	493.27	281.58	0.86	442250	266145	275377	214.6	1.00	-106	-9232	1.00
390	217	19677	0.316	295	493.27	268.90	0.86	442250	241890	274839	195.1	1.00	-379	-32949	1.00
410	217	17874	0.287	295	493.27	256.74	0.86	442250	217883	276841	175.7	1.00	-679	-58958	1.00
421	217	16953	0.273	295	493.27	250.28	0.86	442250	204786	279032	165.1	1.00	-855	-74246	1.00

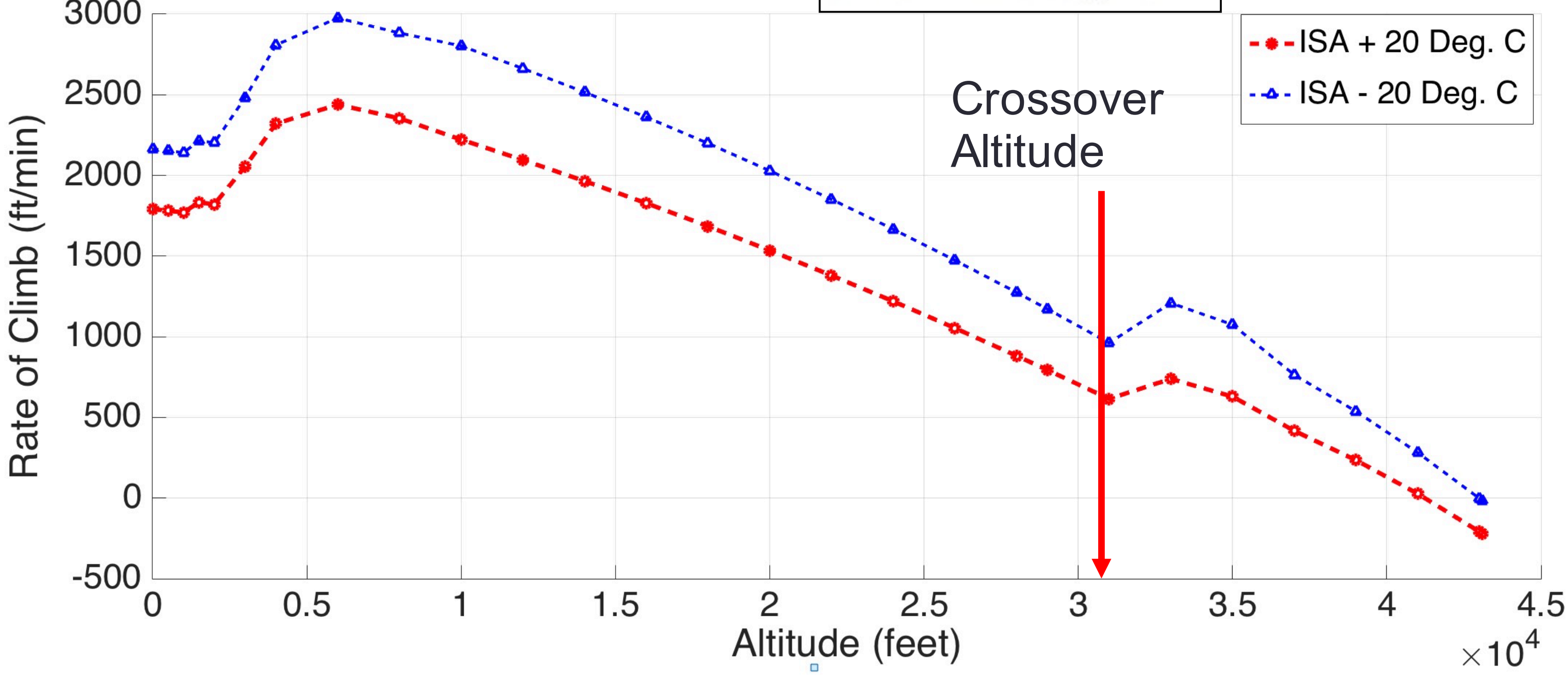
ISA Atmospheric Conditions



source: Airbus

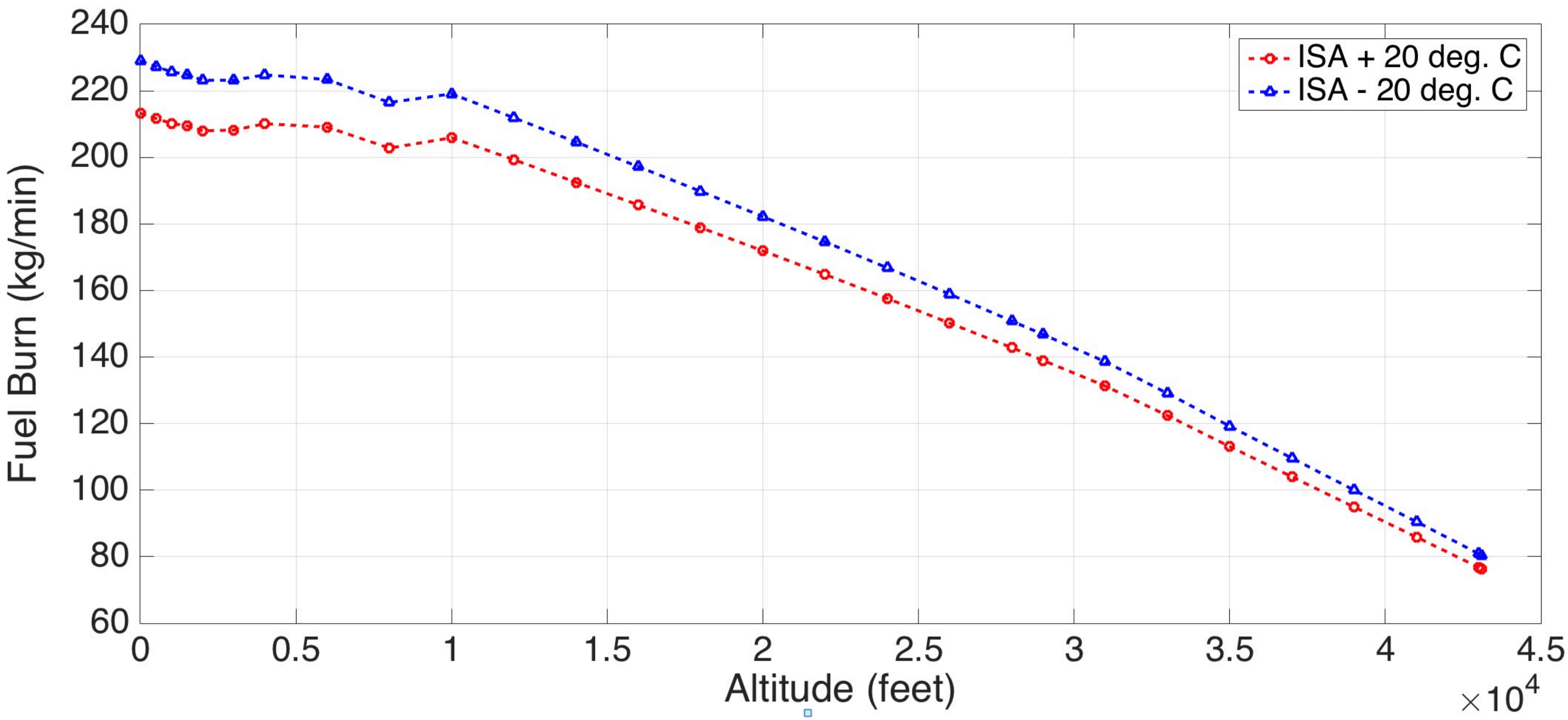
ISA vs off-ISA Conditions

- BADA 3.13 includes 4 off-ISA conditions or performance variations in the PTD files
  - ISA – 10 deg. C
  - ISA – 20 deg. C
  - ISA + 10 deg. C
  - ISA + 20 deg. C



