Airport Planning and Design

Excel Solver

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Demand Function Example

Given data representing demand at an airport (D(t)) we would like to derive the best nonlinear model to fit the data to a model of the form:

$$D(t) = k \cdot a^{b^t} \quad \text{Gompertz Model}$$

$$D(t) = \frac{k}{1 + b \cdot e^{-at}} \quad \text{Logistic Model}$$

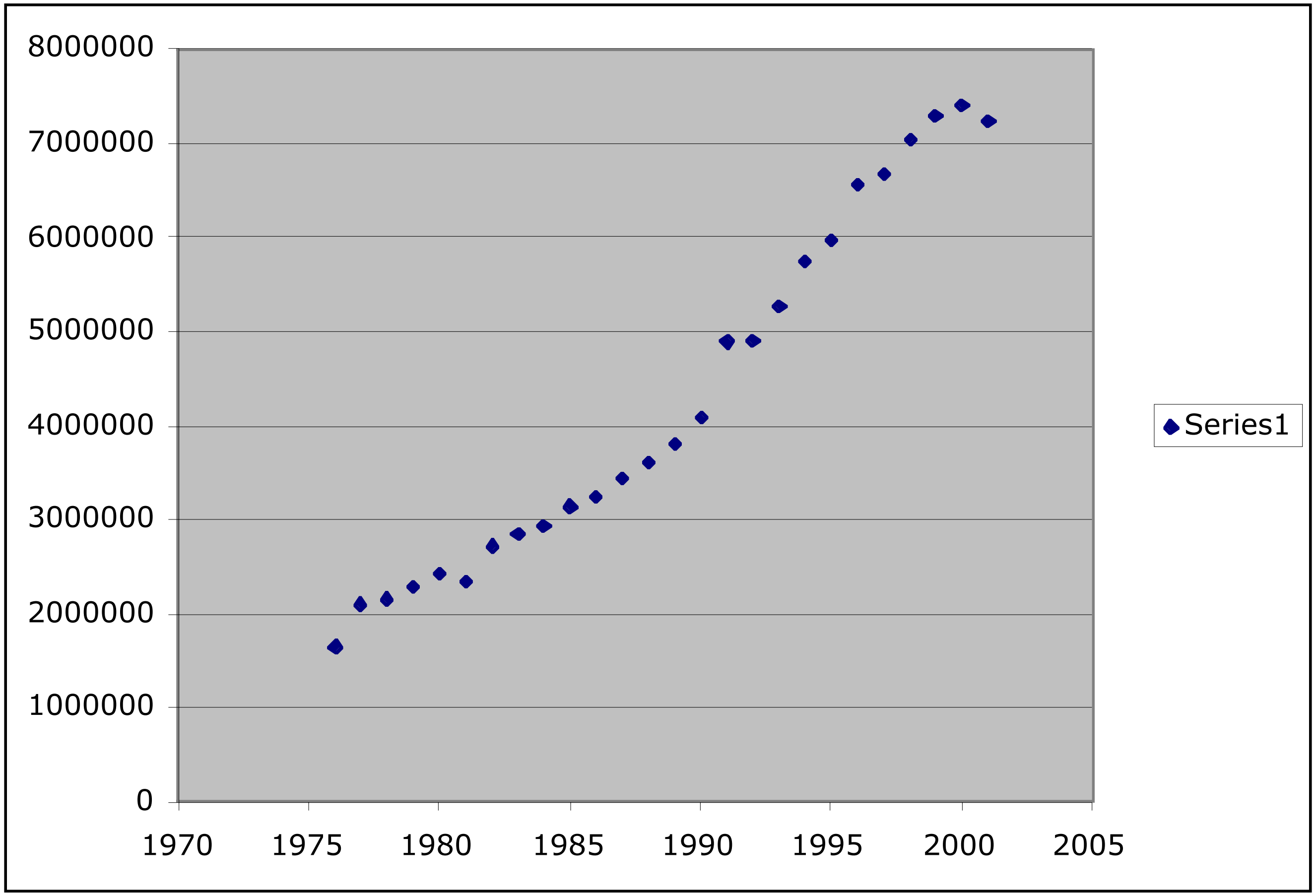
Data

Given: data pairs for time and Demand (D(t))

Find: the best nonlinear regression equation that correlates with the data pairs (t, D(t))

Data File: airport2.xls

Data Set Plot



Setup of Solver Procedure

The idea is to minimize the Sum of Square Errors of the data and an assumed regressions equation

- Create a column with values of the assumed regression equation
- Leave parameters of the model as cells in the spreadsheet (Excel will iterate among any number of parameters)
- Minimize the Sum of the Square Errors (SSE) of the data
- You are done!

Setup of Solver

airport2.xls

	A	B	C	D	E	F
1	Time	Demand		Calculated Demand		Square Errors
2	1976	1650000		1341962.45		94887133035
3	1977	2100342		1505196.89		3.54198E+11
4	1978	2159060		1683267.09		2.26379E+11
5	1979	2289354		1876317.68		1.70599E+11
6	1980	2418506		2084203.58		1.11758E+11
7	1981	2340000		2306447.01		1125802946
8	1982	2723424		2542204.86		32840501643
9	1983	2847846		2790251.16		3317133335
10	1984	2929954		3048979.2		14166907795
11	1985	3138216		3316425.93		31758930175
12	1986	3237440		3590320.11		1.24524E+11
13	1987	3436687		3868152.51		1.86162E+11
14	1988	3613075		4147264.33		2.85358E+11
15	1989	3800849		4424947.52		3.89499E+11
16	1990	4078844		4698548.98		3.84034E+11
17	1991	4890000		4965570.49		5710899368
18	1992	4906601		5223756.2		1.00587E+11
19	1993	5270381		5471161.72		40312794576
20	1994	5753800		5706200.95		2265707736
21	1995	5970459		5927669.25		1830963256
22	1996	6560330		6134744.3		1.81123E+11
23	1997	6669229		6326967.69		1.17143E+11
24	1998	7040655		6504211.56		2.87772E+11
25	1999	7291141		6666635.15		3.90007E+11
26	2000	7412591		6814635.89		3.5755E+11
27	2001	7230000		6948799.21		79073886090
28						
29						3.97399E+12
30						
31	Parameters					
32	a	0				
33	b	5				
34	k	8000000				

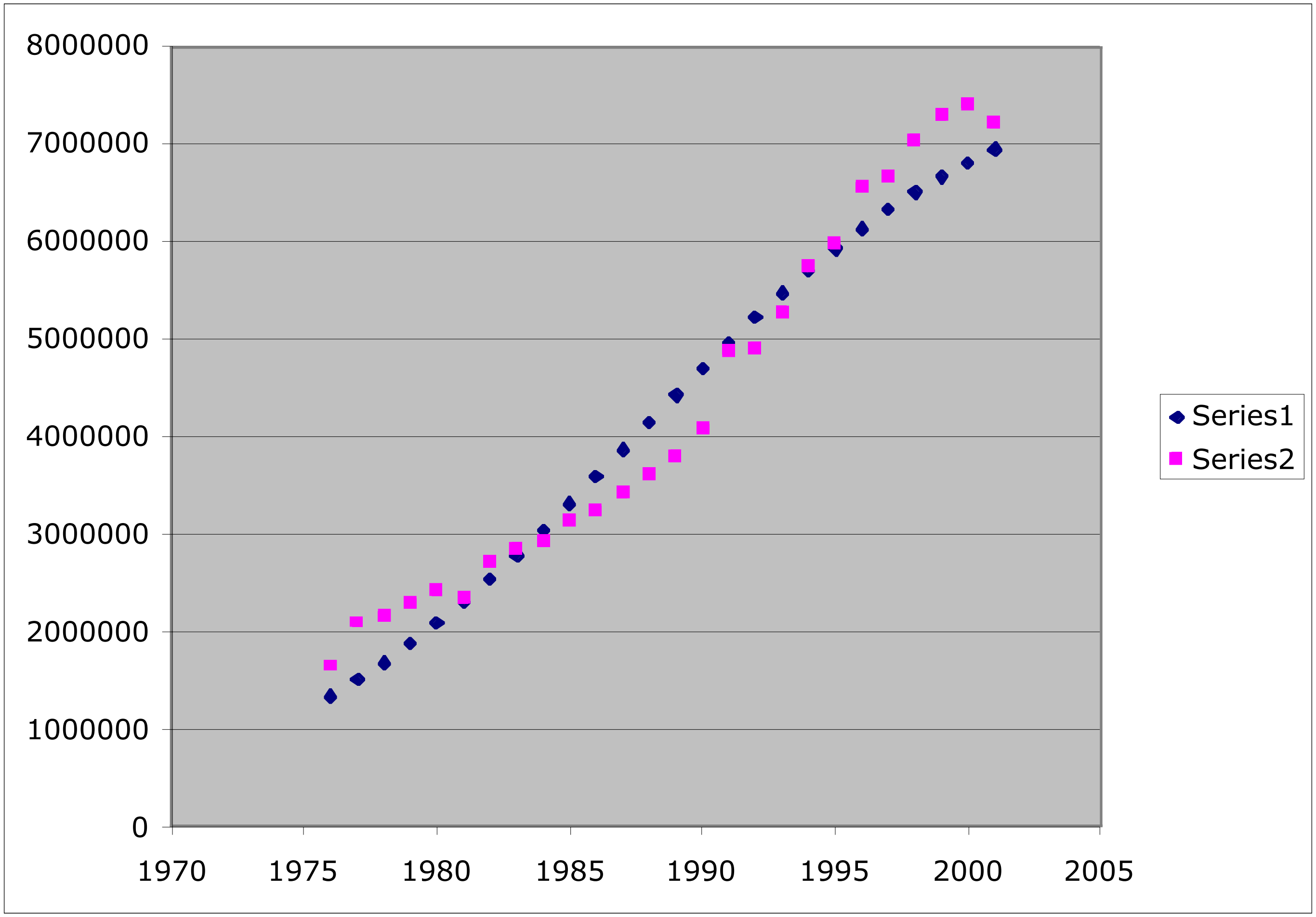
Setup of Solver

Cells to Iterate

20	1994	5753800		5706200.95		2265707736
21	1995	5970459		5927669.25		1830963256
22	1996	6560330		6134744.3		1.81123E+11
23	1997	6669229		6326967.69		1.17143E+11
24	1998	7040655		6504211.56		2.87772E+11
25	1999	7291141		6666635.15		3.90007E+11
26	2000	7412591		6814635.89		3.5755E+11
27	2001	7230000		6948799.21		79073886090
28						
29						3.97399E+12
30						
31	Parameters					
32	a	0				
33	b	5				
34	k	8000000				

