Quiz 1

Solution Instructor: Trani

Honor Code Pledge

The information provided in this exam is my own work. I have not received information from another person while doing this exam.

Your Name ____

Your Signature ____

Write your solutions in a single MSWord file. **Create a PDF file**. Cut and Paste all your answers using screen captures. Show all your work. Label your file with your last name and CEE3804. Email your solutions to vuela@vt.edu and tao81@vt.edu. In the email header use the words CEE 3804 Quiz.

Problem 1 (50 Points)

You are given the task to calculate the volume of earth material to be removed from a hill during the construction of an Interstate Highway . Figures 1 and 2 illustrate the situation. The blue dots in Figure 1 constitute the surveyed elevation points on the hill contour where the cut is to be made. The red line in Figure 2 represent a polynomial approximation of the hill elevations. The elevations and stations of the surveyed points are shown below.

station = [1.75 10 20 30 40 50 60 70 80 90 100];   % in meters

elevation = [5 26 43 51 53 49 40 29 18 8 0];   % in meters

a) Create a Matlab script to estimate the **area under the hill contour** shown in Figure 1. The highway vertical alignment has a slope of 5.09% between stations 0 and 100 (metric) as shown in Figure 1. In the analysis, use the polynomial of your choice to approximate the elevation contour of the hill shown in Figure 1.

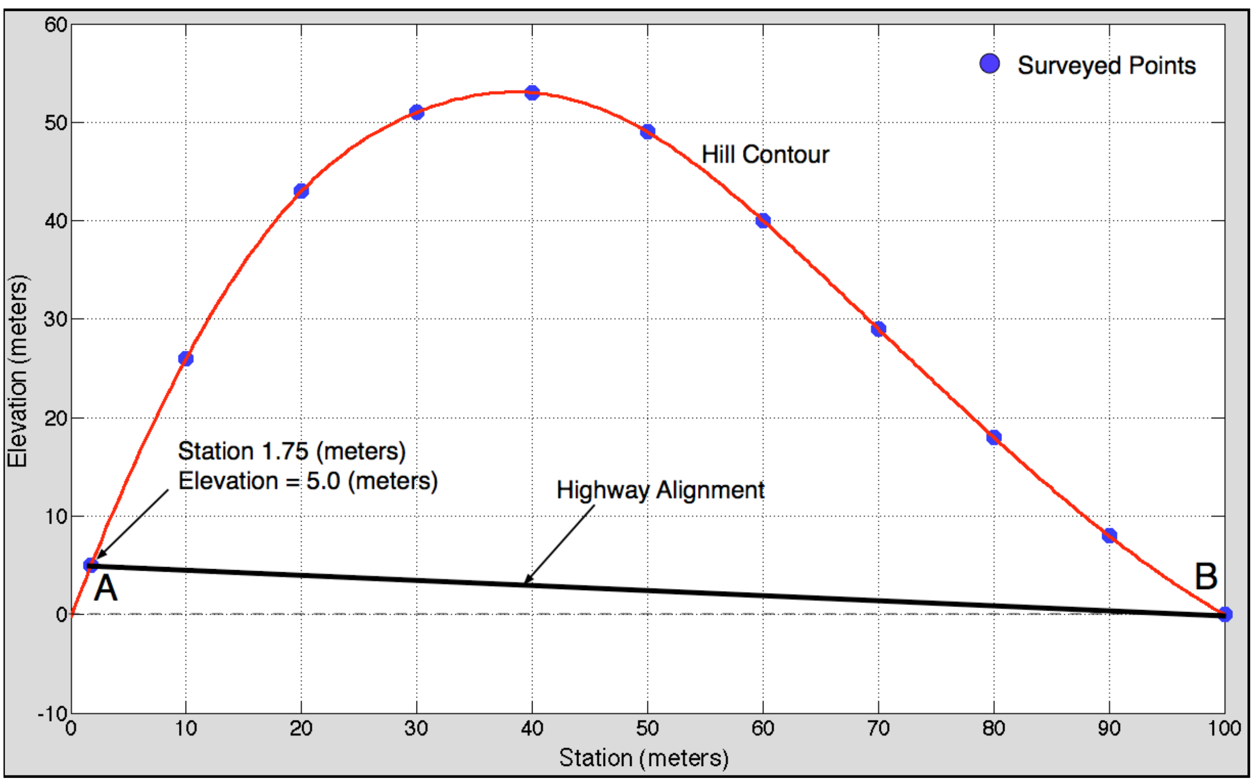


Figure 1. Cross Section of Hill Contour to be Removed for the Road Construction.