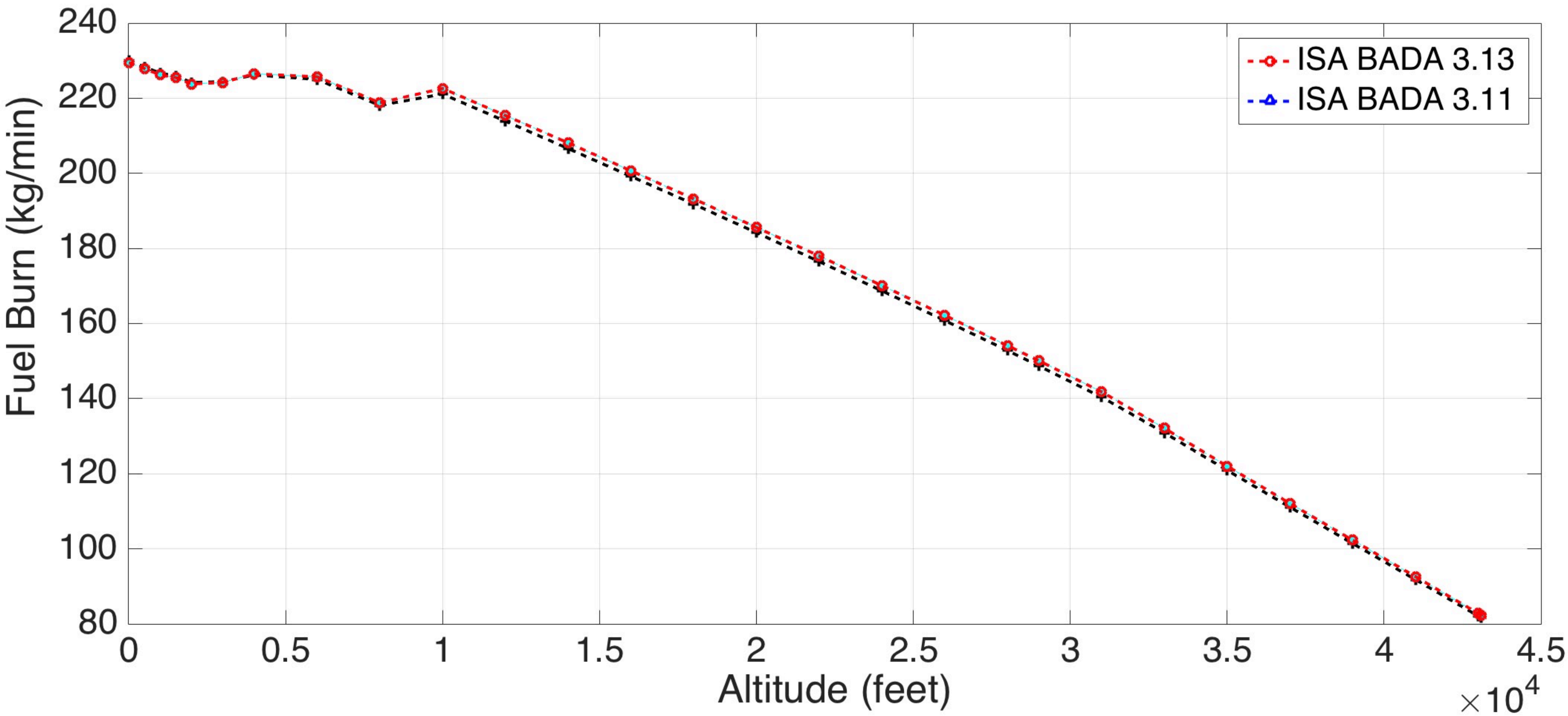
Source: BADA 3.13 model for Boeing 787-8 Dreamliner

Aircraft Performance is Sensitivity to Temperature Conditions



Source: BADA 3.13 model for Boeing 787-8 Dreamliner (Medium Mass)

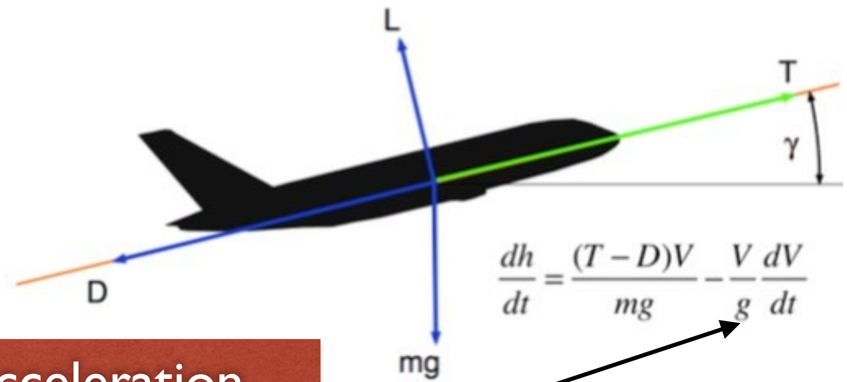
Aircraft Performance Difference Across BADA Versions



Source: BADA 3.13 model for Boeing 787-8 Dreamliner (Medium Mass)

BADA Model Equations

BADA uses a total energy model to derive aircraft performance.



$$mg\frac{dh}{dt} + mV\frac{dV}{dt} = V[T - D]$$

where:

BADA considers the acceleration term in the equation of motion

$\frac{dh}{dt}$ is the rate of climb (m/s)

$\frac{dV}{dt}$ is the acceleration along the flight path (m/s^2)

h is the aircraft altitude (m)

BADA Model Equations (2)

m is the aircraft mass (kg)

V is the aircraft true airspeed (m/s)

g is the gravitational acceleration (9.81 m/s^2)

T is the aircraft thrust (N) and

D is the aircraft drag (N)

BADA Model Equations

- Standard aerodynamic coefficients and equations used to estimate lift and drag coefficients

$$C_l = \frac{2mg}{\rho V^2 S \cos(\phi)}$$

Same as the model explained in CEE 5614 class

where

$C_l =$ is the lift coefficient

$V =$ is the true airspeed

$S =$ is the wing reference area

$\phi =$ is the bank angle (for turning flight)

$mg =$ is the aircraft weight

$\rho =$ density of air

BADA Model Equations (cont.)

- Standard form of aerodynamic drag coefficients (parabolic drag polar)

$$C_d = C_{DO,k} + \boxed{C_{D2,k} C_l^2}$$

Called Cdi in most textbooks (and in class)

where

$Cd =$ total drag coefficient

$C_{DO,k} =$ zero drag coefficient for flight condition k

$C_{D2,k} =$ lift dependent coefficient for flight condition k

for landing configuration BADA assumes

$$C_d = C_{DO,LDG} + C_{DO,\Delta LDG} + C_{D2,LDG} C_l^2$$

where

$C_{DO,\Delta LDG} =$ zero drag coefficient increment due to landing gear

BADA Model Equations (cont.)

- The flight condition k takes 5 different forms for each aircraft wing flap configuration (i.e., landing gear down, approach, initial climb, takeoff or cruise)
- Standard calculation of aerodynamic drag in the BADA model

Universal definition of drag and same relationship explained in CEE 5614 class

$$D = \frac{1}{2}\rho S V^2 C_d$$

where

$C_d =$ total drag coefficient

$D =$ total drag for a given true airspeed (V)