Cell to Minimize

Solution Set and Original Data



Linear Programming Problems

Virginia Tech

General Formulation

Maximize $\sum_{j=1}^{n} c_j x_j$

subject to: $\sum_{j=1}^{n} a_{ij} x_j \le b_i$ for $i = 1, 2, \ldots, m$

$x_j \ge 0$ for $j = 1, 2, \ldots, n$

Linear Programming

$$\sum_{j=1}^{n} c_j x_j$$ Objective Function (OF)

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$ Functional Constraints (*m* of them)

$x_j \geq 0$ Nonnegativity Conditions (*n* of these)

x_j are decision variables to be optimized (min or max)

c_j are costs associated with each decision variable

Linear Programming

a_{ij} are the coefficients of the functional constraints

b_i are the amounts of the resources available (RHS)

LP Example (Construction)

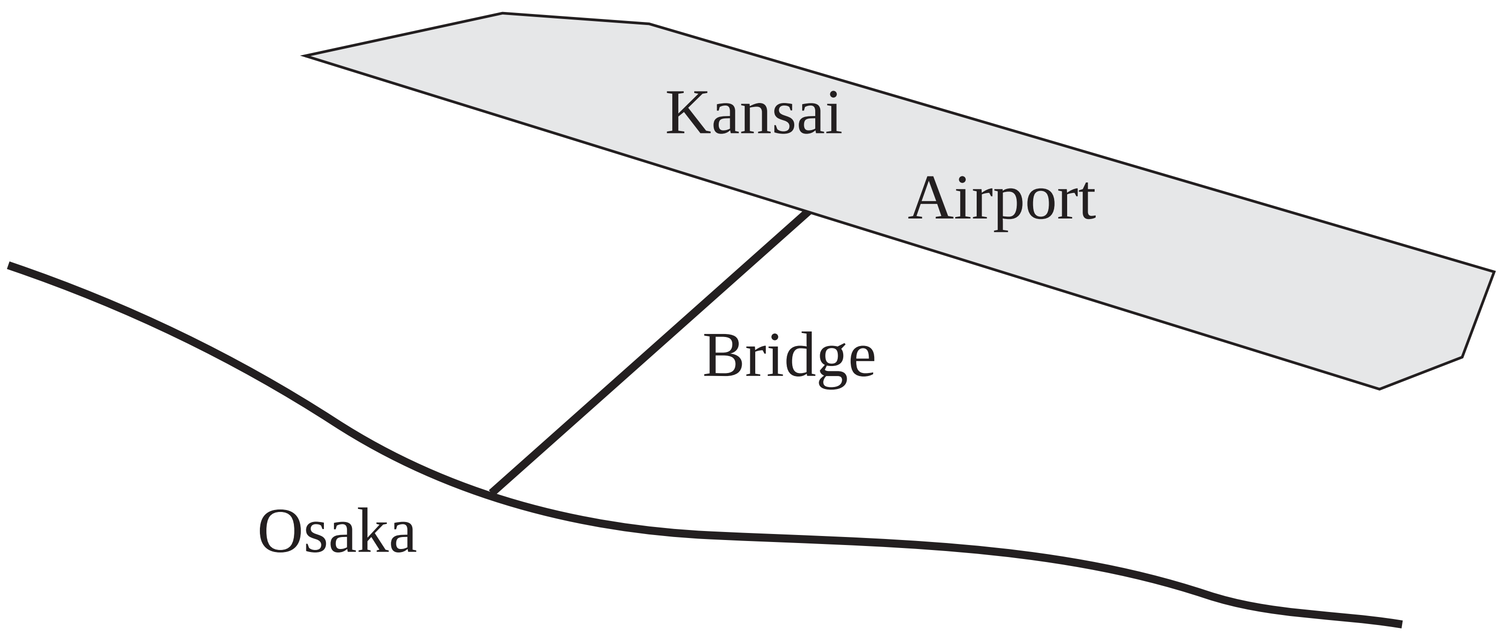
During the construction of an off-shore airport in Japan the main contractor used two types of cargo barges to transport materials from a fill collection site to the artificial island built to accommodate the airport.

The types of cargo vessels have different cargo capacities and crew member requirements as shown in the table:

Vessel Type	**Capacity (m-ton)**	**Crew required**	**Number available**
Fuji	300	3	40
Haneda	500	2	60

Osaka Bay Model

According to company records there are 180 crew members in the payroll and all crew members are trained to either manage the "Haneda" or "Fuji" vessels.