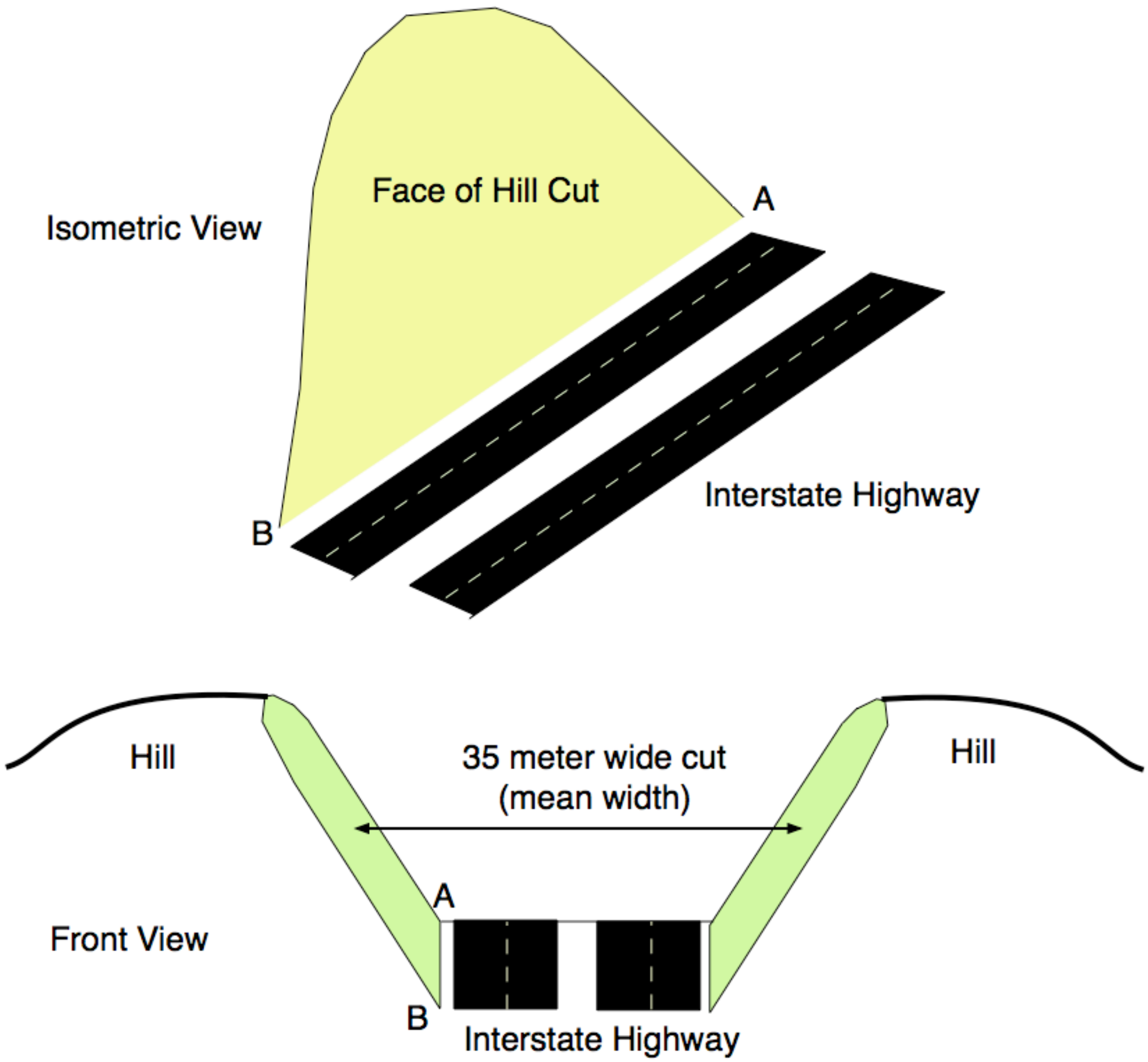


Figure 2. Isometric and Front Views of Hill Cut Required for Road Construction.