Others as defined before

Recall BADA Aerodynamic Coefficients

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC B748__.OPF CCCCCCCCCCCCCCCCCCCCCCCCCCCCC/
CC                                                                    /
CC                AIRCRAFT PERFORMANCE OPERATIONAL FILE               /
CC                                                                    /
CC                                                                    /
CC       File_name: B748__.OPF                                        /
CC                                                                    /
CC       Creation_date: Jan 30 2012                                   /
CC                                                                    /
CC       Modification_date: Apr 10 2012                               /
CC                                                                    /
CC                                                                    /
CC====== Actype ======================================================/
CD    B748__          4 engines    Jet                      H         /
CC   B747-8F with GENX-2B67 engines                        wake       /
CC                                                                    /
CC====== Mass (t) ====================================================/
CC     reference     minimum      maximum     max payload  mass grad  /
CD     .36633E+03   .19105E+03   .44225E+03   .13390E+03  .62663E-01  /
CC====== Flight envelope =============================================/
CC      VMO(KCAS)       MMO        Max.Alt        Hmax      temp grad /
CD     .36500E+03   .90000E+00   .42100E+05   .32973E+05  -.1492E+03  /
CC====== Aerodynamics ================================================/
CC Wing Area and Buffet coefficients (SIM)                            /
CCndrst Surf(m2)    Clbo(M=0)       k          CM16                   /
CD 5    .51097E+03   .10815E+01   .30373E+00   .00000E+00             /
CC    Configuration characteristics                                   /
CC n Phase  Name    Vstall(KCAS)     CD0          CD2        unused   /
CD 1 CR    CLEAN     .18400E+03   .25669E-01   .39082E-01  .00000E+00 /
CD 2 IC    F1        .14600E+03   .19021E-01   .60437E-01  .00000E+00 /
CD 3 TO    F5        .14200E+03   .22667E-01   .56399E-01  .00000E+00 /
CD 4 AP    F20       .13100E+03   .48043E-01   .47344E-01  .00000E+00 /
CD 5 LD    F30       .12400E+03   .10733E+00   .43809E-01  .00000E+00 /
```

OPF file

$$C_d = C_{DO,k} + C_{D2,k} C_l^2$$

k = phase of flight phase
Example: 4 = approach phase

BADA Model Fuel Consumption

The aircraft thrust specific fuel consumption (η) is estimated as follows:

$$\eta = C_{f1}(1 + V/C_{f2})$$

where:

η is the aircraft thrust specific fuel consumption (kg/min./kN)

V is the aircraft true airspeed (knots)

C_{f1} and C_{f2} are model coefficients

BADA Model Fuel Consumption

$$f_{nom} = \eta T$$

where:

f_{nom} is the nominal aircraft fuel consumption (kg/min.)

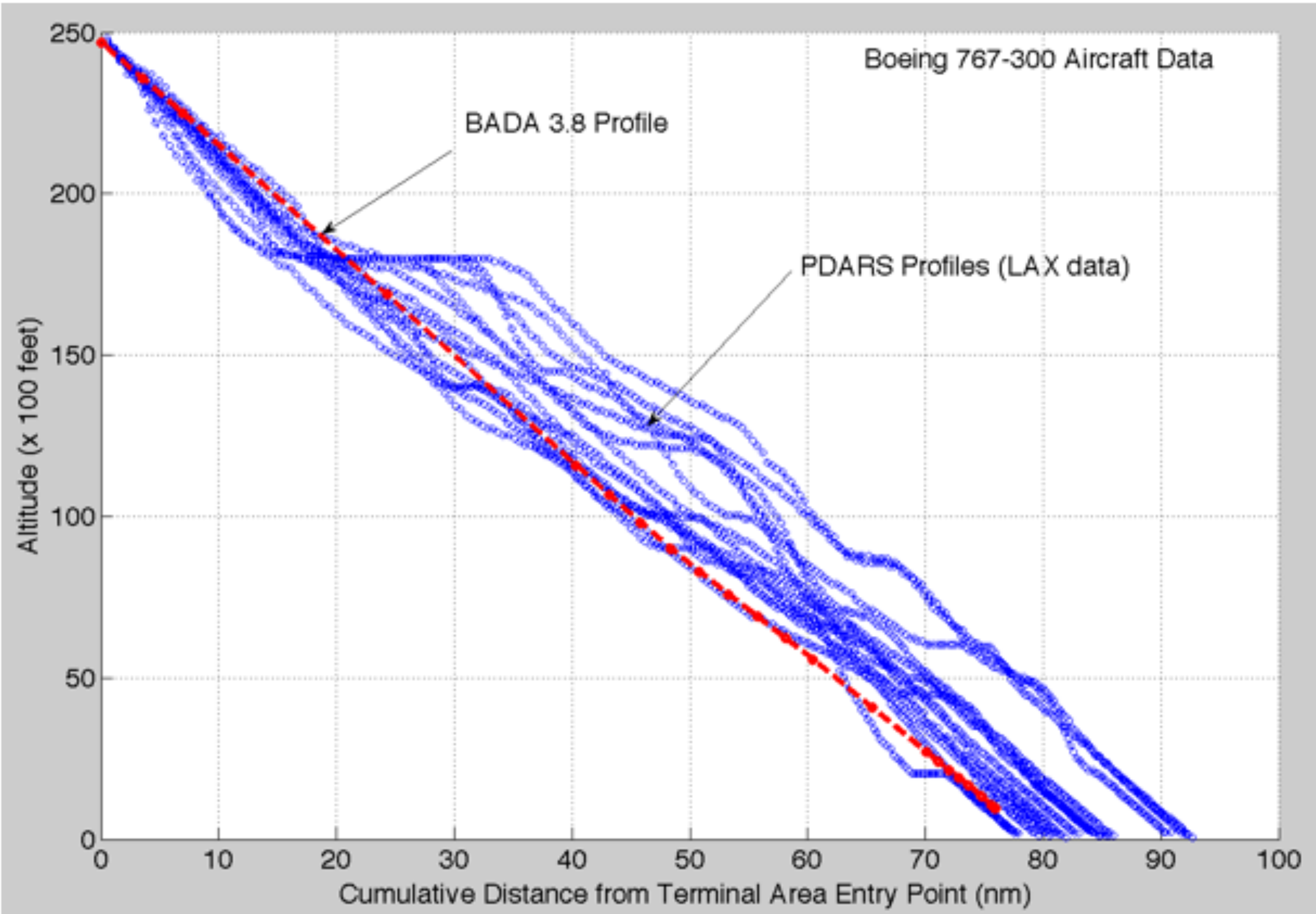
The Specific Air Range (SAR) a measure of aircraft fuel efficiency is,

$$SAR = \frac{\Delta d}{\Delta f}$$

SAR units are: nm/kg

where: Δd is the change in position over time t and Δf is the fuel consumed traveling distance Δd.

Comparison of BADA and Real Radar Profiles for Boeing 767-300 Aircraft



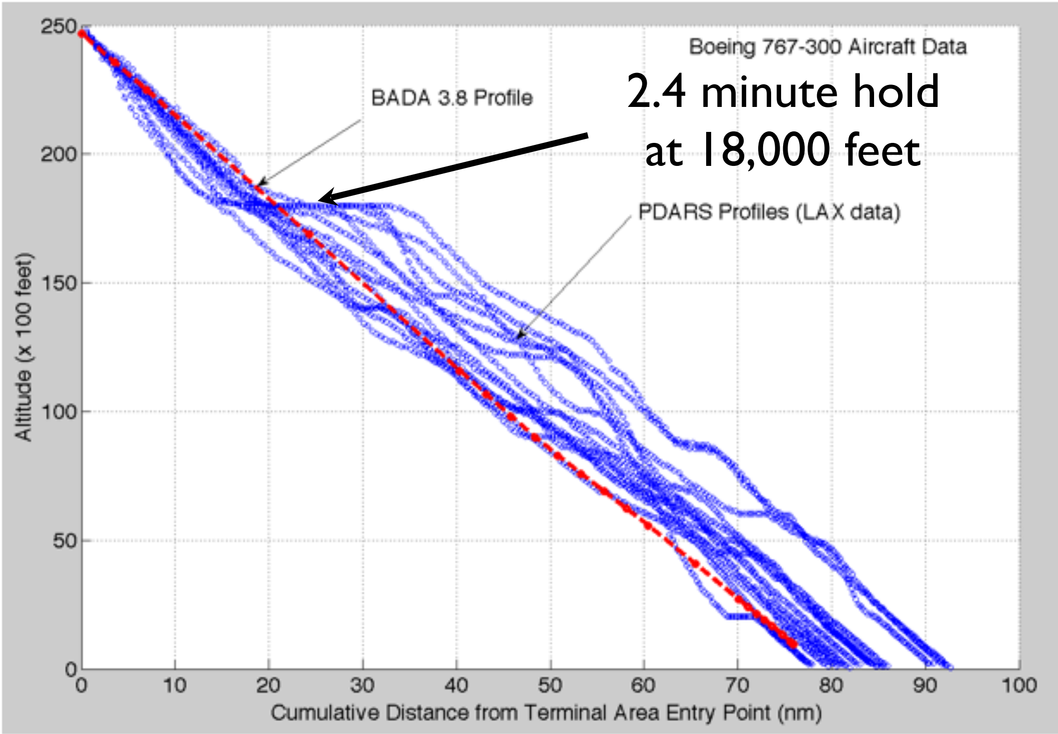
Sample BADA Coefficients

- Aircraft aerodynamic parameters are found in the operational performance file (OPF)

```
CC====== Actype ============================================================/
CD   B763__          2 engines    Jet                        H             /
CC  B767-300ER with PW4060 engines                          wake           /
CC                                                                         /
CC====== Mass (t) ==========================================================/
CC    reference      minimum      maximum     max payload  mass grad       /
CD     .15459E+03   .90011E+02   .18688E+03   .43799E+02   .11823E+00      /
CC====== Flight envelope ===================================================/
CC     VMO(KCAS)       MMO        Max.Alt       Hmax       temp grad       /
CD     .36000E+03   .86000E+00   .43100E+05   .36502E+05   -.4112E+02      /
CC====== Aerodynamics ======================================================/
CC Wing Area and Buffet coefficients (SIM)                                 /
CCndrst Surf(m2)     Clbo(M=0)        k          CM16                      /
CD 5    .28335E+03   .21687E+01   .17102E+01   .00000E+00                  /
CC   Configuration characteristics                                         /
CC n Phase  Name    Vstall(KCAS)    CD0          CD2          unused       /
CD 1 CR     CLEAN    .16700E+03   .21112E-01   .42118E-01   .00000E+00     /
CD 2 IC     F1       .13800E+03   .16120E-01   .53848E-01   .00000E+00     /
CD 3 TO     F15      .12400E+03   .27554E-01   .47458E-01   .00000E+00     /
CD 4 AP     F20      .12200E+03   .30382E-01   .48029E-01   .00000E+00     /
CD 5 LD     F30      .11800E+03   .93121E-01   .39805E-01   .00000E+00     /
```

Sample Calculation : Boeing 767-300 Holding at 18,000 feet

- Boeing 767-300
- PW 4042 engines
- 154.590 metric tons of mass
- 401 knots (true airspeed) or 206.29 m/s
- Hold for 2.4 minutes at 18,000 feet



Calculations

- At 18,000 feet (assume ISA conditions and no turning - straight and level flight)

$$\rho = 0.698 \text{ kg/m}^3$$

$$a = 318.5 \text{ m/s}$$

$$S = 283.33 \text{ m}^2$$

Boeing 767-300 per BADA data

$$V = 206.3 \text{ m/s}$$

Assume Nominal Mass

$$C_l = \frac{2mg}{\rho V^2 S \cos(\phi)} = \frac{2(154{,}590)9.81}{(0.698)(206.3)^2(283.22)\cos(0)}$$

$$C_l = 0.3605$$

Using "clean" configuration parabolic drag polar Called Phase 1 in BADA

$$C_d = C_{DO,CR} + C_{D2,CR} C_l^{\,2}$$

$$C_d = 0.02111 + 0.04212(0.3605)^2$$

$$C_d = 0.0266$$

B767-300

CC	n	Phase	Name	Vstall(KCAS)	CD0	CD2	unused	
CC		Configuration characteristics						/
CD	1	CR	CLEAN	.16700E+03	.21112E-01	.42118E-01	.00000E+00	/
CD	2	IC	F1	.13800E+03	.16120E-01	.53848E-01	.00000E+00	/
CD	3	TO	F15	.12400E+03	.27554E-01	.47458E-01	.00000E+00	/
CD	4	AP	F20	.12200E+03	.30382E-01	.48029E-01	.00000E+00	/
CD	5	LD	F30	.11800E+03	.93121E-01	.39805E-01	.00000E+00	/

Calculations (Continuation)

- The aircraft drag is the estimated to be:

$$D = \frac{1}{2}\rho S V^2 C_d = 0.5(0.698)(206.3)^2(283.22)(0.0266)$$

$$D = 111{,}900 \text{ Newtons}$$

- The aircraft fuel burn under the holding condition is,

$$F_{cr} = \eta T_{HR} C_{fcr}$$

$$F_{cr} = \text{fuel flow (kg/min)}$$

$$\eta = \text{thrust specific fuel consumption (kg/(min*kN))}$$

$$T_{HR} = \text{thrust developed by the engines (kN)}$$

$$C_{fcr} = \text{cruise fuel flow factor}$$

Calculations (Continuation)

- Per BADA model, the aircraft fuel burn is defined as,

$$F_{cr} = \eta T_{HR} C_{fcr}$$

F_{cr} = fuel flow (kg/min)

η = thrust specific fuel consumption(kg/(min*kN)) - TSFC

T_{HR} = total thrust (kN)

C_{fcr} = cruise fuel flow factor (dim)

The calculation of TSFC is,

$$\eta = C_{f1}\left(1 + \frac{V}{C_{f2}}\right)$$

$$F_{cr} = T_{HR} C_{fcr} C_{f1}\left(1 + \frac{V}{C_{f2}}\right)$$