Osaka Bay Model

Mathematical Formulation

Maximize $Z = 300x_1 + 500x_2$

subject to: $3x_1 + 2x_2 \leq 180$

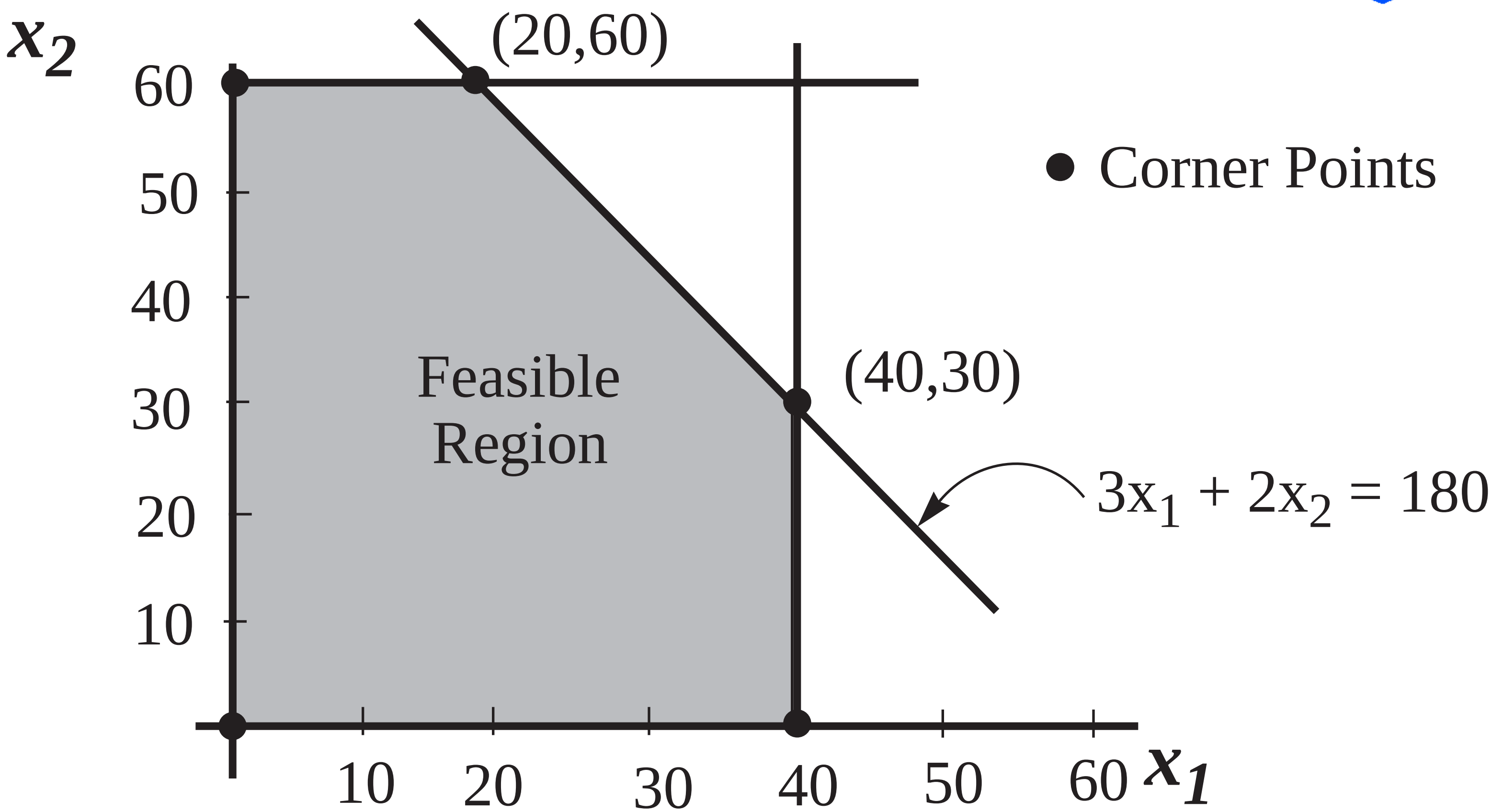
$x_1 \leq 40$

$x_2 \leq 60$

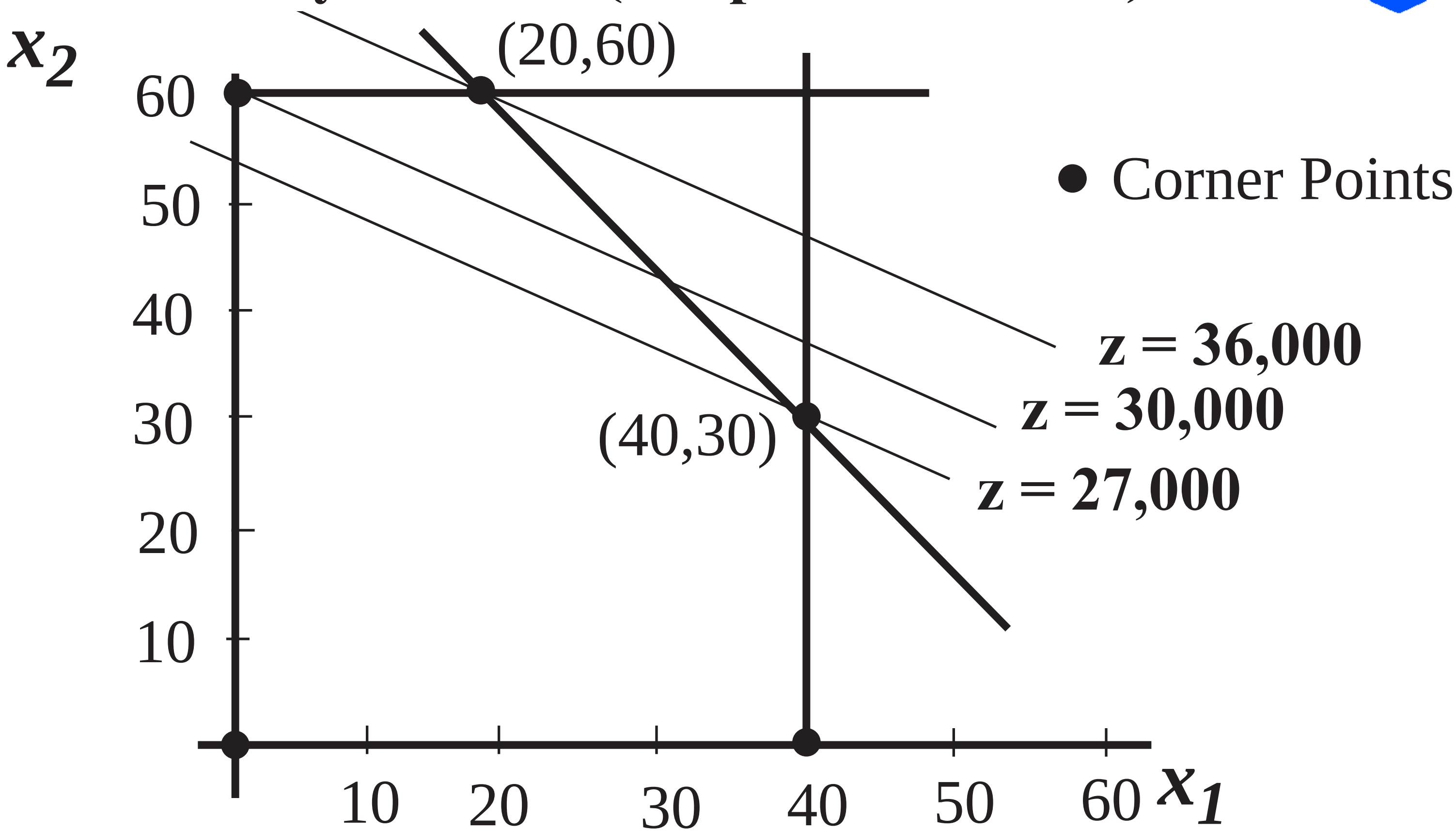
$x_1 \geq 0$ and $x_2 \geq 0$

Note: let x_1 and x_2 be the no. “Fuji” and “Haneda” vessels

Osaka Bay Problem (Graphical Solution)



Osaka Bay Problem (Graphical Solution)



Note: Optimal Solution (x_1, x_2) = (20,60) vessels

Solution Using Excel Solver

- Solver is a Generalized Reduced Gradient (GRG2) nonlinear optimization code
- Developed by Leon Lasdon (UT Austin) and Allan Waren (Cleveland State University)
- Optimization in Excel uses the Solver add-in.
- Solver allows for one function to be minimized, maximized, or set equal to a specific value.
- Convergence criteria (convergence), integer constraint criteria (tolerance), and are accessible through the OPTIONS button.

Excel Solver

- Excel can solve simultaneous linear equations using matrix functions
- Excel can solve one nonlinear equation using Goal Seek or Solver
- Excel does not have direct capabilities of solving n multiple nonlinear equations in n unknowns, but sometimes the problem can be rearranged as a minimization function

Osaka Bay Problem in Excel

Optimization Problem for Osaka Bay

Decision Variables		
x1	20	Number of Ships Type 1
x2	60	Number of Ships Type 2

Objective Function	
300 x1 + 500 x2	36000

Objective function
Stuff to be solved

Constraint Equations			
	Formula		
3 x1 + 2 x2 <= 180	180	<=	180
x1 <= 40	20	<=	40
x2 <= 60	60	<=	60
x1 >= 0	20	>=	0
x2 >= 0	60	>=	0

Osaka Bay Problem in Excel

Optimization Problem for Osaka Bay

Decision variables (what your control)

Decision Variables			
x1	20		Number of Ships Type 1
x2	60		Number of Ships Type 2

Objective Function	
300 x1 + 500 x2	36000

Constraint Equations			
	Formula		
3 x1 + 2 x2 <= 180	180	<=	180
x1 <= 40	20	<=	40
x2 <= 60	60	<=	60
x1 >= 0	20	>=	0
x2 >= 0	60	>=	0

Osaka Bay Problem in Excel

Optimization Problem for Osaka Bay

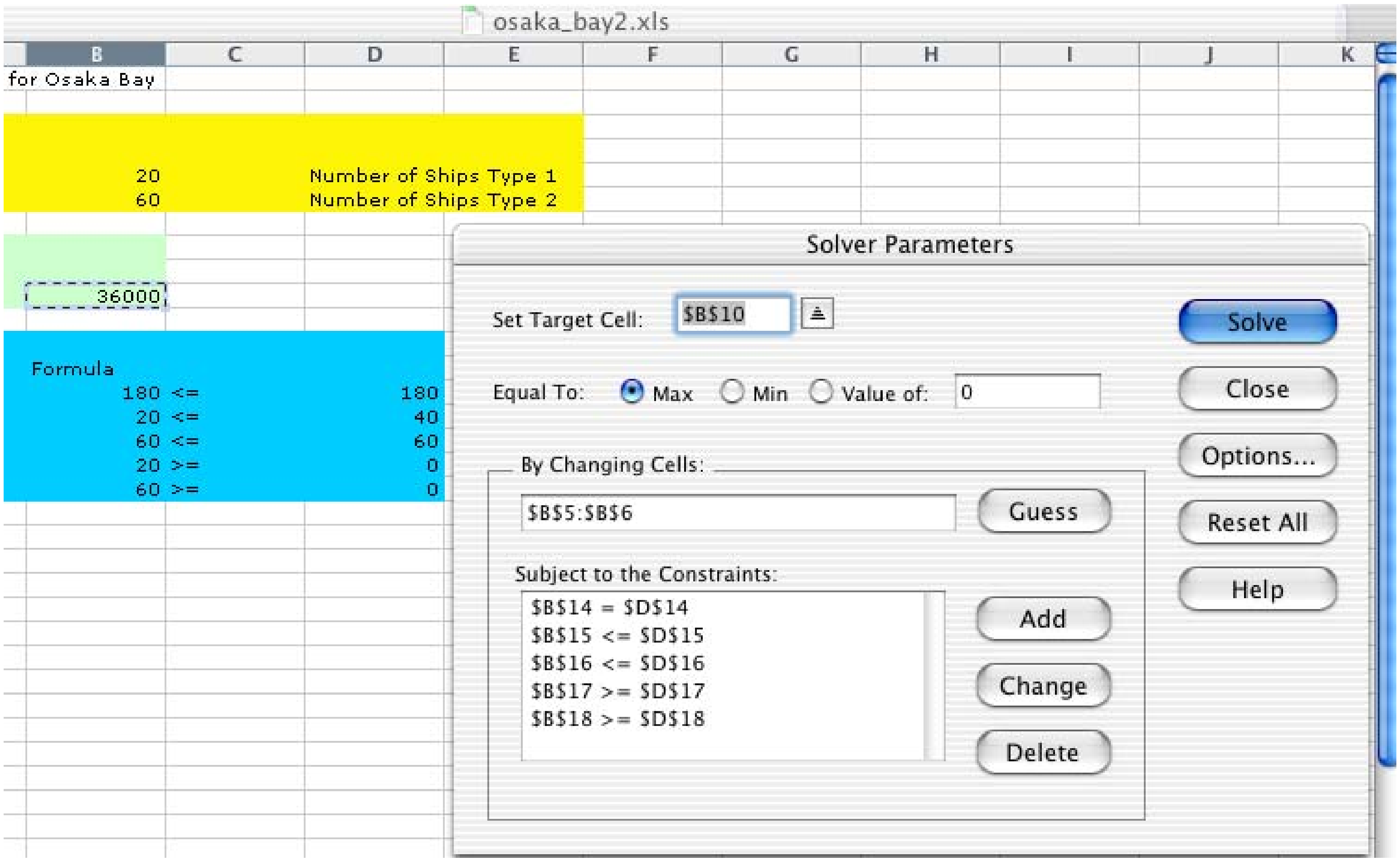
Decision Variables		
x1	20	Number of Ships Type 1
x2	60	Number of Ships Type 2