```
CC====== Engine Thrust ===============================================/
CC          Max climb thrust coefficients (SIM)                       /
CD       .32241E+06   .56718E+05   .13638E-10   .95535E+01   .37598E-02 /
CC       Desc(low)    Desc(high)   Desc level   Desc(app)    Desc(ld)   /
CD       .55988E-01   .64359E-01   .26418E+05   .12475E+00   .32981E+00 /
CC       Desc CAS     Desc Mach    unused       unused       unused     /
CD       .31000E+03   .80000E+00   .00000E+00   .00000E+00   .00000E+00 /
CC====== Fuel Consumption ============================================/
CC     Thrust Specific Fuel Consumption Coefficients                  /
CD       .74220E+00   .20605E+04                                      /
CC     Descent Fuel Flow Coefficients                                 /
CD       .15902E+02   .14538E+06                                      /
CC     Cruise Corr.   unused       unused       unused       unused     /
CD       .90050E+00   .00000E+00   .00000E+00   .00000E+00   .00000E+00 /
CC====== Ground ======================================================/
CC          TOL          LDL          span         length       unused   /
CD       .26520E+04   .16770E+04   .47570E+02   .54940E+02   .00000E+00 /
CC====================================================================/
```

Calculations (Continuation)

$$\eta = C_{f1}\left(1+\frac{V}{C_{f2}}\right)$$

where:

$C_{f1} =$ first thrust specific fuel consumption parameter (kg/(min*kN))

$C_{f2} =$ second thrust specific fuel consumption parameter

$V =$ true airspeed (knots)

Boeing 767-300 per BADA data

```
CC====== Fuel Consumption ============================================/
CC   Thrust Specific Fuel Consumption Coefficients                    /
CD     .74220E+00   .20605E+04                                        /
CC   Descent Fuel Flow Coefficients                                   /
CD     .15902E+02   .14538E+06                                        /
CC   Cruise Corr.    unused        unused        unused        unused /
CD     .90050E+00   .00000E+00    .00000E+00    .00000E+00   .00000E+00 /
```

$$\eta = C_{f1}\left(1+\frac{V}{C_{f2}}\right)$$

$$C_{f1} = 0.7422$$

$$C_{f2} = 2060.5$$

$$\eta = 0.7422\left(1+\frac{400}{2060.5}\right) = 0.8863$$

Calculations (Continuation)

$$F_{cr} = \eta T_{HR} C_{fcr}$$

$$F_{cr} = \eta D C_{fcr} \text{ use Drag since T=D holding altitude}$$

$$F_{cr} = (0.8863)(111{,}900/1000)(0.90)$$

$$F_{cr} = 89.2 \text{ kg/minute}$$

Boeing 767-300 per BADA data

- The value of fuel consumption assumes the thrust or drag to be in kiloNewtons (hence the division by 1000)
- The fuel burn for a 2.4 minute hold at 18,000 feet is estimated to be 200.25 kg above the usual unrestricted descent profile (i.e., continuous descent)
- 639 kg of CO_2 added to each descent profile

Sanity Check Against Published BADA Data in PTF Files

```
BADA PERFORMANCE FILE                                    Apr 10 2012

AC/Type: B763__
                              Source OPF File:           Apr 10 2012
                              Source APF file:           Mar 02 2010

 Speeds:   CAS(LO/HI)  Mach   Mass Levels [kg]      Temperature:  ISA
 climb   - 250/310     0.80   low      -  108013
 cruise  - 250/310     0.80   nominal  -  154590    Max Alt. [ft]:  43100
 descent - 250/310     0.80   high     -  186880
```

FL	CRUISE TAS [kts]	CRUISE fuel [kg/min] lo	CRUISE fuel [kg/min] nom	CRUISE fuel [kg/min] hi	CLIMB TAS [kts]	CLIMB ROCD [fpm] lo	CLIMB ROCD [fpm] nom	CLIMB ROCD [fpm] hi	CLIMB fuel [kg/min] nom	DESCENT TAS [kts]	DESCENT ROCD [fpm] nom	DESCENT fuel [kg/min] nom
0					166	2870	2235	1908	258.6	158	1025	85.0
5					167	2857	2220	1893	256.5	160	1041	84.3
10					169	2843	2206	1877	254.3	166	1125	83.8
160	389	79.9	89.2	97.6	389	3603	2483	1972	205.2	389	2155	14.2
180	401	79.8	89.2	97.7	401	3378	2306	1814	196.4	401	2198	13.9
200	413	79.7	89.2	97.8	413	3144	2123	1649	187.5	413	2240	13.7

Manual calculations agree with BADA PTF file table

$$F_{cr} = \eta T_{HR} C_{fcr}$$

$$F_{cr} = \eta D C_{fcr} \quad \text{use Drag since T=D holding altitude}$$

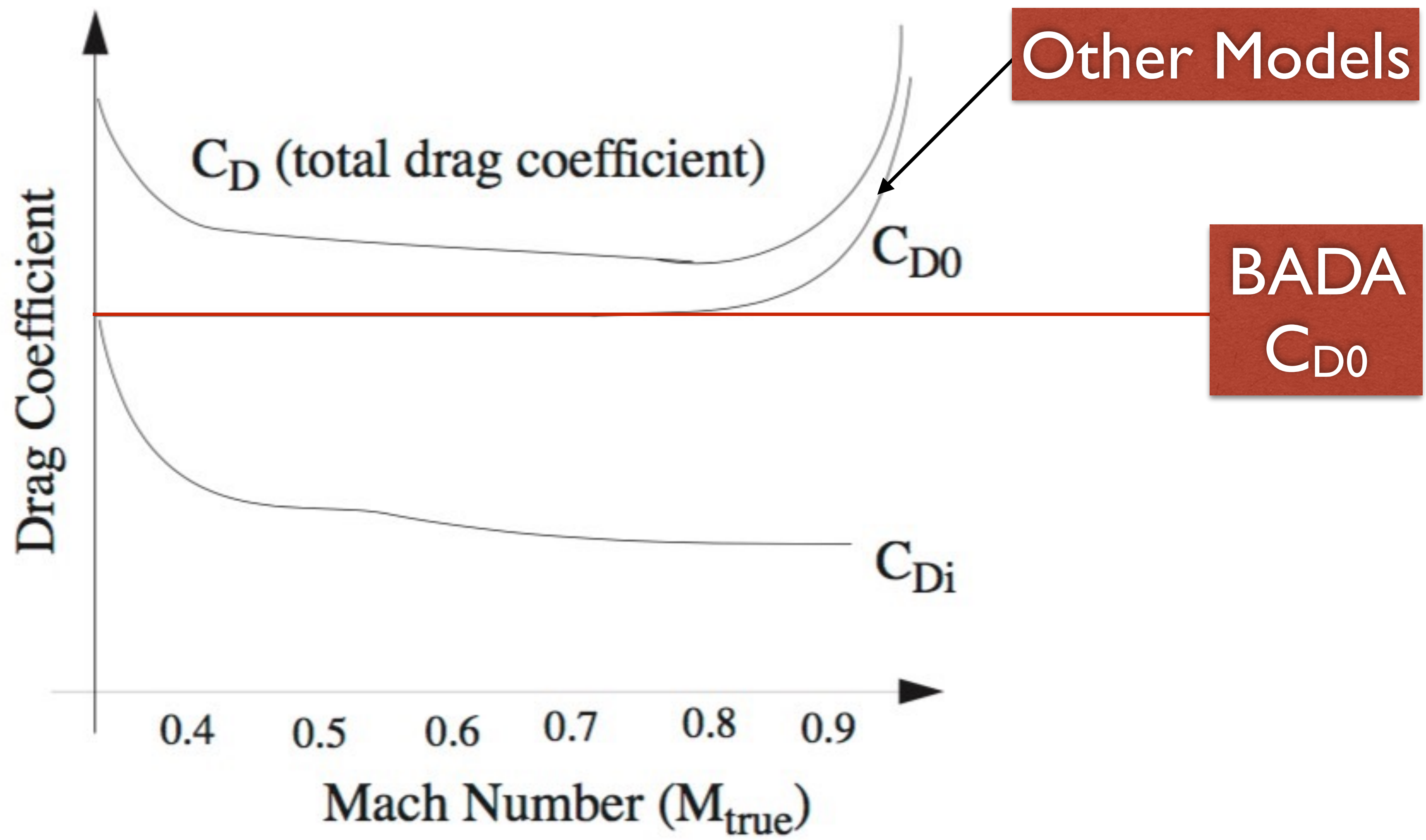
$$F_{cr} = (0.8863)(111{,}900 / 1000)(0.90)$$

$$F_{cr} = 89.2 \text{ kg/minute}$$

General Trends and Differences with Models Presented in Class

- BADA model uses a parabolic drag polar (i.e., variation of drag coefficient is quadratic with lift coefficient)
- The BADA model assumes the zero lift draft coefficient (C_{d0}) is a constant
- The assumption in CEE 5614 is that (C_{d0}) is a function of Mach number and nonlinear in the high-speed regime
- The last assumption is typical in most aerodynamic textbooks and supporter by wind tunnel and flight testing

BADA Model vs. Other Aero Models (included the one presented in CEE 5614)



BADA Climb Profile Calculations

- Fundamental equation

$$\frac{dh}{dt} = \frac{(Thr - D) \cdot V_{TAS}}{m g_0}\left[1 + \left(\frac{V_{TAS}}{g_0}\right)\left(\frac{dV_{TAS}}{dh}\right)\right]^{-1}$$

Energy Share Factor (ESF)

$$f\{M\} = \left[1 + \left(\frac{V_{TAS}}{g_0}\right) \cdot \left(\frac{dV_{TAS}}{dh}\right)\right]^{-1}$$

Different Aircraft Behaviors for Different Altitude Conditions

Low mass CLIMBS
===============

PTD files

FL[-]	T[K]	p[Pa]	rho[kg/m3]	a[m/s]	TAS[kt]	CAS[kt]	M[-]	mass[kg]	Thrust[N]	Drag[N]	Fuel[kgm]	ESF[-]	ROC[fpm]	TDC[N]	PWC[-]
0	288	101325	1.225	340	138.54	138.54	0.21	68616	193530	48754	156.7	0.98	2590	127289	0.88
5	287	99508	1.207	340	139.55	138.54	0.21	68616	191706	48760	155.4	0.98	2575	125680	0.88
10	286	97717	1.190	339	140.56	138.54	0.21	68616	189886	48766	154.1	0.98	2560	124075	0.88
15	285	95952	1.172	339	146.70	143.54	0.22	68616	188071	46814	153.5	0.97	2668	124195	0.88
20	284	94213	1.155	338	147.77	143.54	0.22	68616	186261	46820	152.2	0.97	2652	122598	0.88
30	282	90812	1.121	337	170.81	163.54	0.26	68616	182656	41587	152.5	0.96	3072	124029	0.88
40	280	87511	1.088	336	205.04	193.54	0.31	68616	179070	38986	154.2	0.95	3605	123163	0.88