Objective Function	
300 x1 + 500 x2	36000

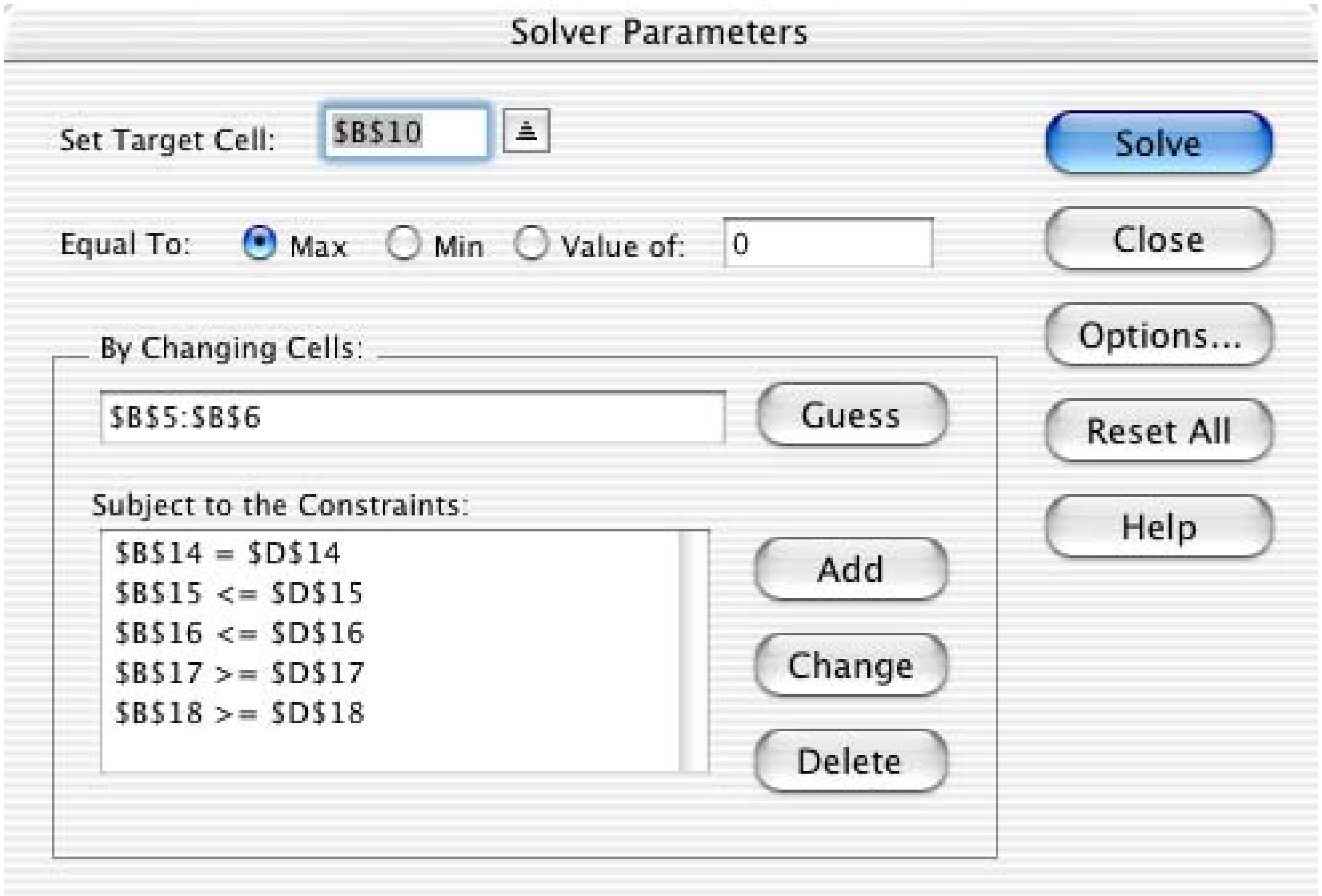
Constraint equations (limits to the problem)

Constraint Equations			
	Formula		
3 x1 + 2 x2 <= 180	180	<=	180
x1 <= 40	20	<=	40
x2 <= 60	60	<=	60
x1 >= 0	20	>=	0
x2 >= 0	60	>=	0