60	276	81200	1.024	333	272.30	250.00	0.42	68616	171956	43498	157.0	0.91	4227	112942	0.88
80	272	75262	0.963	331	280.34	250.00	0.44	68616	164919	43435	151.6	0.91	4090	106811	0.88
100	268	69682	0.905	328	345.37	300.00	0.54	68616	157958	53149	153.1	0.87	4157	92151	0.88

Rate of climb term $\boxed{mg\frac{dh}{dt}}$

Energy Share Factor (ESF)

Acceleration term $\boxed{mV\frac{dV}{dt}}$

Interpretation: from 0-1000 feet the aircraft uses 98% of its power to climb and 2% to accelerate

BADA Flight Envelope

- BADA prescribes how the aircraft "flies" across the flight envelope
- Speed profiles are defined for each aircraft

The maximum speed and altitude for an aircraft are expressed in terms of the following six parameters:

V_{MO}	-	maximum operating speed (CAS) [kt]
M_{MO}	-	maximum operational Mach number
h_{MO}	-	maximum operating altitude [ft] above standard MSL
h_{max}	-	maximum altitude [ft] above standard MSL at MTOW under ISA conditions (allowing about 300 ft/min of residual rate of climb)
G_w	-	mass gradient on h_{max} [ft/kg]
G_t	-	temperature gradient on h_{max} [ft/K]

Project BADA – EEC Technical/Scientific Report No. 15/04/02-43

BADA Flight Envelope (Climb)

The following parameters are defined for each aircraft type to characterise the climb phase:

$V_{cl,1}$	-	standard climb CAS [knots] between 1,500/6,000 and 10,000 ft
$V_{cl,2}$	-	standard climb CAS [knots] between 10,000 ft and Mach transition altitude
M_{cl}	-	standard climb Mach number above Mach transition altitude

- For jet aircraft the following CAS schedule is assumed, based on the parameters mentioned above and the take-off stall speed:

from 0 to 1,499 ft	$C_{Vmin} \cdot (V_{stall})_{TO} + Vd_{CL,1}$	(4.1-1)
from 1,500 to 2,999 ft	$C_{Vmin} \cdot (V_{stall})_{TO} + Vd_{CL,2}$	(4.1-2)
from 3,000 to 3,999 ft	$C_{Vmin} \cdot (V_{stall})_{TO} + Vd_{CL,3}$	(4.1-3)
from 4,000 to 4,999 ft	$C_{Vmin} \cdot (V_{stall})_{TO} + Vd_{CL,4}$	(4.1-4)
from 5,000 to 5,999 ft	$C_{Vmin} \cdot (V_{stall})_{TO} + Vd_{CL,5}$	(4.1-5)
from 6,000 to 9,999 ft	$\min(V_{cl,1}, 250\text{ kt})$	
from 10,000 ft to Mach transition altitude	$V_{cl,2}$	
above Mach transition altitude	M_{cl}	

Project BADA – EEC Technical/Scientific Report No. 15/04/02-43

BADA Flight Envelope (Descent)

The following parameters are defined for each aircraft type to characterise the descent phase:

$V_{des,1}$	-	standard descent CAS [knots] between 3,000/6,000 and 10,000 ft
$V_{des,2}$	-	standard descent CAS [knots] between 10,000 ft and Mach transition altitude
M_{des}	-	standard descent Mach number above Mach transition altitude

- For jet and turboprop aircraft the following CAS schedule is assumed, based on the above parameters and the landing stall speed:

from 0 to 999 ft	$C_{Vmin} \cdot (V_{stall})_{LD} + Vd_{DES,1}$	(4.3-1)
from 1,000 to 1,499 ft	$C_{Vmin} \cdot (V_{stall})_{LD} + Vd_{DES,2}$	(4.3-2)
from 1,500 to 1,999 ft	$C_{Vmin} \cdot (V_{stall})_{LD} + Vd_{DES,3}$	(4.3-3)
from 2,000 to 2,999 ft	$C_{Vmin} \cdot (V_{stall})_{LD} + Vd_{DES,4}$	(4.3-4)
from 3,000 to 5,999 ft	$\min(V_{des,1}, 220)$	
from 6,000 to 9,999 ft	$\min(V_{des,1}, 250)$	
from 10,000 ft to Mach transition altitude	$V_{des,2}$	
above Mach transition altitude	M_{des}	

Project BADA – EEC Technical/Scientific Report No. 15/04/02-43

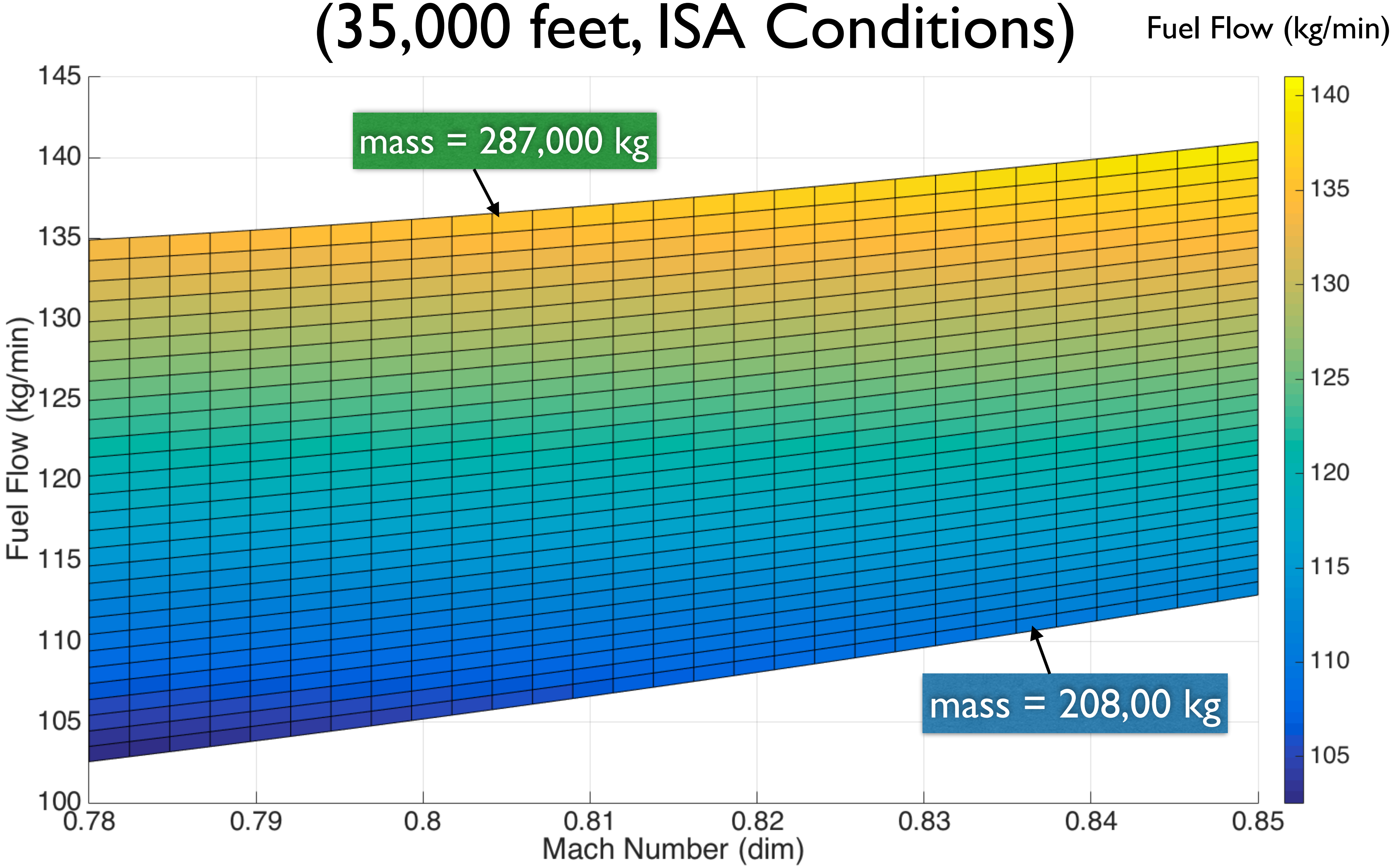
BADA Operations Performance

Model Category	Symbols	Units	Description
Aircraft type (3 values)	n_{eng}	dimensionless	number of engines
	engine type	string	either Jet, Turboprop or Piston
	wake category	string	either J, H, M or L
Mass (4 values)	m_{ref}	tonnes	reference mass
	m_{min}	tonnes	minimum mass
	m_{max}	tonnes	maximum mass
	m_{pyld}	tonnes	maximum payload mass
Flight envelope (6 values)	V_{MO}	knots (CAS)	maximum operating speed
	M_{MO}	dimensionless	maximum operating Mach number
	h_{MO}	feet	maximum operating altitude
	h_{max}	feet	max. altitude at MTOW and ISA
	G_w	feet/kg	weight gradient on max. altitude
	G_t	feet/K	temperature gradient on max. altitude

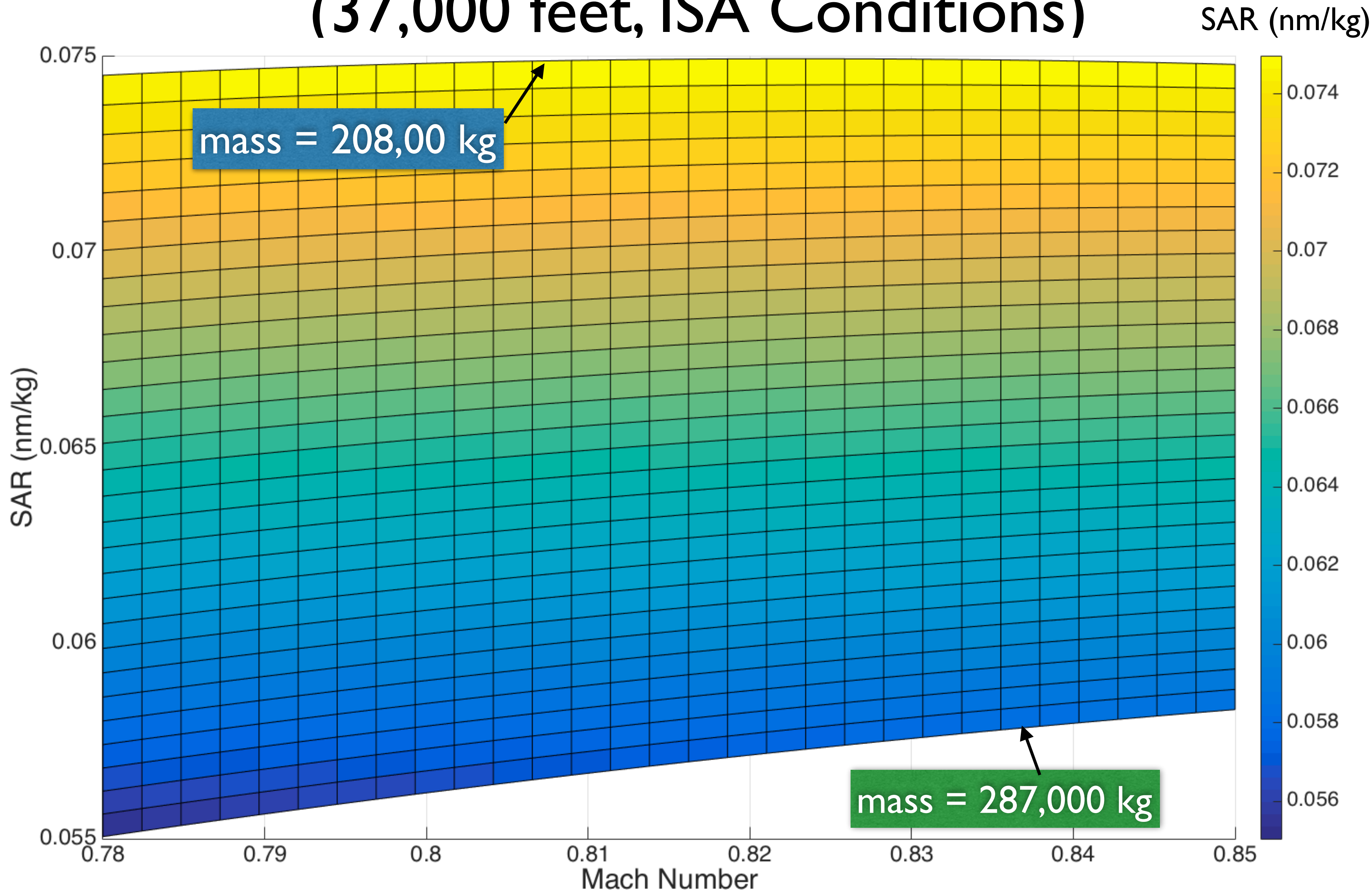
BADA Operations Performance

Aerodynamics (16 values for jet aircraft, only 14 values for others)	**S**	m^2	**reference wing surface area**
	$C_{D0,CR}$	dimensionless	**parasitic drag coefficient (cruise)**
	$C_{D2,CR}$	dimensionless	**induced drag coefficient (cruise)**
	$C_{D0,AP}$	dimensionless	**parasitic drag coefficient (approach)**
	$C_{D2,AP}$	dimensionless	**induced drag coefficient (approach)**
	$C_{D0,LD}$	dimensionless	**parasitic drag coefficient (landing)**
	$C_{D2,LD}$	dimensionless	**induced drag coefficient (landing)**
	$C_{D0,\Delta LDG}$	dimensionless	**parasite drag coef. (landing gear)**
	$(V_{stall})_i$	knots (CAS)	**stall speed [TO, IC, CR, AP, LD]**
	$C_{Lbo\ (M=0)}$	dimensionless	**Buffet onset lift coef. (jet only)**
	K	dimensionless	**Buffeting gradient (jet only)**

Boeing 777-200 Fuel Flow Profiles (35,000 feet, ISA Conditions)



Boeing 777-200 SAR Profiles (37,000 feet, ISA Conditions)



How Good is the BADA Model?

- 3% accurate for most flight conditions
- Weak in the very high-speed regime (due to fixed value of C_{D0}) - BADA 4 fixes that issue
- Goodness should be judged in terms of broad model coverage (hundreds of aircraft modeled)
- Most FAA and Eurocontrol studies use the BADA model

Eurocontrol Provides Accuracy Tables for Each Aircraft

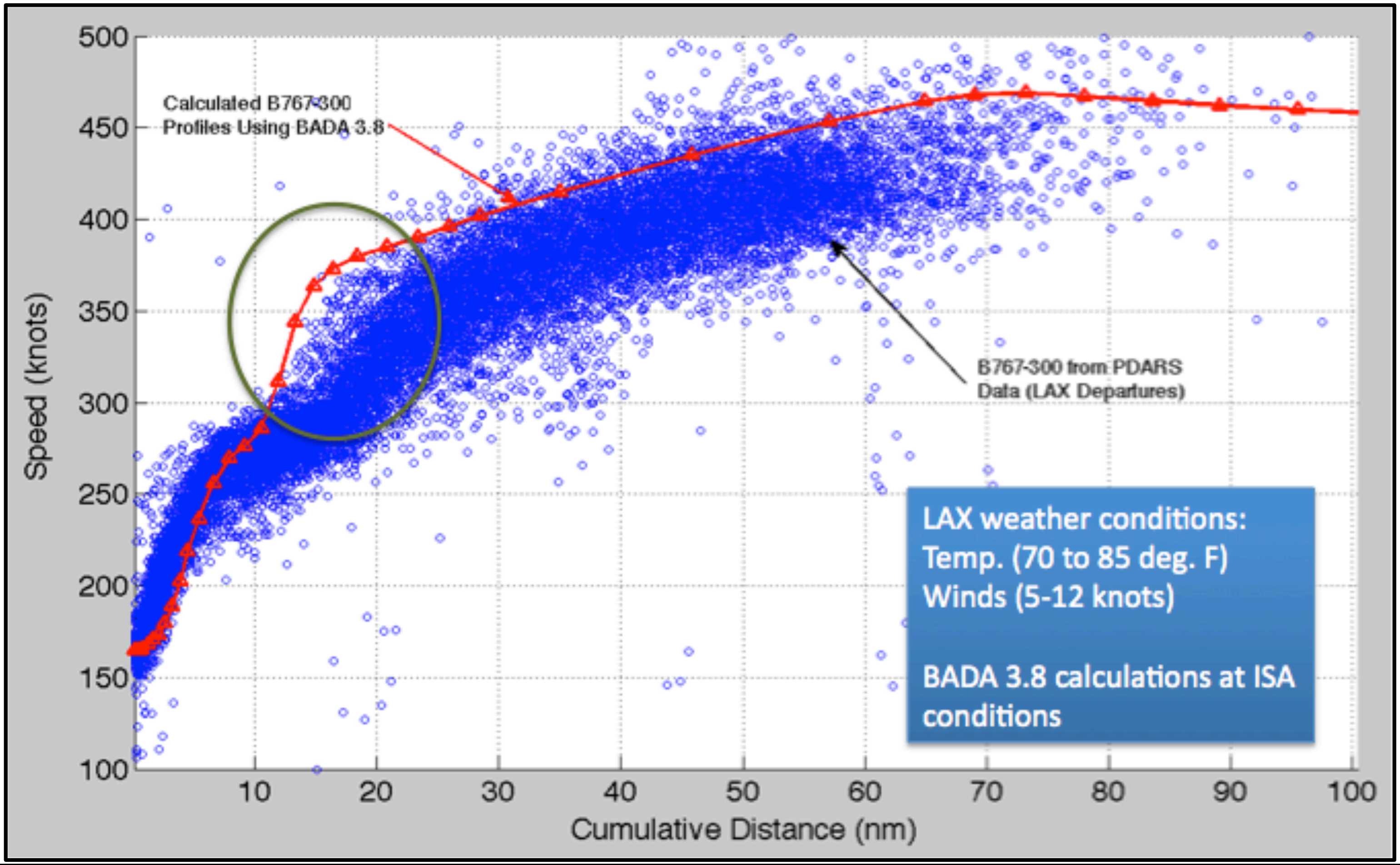
Airbus A320

Project BADA – EEC Technical/Scientific Report No. 15/04/02-46

TRJ ID	TRJ Type	CAS <FL100	CAS >FL100	M	Aircraft mass	Delta ISA	Comment	RMS [ft/min]	MEAN [ft/min]	STD [ft/min]	MAX [ft/min]	RMS [kg/min]	MEAN [kg/min]	STD [kg/min]	MAX [kg/min]
1	CMB	250	250	0.6	47400	0		95.46238	-19.50276	93.44897	-267.16163	1.82251	-0.97692	1.53856	-3.9835