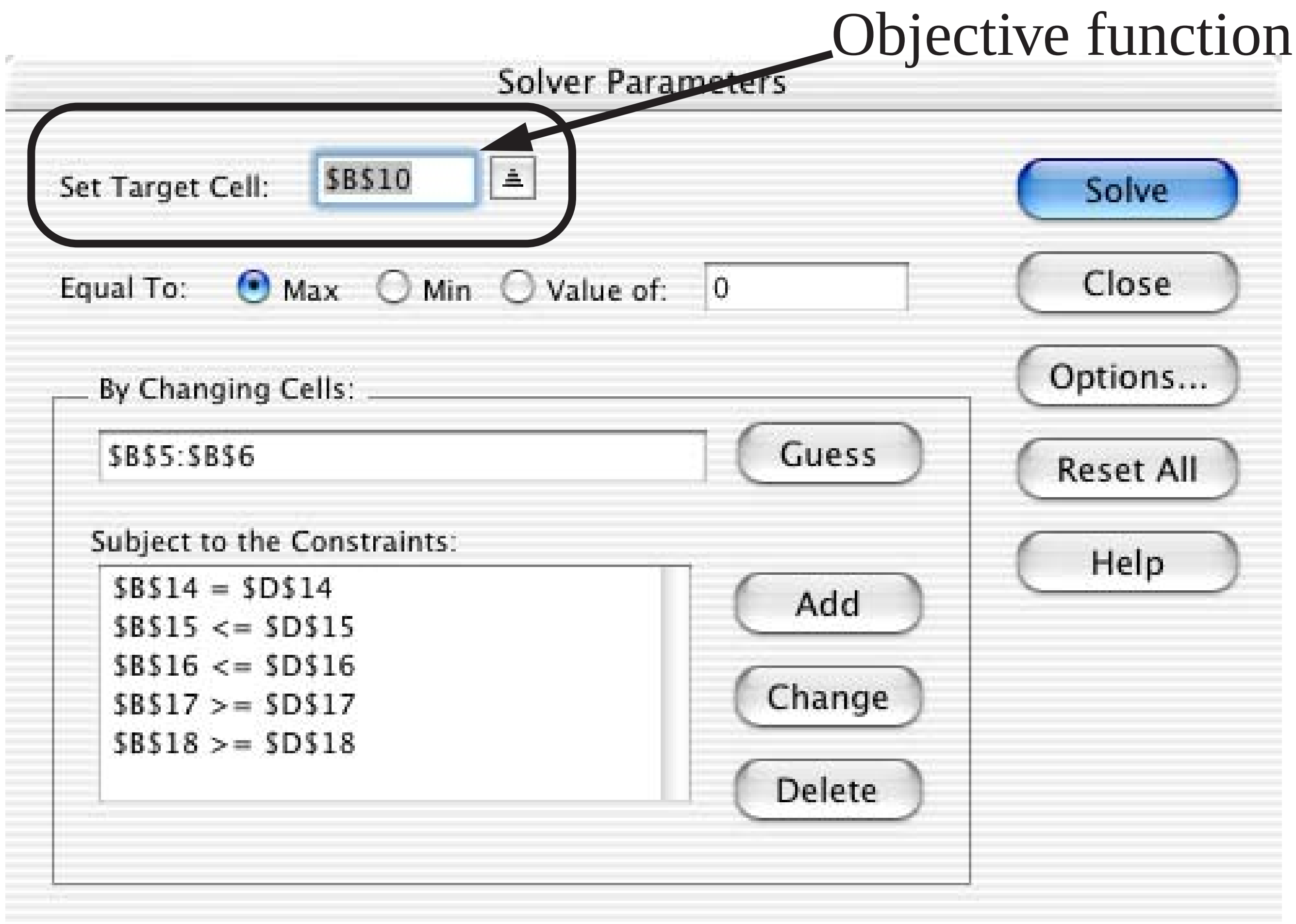
Solver Panel in Excel



Solver Panel in Excel

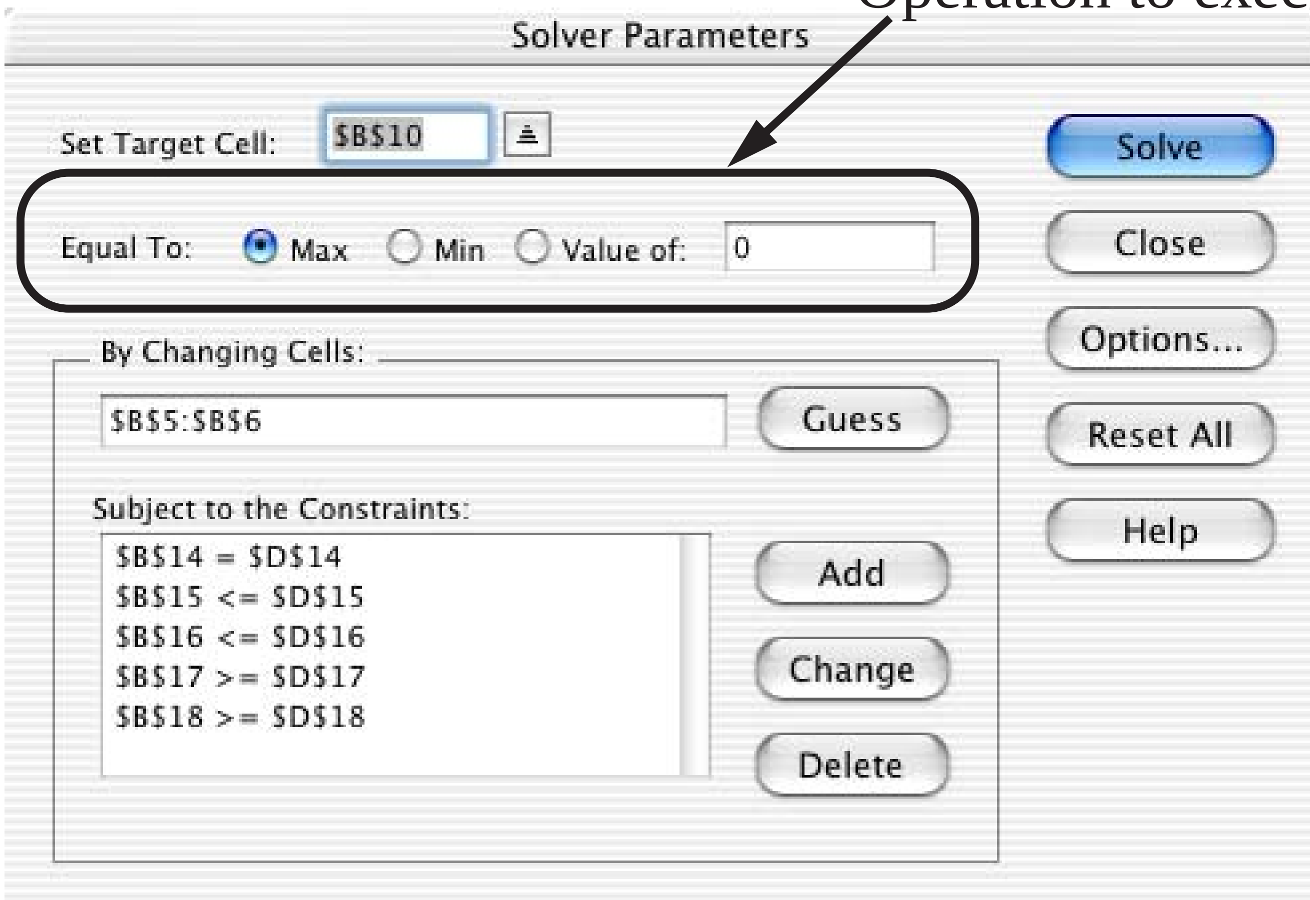


Solver Panel in Excel

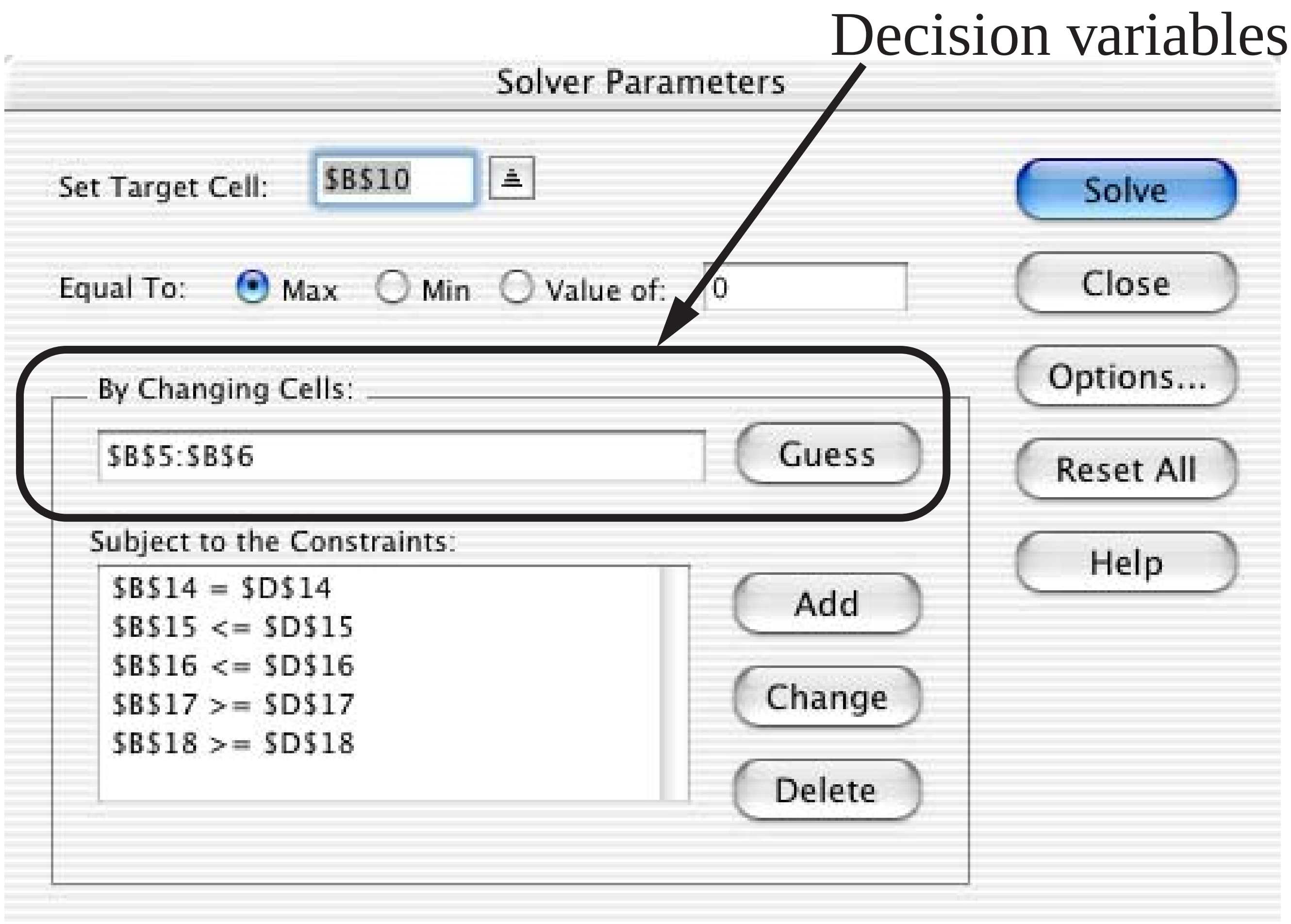


Solver Panel in Excel

Operation to execute

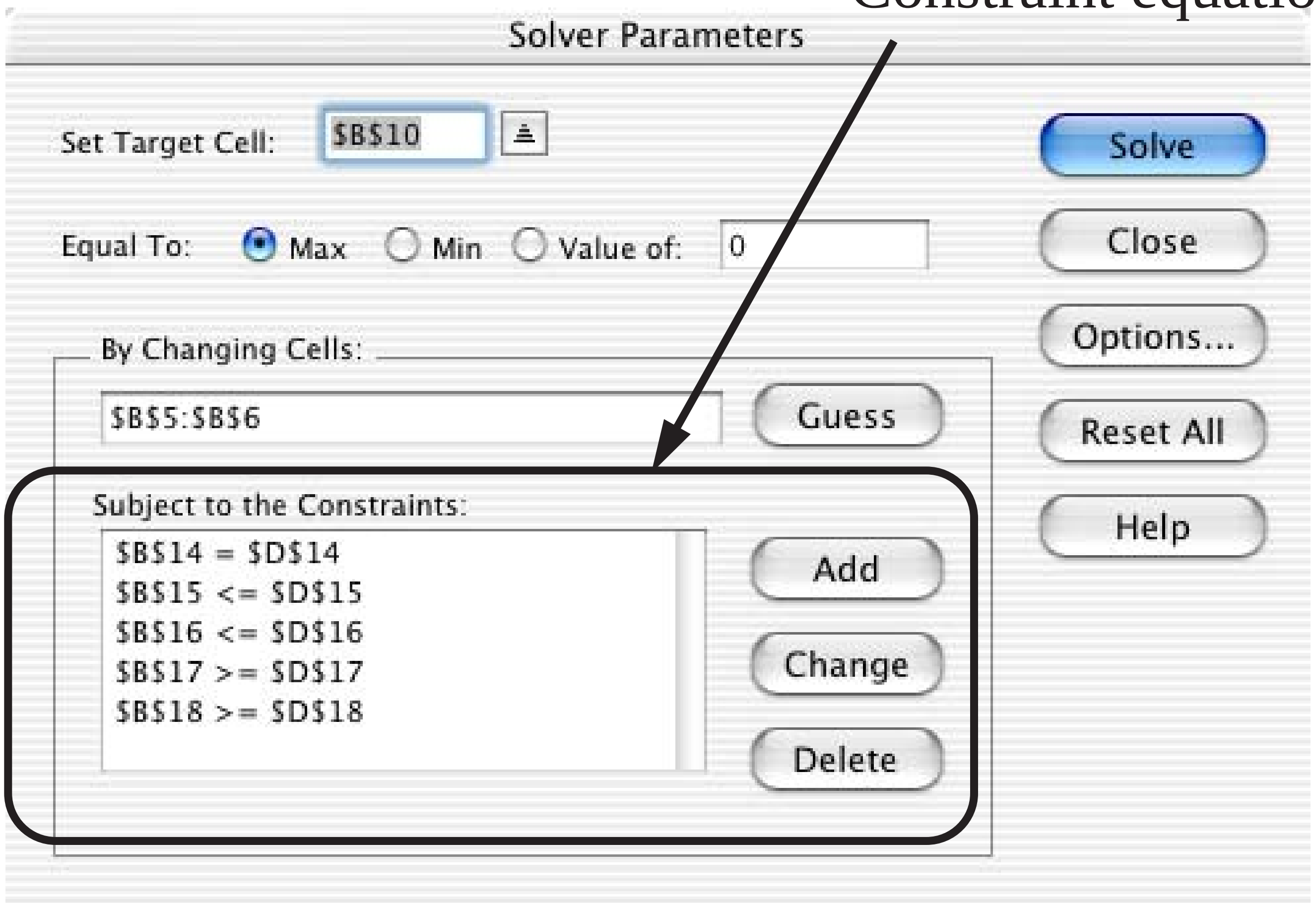


Solver Panel in Excel

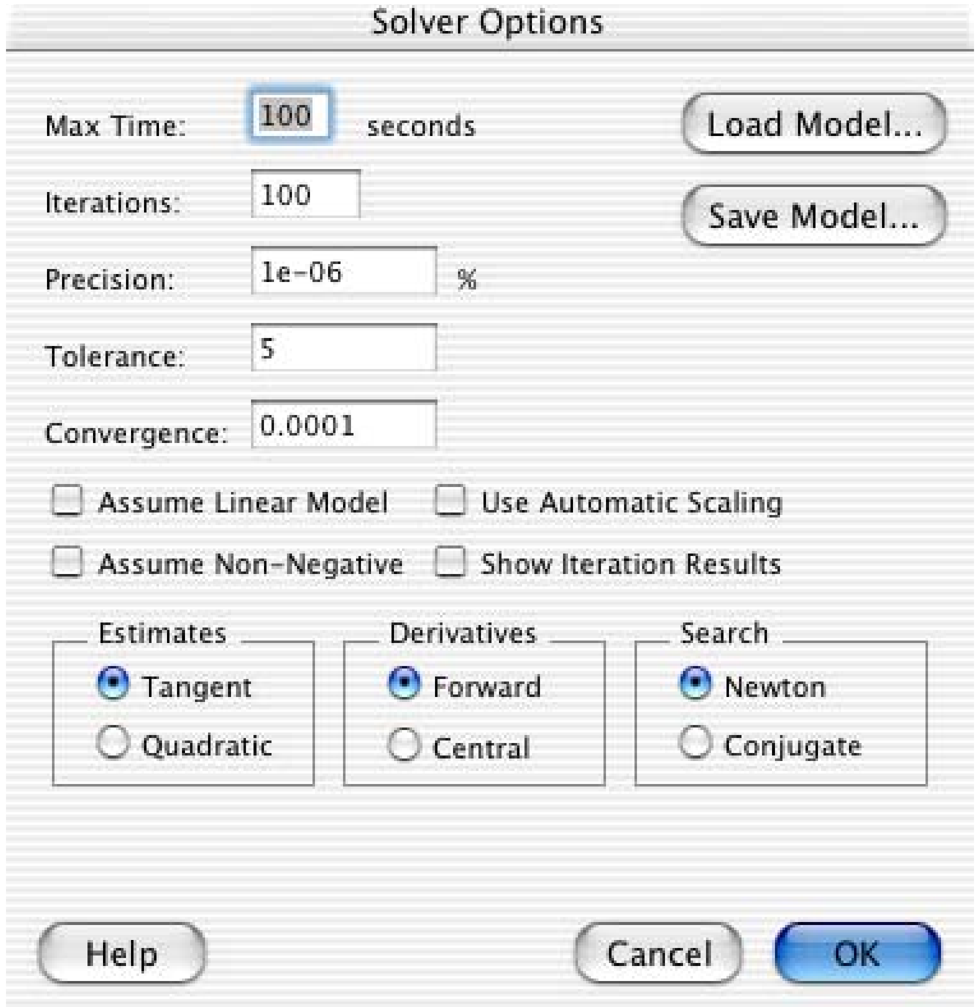


Solver Panel in Excel

Constraint equations



Solver Options Panel Excel



Excel Solver Limits Report

- Provides information about the limits of decision variables

Microsoft Excel 10.1 Limits Report
Worksheet: [Workbook1]Sheet1
Report Created: 3/10/2003 5:04:26 AM

Cell	Target Name	Value
B10	300 x1 + 500 x2	36000

Cell	Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
B5	x1	20	0	30000	20	36000
B6	x2	60	1.33227E-15	6000	60	36000

Excel Solver Sensitivity Report

- Provides information about shadow prices of decision variables