2	CMB	250	250	0.6	65900	0		53.85403	0.42777	53.85233	-106.7547	1.32019	-0.68519	1.12846	-2.68895
3	CMB	250	250	0.6	77000	0		51.5305	-5.19022	51.26845	-98.46809	1.21749	-0.7473	0.96116	-2.87238
4	CMB	340	340	0.8	47400	0		112.77173	-36.66326	106.64553	-351.67168	1.9895	-0.62706	1.88809	-8.16375
5	CMB	340	340	0.8	65900	0		76.44849	-6.29307	76.18904	-234.42772	1.46273	-0.39887	1.4073	2.74534
6	CMB	340	340	0.8	77000	0		65.60937	-5.15927	65.4062	-167.07799	1.41206	-0.50975	1.31684	-2.45286
7	CMB	310	310	0.78	47400	0		65.67395	12.70318	64.43366	167.88802	1.65015	-0.37061	1.608	3.83497
8	CMB	310	310	0.78	65900	0		50.25753	27.47948	42.07966	108.42289	1.6033	-0.39591	1.55365	5.26354
9	CMB	310	310	0.78	77000	0		36.8917	18.69244	31.8055	-78.95411	1.31089	-0.4663	1.22515	-2.71884
10	CMB	310	310	0.78	47400	10		63.46123	-0.35359	63.46025	154.54491	1.8501	1.2859	1.33016	4.5214
11	CMB	310	310	0.78	65900	10		45.58419	18.47384	41.67296	-113.29937	1.81013	1.17683	1.37537	3.5012
12	CMB	310	310	0.78	77000	10		33.56252	11.00403	31.70732	-86.13185	1.70065	1.22063	1.18418	3.61904
13	CMB	310	310	0.78	65900	20		37.59368	1.03798	37.57935	99.28708	2.1887	1.8089	1.2322	4.7528
14	DES	310	310	0.78	47400	0		75.48633	-4.50696	75.35166	175.76834	1.83428	0.08875	1.83213	6.99055
15	DES	310	310	0.78	65900	0		33.53863	14.34174	30.31756	80.42327	1.41548	-0.00908	1.41546	3.2487
16	DES	310	310	0.78	77000	0		39.952	8.22195	39.09683	-84.69529	1.49947	-0.08423	1.4971	5.48679
17	CRZ	/	/	0.78	64000	0						0.46342	0.02711	0.46262	1.39843

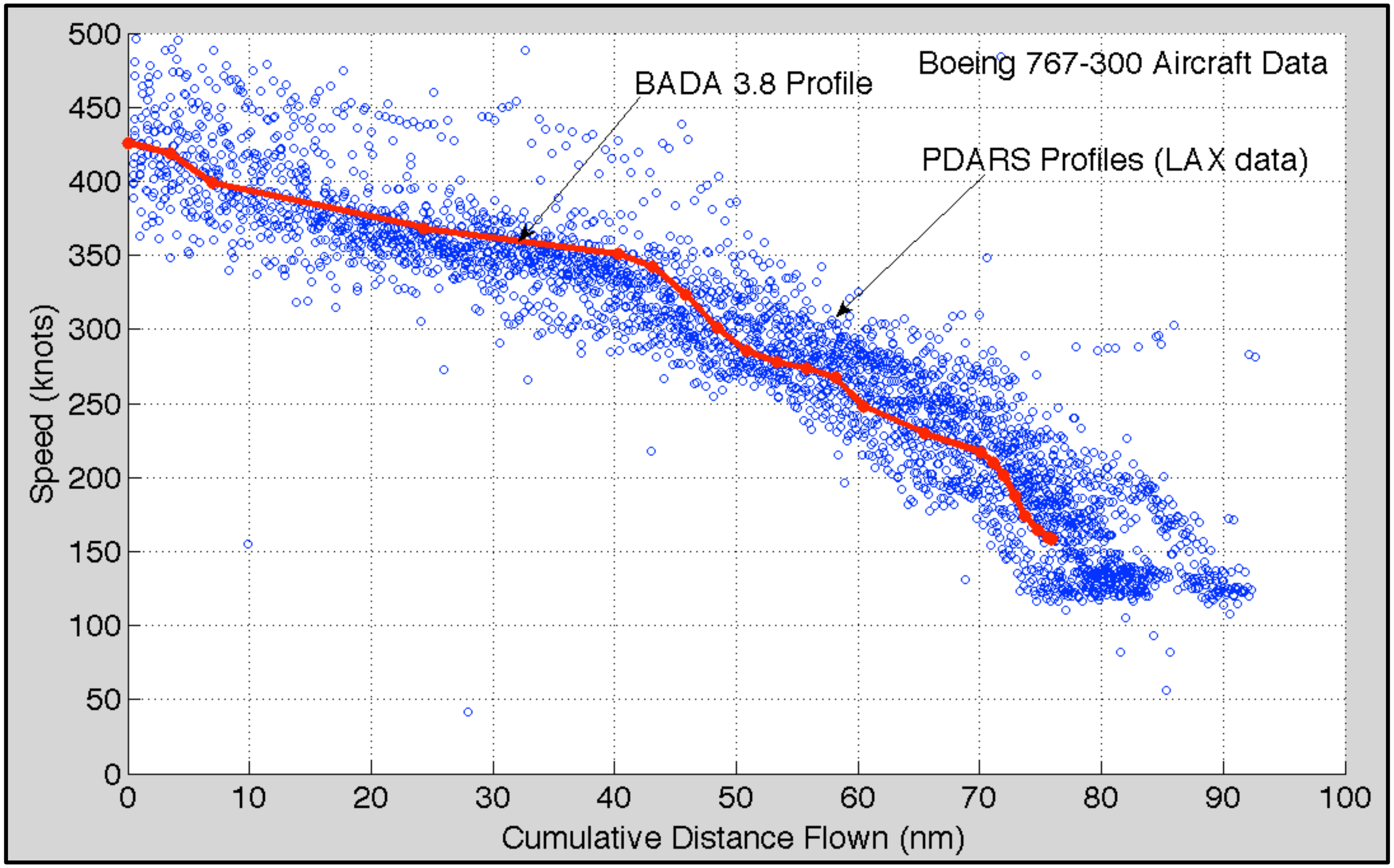
Comparing of BADA vs. Radar Data

- BADA profiles are based on European flight data
- Note some of the deviations shown compared to US radar data (PDARS)



BADA Descent Profiles

- BADA profiles are based on European flight data
- Note some of the deviations shown compared to US radar data (PDARS)

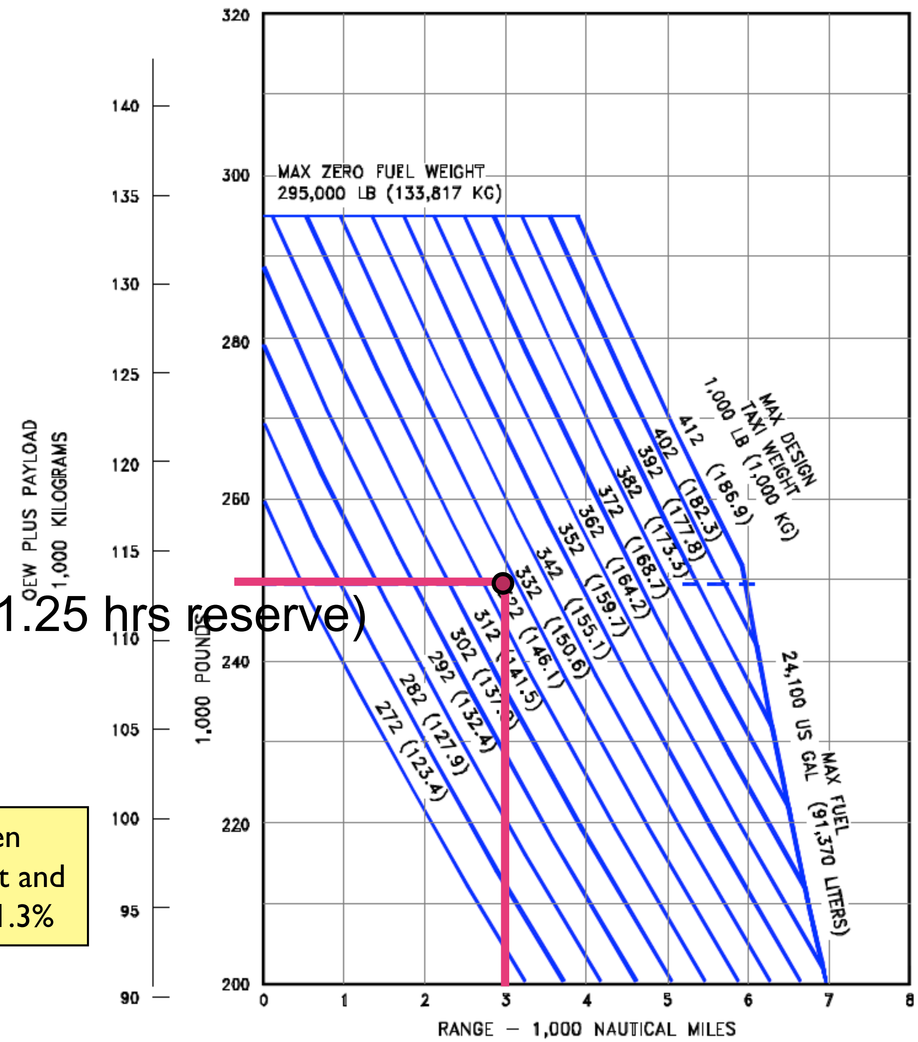


Reality Check with Boeing Data

- **Example:**
- 3,000 nm trip across the Atlantic
- 269 passengers + baggage
- OEW = 85,235 kg
- OEW + PYL = 113,500 kg
- 148,800 kg takeoff mass
- Fuel mass = 35,300 kg (includes 1.25 hrs reserve)
- 7.2 hour trip + 1.25 hr. reserve
- Reserve = 5,200
- **Trip fuel =~ 30,7**
- **Regression model = 30,300 kg**

The difference between Boeing planning document and the regression model is 1.3%

B767-300ER Payload-Range

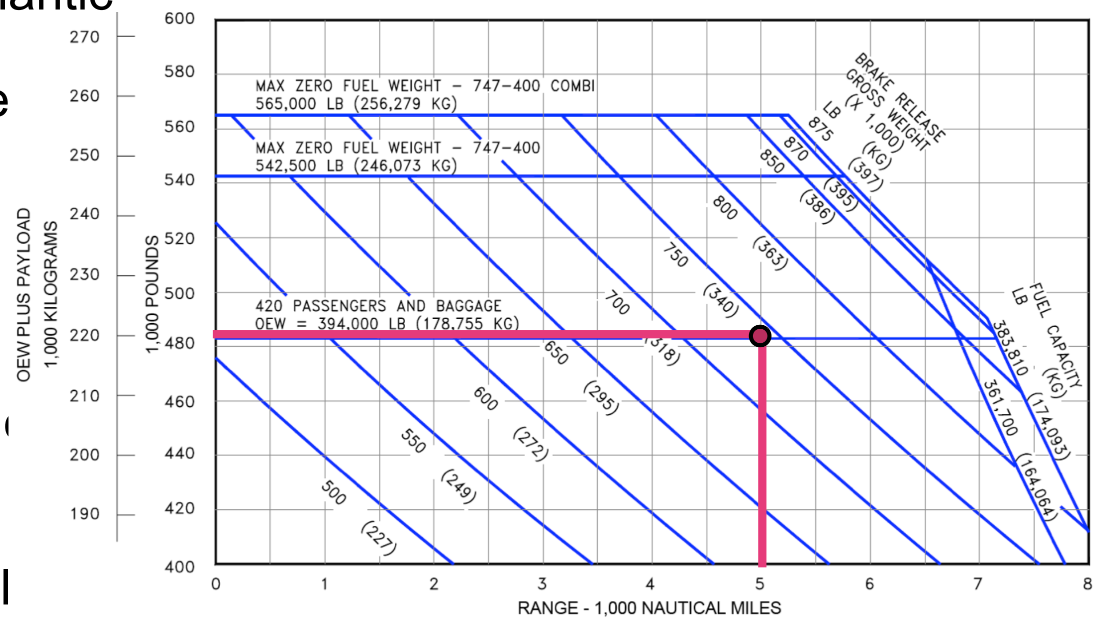


source: Boeing 767-300ER Docs.

Reality Check with Boeing Data

- **Example:**
- 5,000 nm trip across the Atlantic
- 420 passengers + baggage
- OEW + PYL = 220,000 kg
- 335,000 kg takeoff mass
- Fuel mass = 110,000 kg (in
  hr holding at 1500 feet)
- 11.2 hour trip + reserve fuel
- Reserve = 16,000 kg
- **Trip fuel =~ 94,000 kg**
- **Regression model = 95,170 kg**

B747-400 Payload-Range



source: Boeing 747-400 Docs.

The difference between Boeing planning document and the regression model is 1.2%