Microsoft Excel 10.1 Sensitivity Report
Worksheet: [osaka_bay2.xls]Sheet1
Report Created: 3/10/2003 5:47:49 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Gradient
B5	x1	20	0
B6	x2	60	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
B18	x2 >= 0 Formula	60	0
B15	x1 <= 40 Formula	20	0
B16	x2 <= 60 Formula	60	300
B17	x1 >= 0 Formula	20	0
B14	3 x1 + 2 x2 <= 180 Formula	180	100

Unconstrained Optimization Problems

- Common in engineering applications
- Can be solved using Excel solver as well
- The idea is to write an equation (linear or nonlinear) and then use solver to iterate the variable (or variables) to solve the problem

Simple One Dimensional Unconstrained Optimization

- Given the quadratic equation

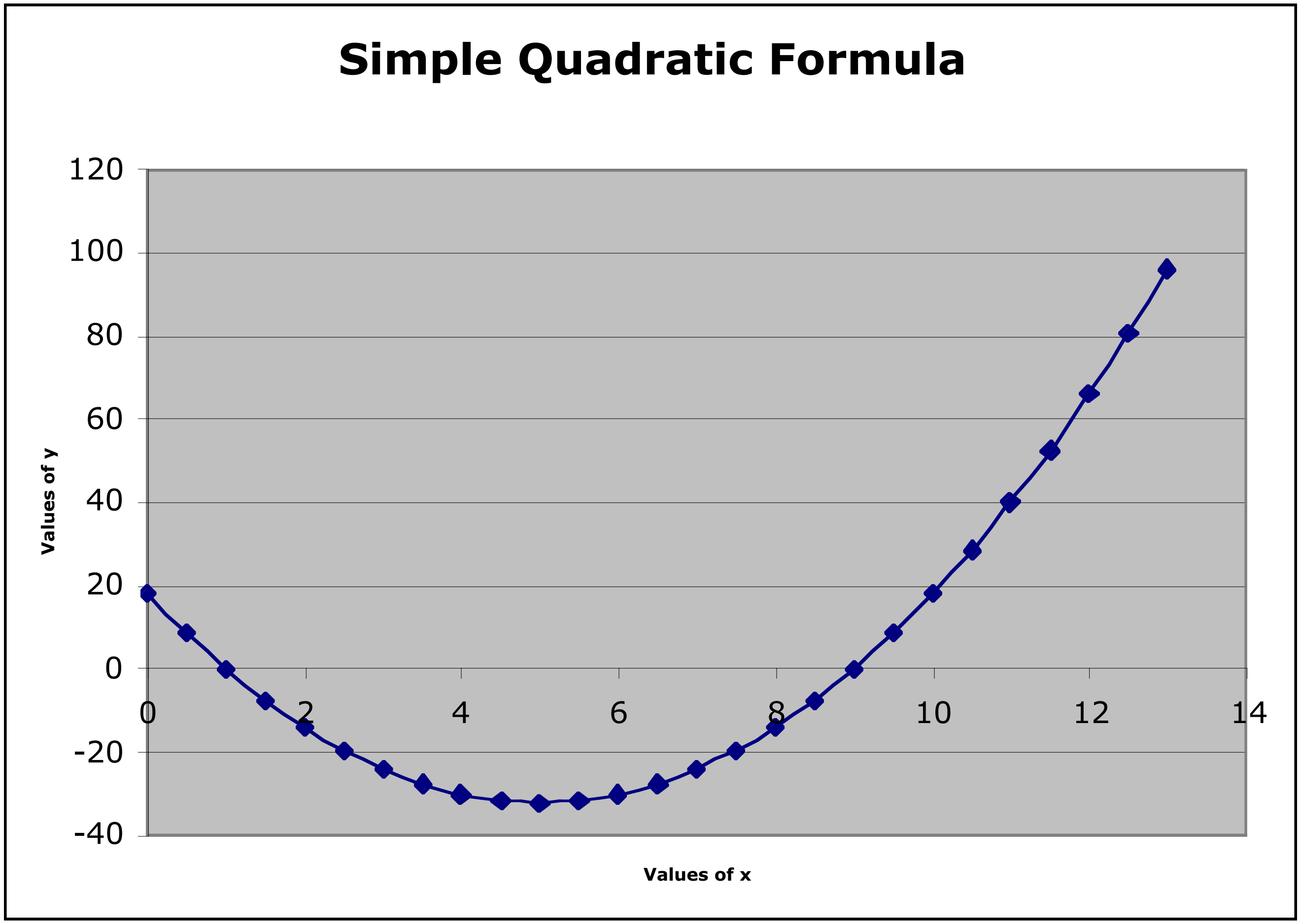
$$y = 2x^2 - 20x + 18$$

- Find the minima of the equation for all values of x

Solution:

- Lets try the Excel Solver

Plot of Equation to be Solved

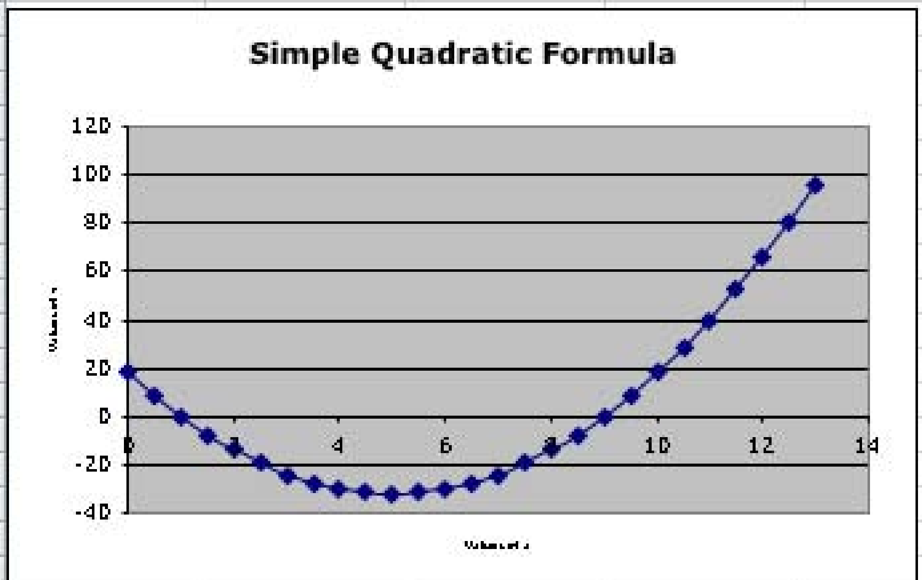


Excel Solver Procedure

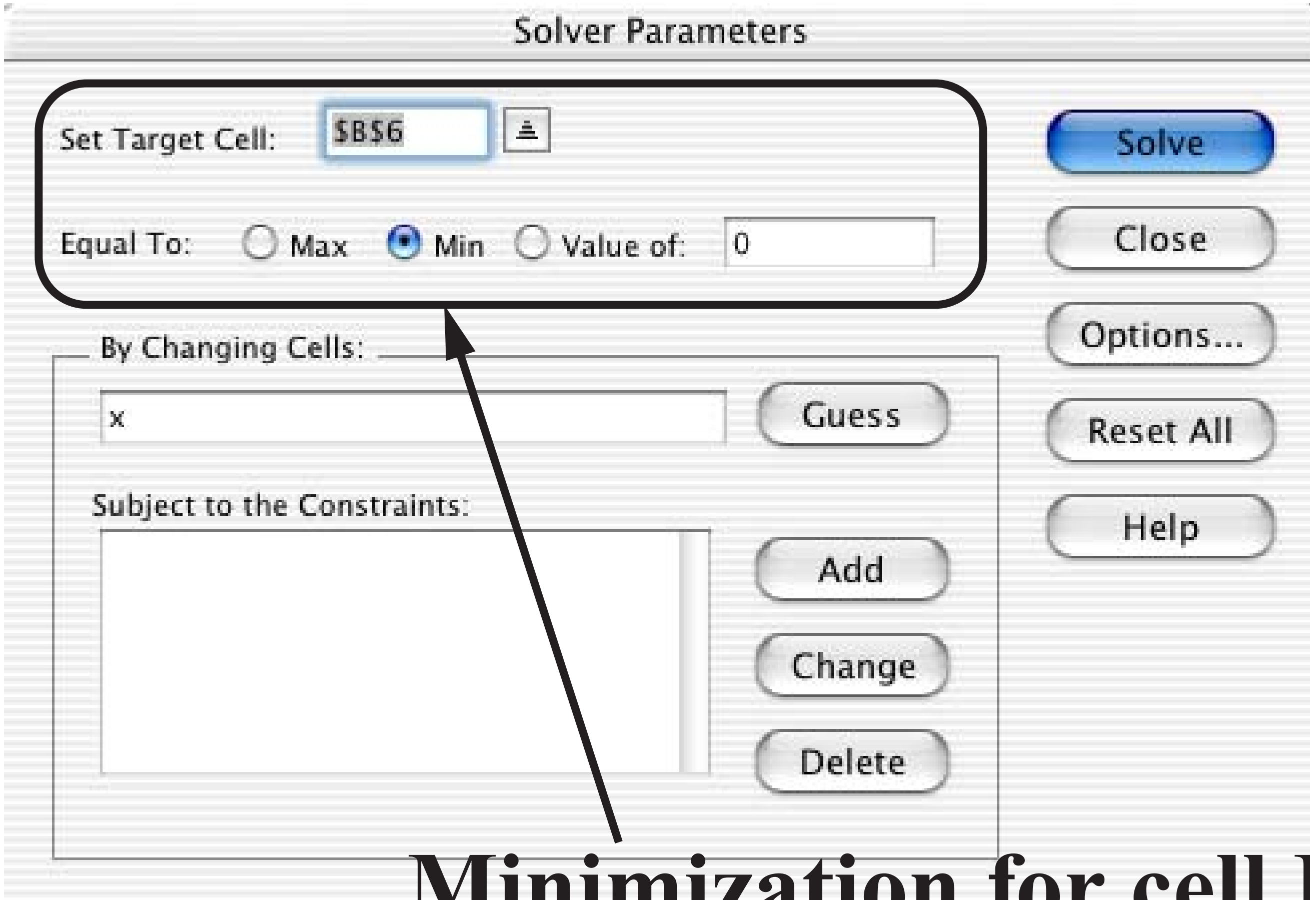
minima_example.xls

	A	B	C	D	E	F	G
1	Finding the Minima of a function						
2							
3	Function	y=2*x^2 - 20 * x + 18					
4							
5	Guess	5			Values of x	y	
6	Equation	-32			0	18	
7					0.5	8.5	
8					1	0	
9					1.5	-7.5	
10					2	-14	
11					2.5	-19.5	
12					3	-24	
13					3.5	-27.5	
14					4	-30	
15					4.5	-31.5	
16					5	-32	
17					5.5	-31.5	
18					6	-30	
19					6.5	-27.5	
20					7	-24	
21					7.5	-19.5	
22					8	-14	
23					8.5	-7.5	
24					9	0	
25					9.5	8.5	
26					10	18	
27					10.5	28.5	

Guess value of x



Excel Solver Panel



Minimization for cell B6

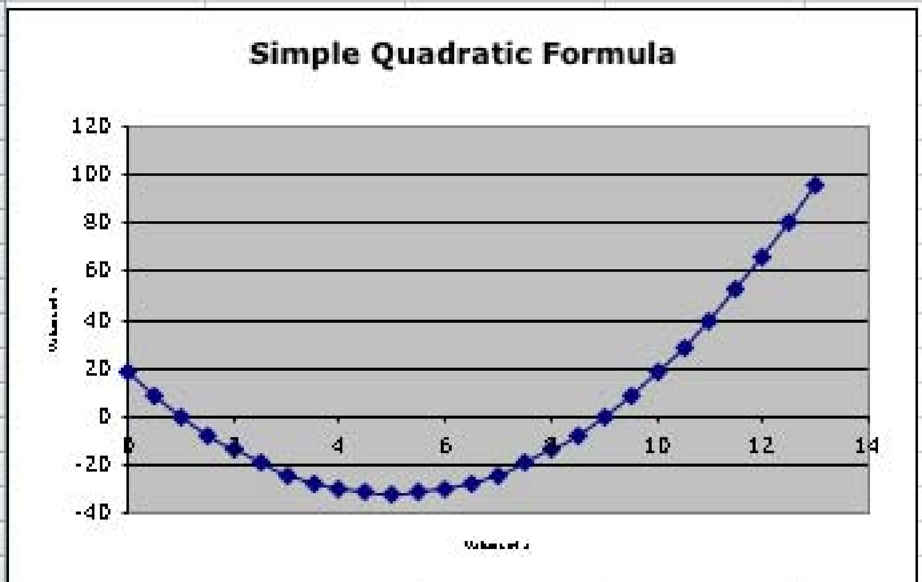
Excel Solver Procedure

minima_example.xls

	A	B
1	Finding the Minima of a function	
3	Function	y=2*x^2 - 20 * x + 18
5	Guess	5
6	Equation	-32

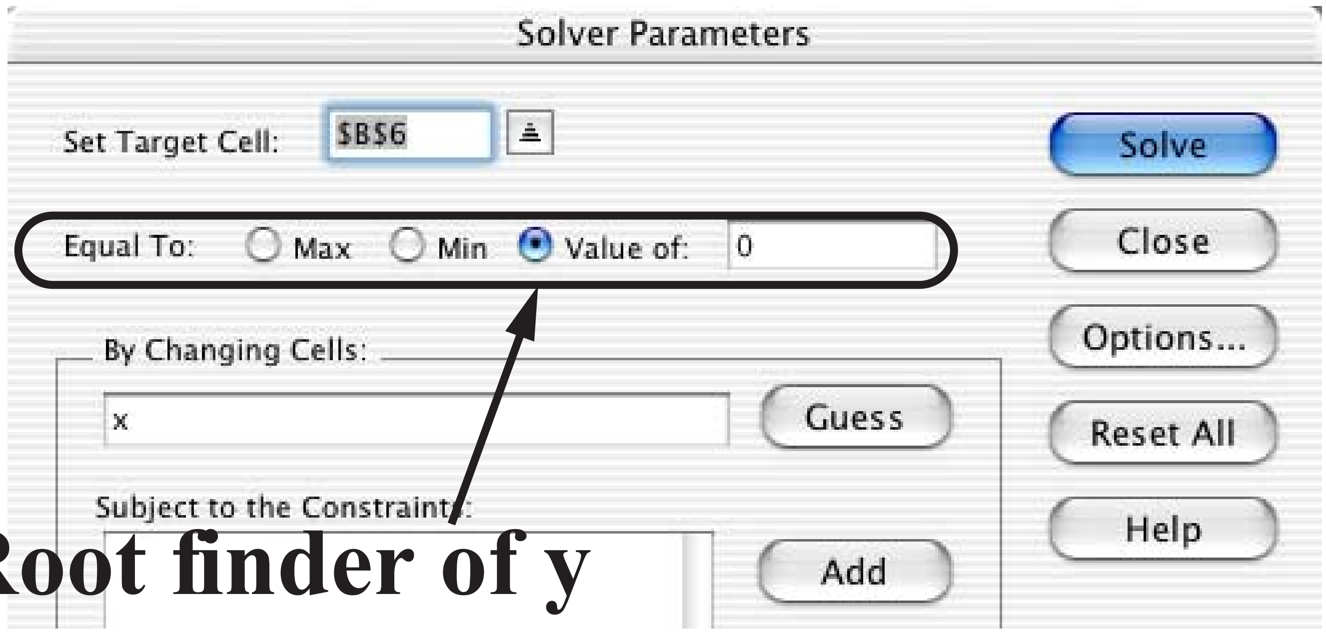
Minimum of y

Values of x	y
0	18
0.5	8.5
1	0
1.5	-7.5
2	-14
2.5	-19.5
3	-24
3.5	-27.5
4	-30
4.5	-31.5
5	-32
5.5	-31.5
6	-30
6.5	-27.5
7	-24
7.5	-19.5
8	-14
8.5	-7.5
9	0
9.5	8.5
10	18



Finding the Roots of y Using Excel Solver

- Easily change the minimimzation problem into a root finder by changing the character of the operation in Excel Solver

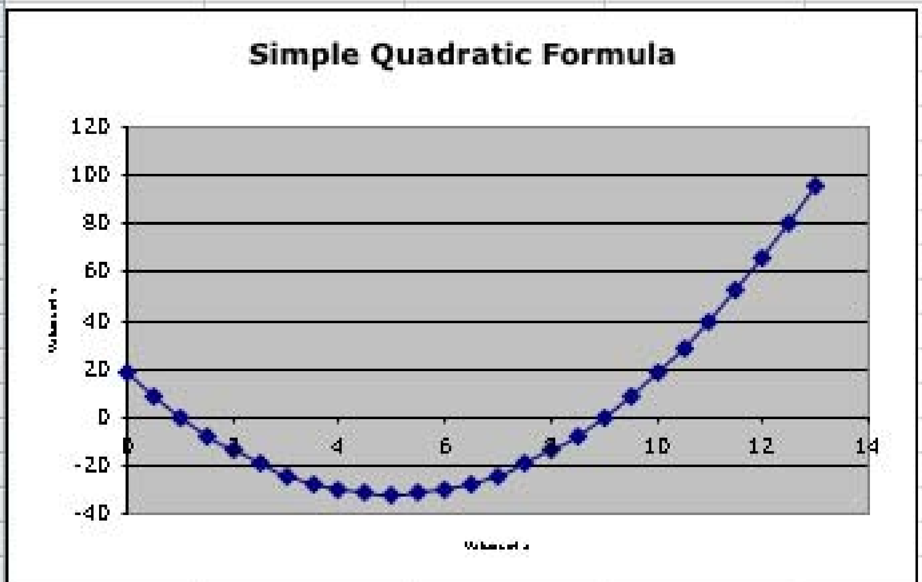


Root finder of y

Root Finder for y

minima_example.xls

	A	B	C	D	E	F	G
1	Finding the Minima of a function						
2							
3	Function	y=2*x^2 - 20 * x + 18					
4							
5	Guess	9			Values of x	y	
6	Equation	-7.031E-08			0	18	
7					0.5	8.5	
8					1	0	
9					1.5	-7.5	
10					2	-14	
11					2.5	-19.5	
12					3	-24	
13					3.5	-27.5	
14					4	-30	
15					4.5	-31.5	
16					5	-32	
17					5.5	-31.5	
18					6	-30	
19					6.5	-27.5	
20					7	-24	
21					7.5	-19.5	
22					8	-14	
23					8.5	-7.5	
24					9	0	
25					9.5	8.5	
26					10	18	



Example for Class Practice

- Minimization example (mixing problem)
- Airline fleet assignment problem

Minimization LP Example

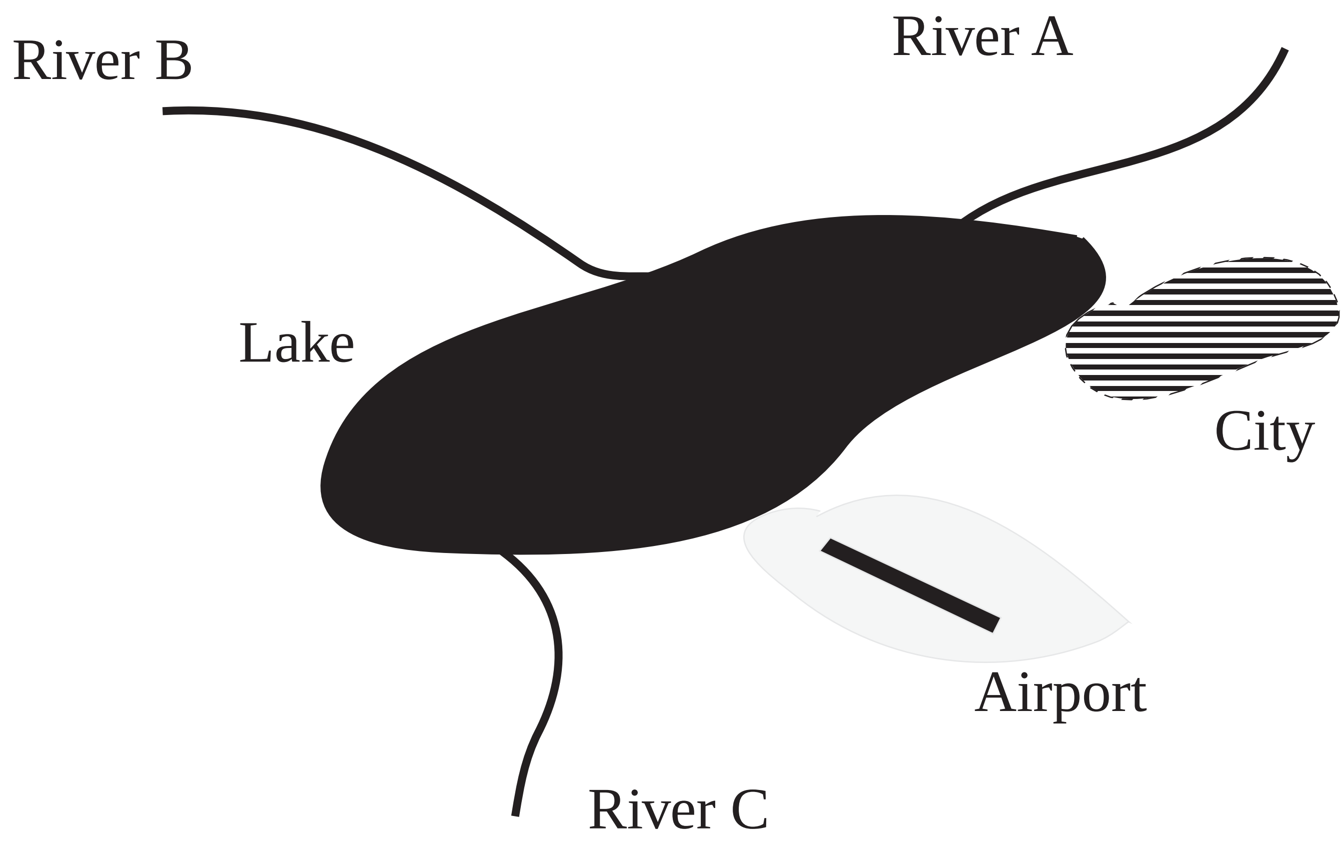
A construction site requires a minimum of 10,000 cu. meters of sand and gravel mixture. The mixture must contain no less than 5,000 cu. meters of sand and no more than 6,000 cu. meters of gravel.

Materials may be obtained from two sites: 30% of sand and 70% gravel from site 1 at a delivery cost of $5.00 per cu. meter and 60% sand and 40% gravel from site 2 at a delivery cost of $7.00 per cu. meter.

a) Formulate the problem as a Linear Programming problem

b) Solve using Excel Solver

Application to Water Pollution



Water Pollution Management

The following are pollution loadings due to five sources:

Note: Pollution removal schemes vary in cost dramatically.

Source	Pollution Loading (kg/yr)	Unit Cost of Removal ($/kg)
River A	18,868	1.2
River B	20,816	1.0
River C	37,072	0.8
Airport	28,200	2.2
City	12,650	123.3

Water Pollution Management

It is desired to reduce the total pollution discharge to the lake to 70,000 kg/yr. Therefore the target pollution reduction is 117,606-70,000 = 47,606 kg/yr.

Solution:

Let x_1, x_2, x_3, x_4, x_5 be the pollution reduction values expected in (kg/yr). The costs of unit reduction of pollution are given in the previous table.

The total pollution reduction from all sources should be at least equal to the target reduction of 47,606 kg.

LP Applications - Water Pollution Management

The reductions for each source cannot be greater than the present pollution levels. Mathematically,

$x_1 \leq 18868$ constraint for River A

$x_2 \leq 20816$ constraint for River B

$x_3 \leq 37072$ constraint for River C

$x_4 \leq 28200$ airport constraint

$x_5 \leq 12650$ city constraint

Water Pollution Management

The reductions at each source should also be non negative.

Using this information we characterize the problem as follows:

Min $z = 1.2x_1 + 1.0x_2 + 0.8x_3 + 2.2x_4 + 123.3x_5$

s.t. $x_1 + x_2 + x_3 + x_4 + x_5 \geq 47606$

$$x_1 \leq 18868$$

$$x_2 \leq 20816$$

$$x_3 \leq 37072$$

$x_4 \leq 28200$ and $x_5 \leq 12650$

Water Resource Management

Rewrite the objective function as follows:

Max $-z + 1.2x_1 + 1.0x_2 + 0.8x_3 + 2.2x_4 + 123.3x_5 + Mx_{12}$

st. $x_1 + x_2 + x_3 + x_4 + x_5 - x_6 + x_{12} = 47606$

$x_1 + x_7 = 18868$

$x_2 + x_8 = 20816$

$x_3 + x_9 = 37072$

$x_4 + x_{10} = 28200$

$x_5 + x_{11} = 12650$