Solution:

Create a polynomial that fits the elevation data points. I used a 4th order polynomial. A third order is also a good choice. A fourth order polynomial that fits the data is:

elevation = 3.931e-7 * (station).^4 + 0.0001852 * (station).^3 - 0.05412* (station).^2 + 3.173 * station - 0.4415

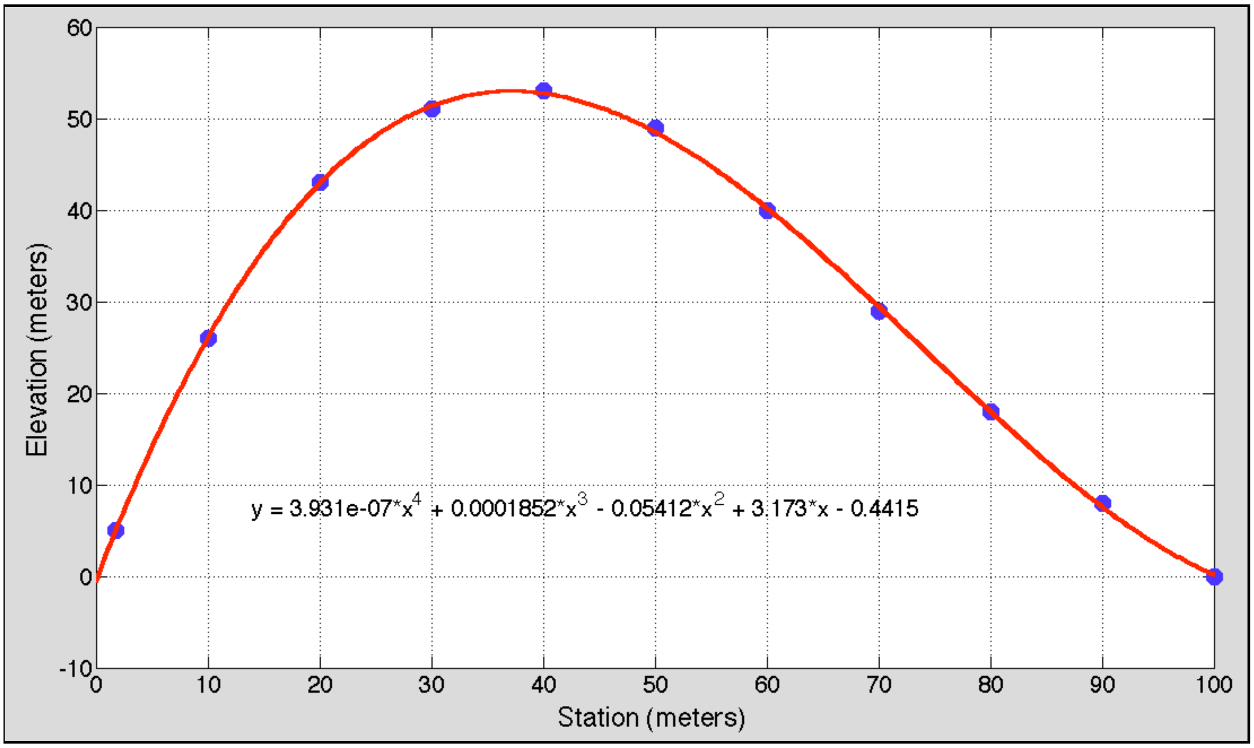


Figure 3. Polynomial Approximation of the Hill Elevation Contour.

Figures 4 and 5 illustrate the Matlab scripts to estimate the area and the volume of Earth material to be removed. Figure 4 is the main script that defines the problem and calls a function called **elevationCalculation** (see Figure 5) needed to do the calculation. The function **elevationCalculation** contains the polynomial to be integrated.

The value of the area under the hill contour is **2947.44 square meters**. The volume of the Earth material to be removed is **103160.23 cubic meters.** Note that the area under highway alignment is 245.625 sq. meters. The area under the polynomial is 3,193.1 sq. meters.

```
% Script to estimate the area under a hill contour
clear; clc
% Function calls: elevationCalculation

initialStation = 1.75;              % initial station to compute area (m)
finalStation = 100;             % final station to compute area (m)
widthOfCut = 35;              % average width of cut to compute volume (m)
heightHighwayAtInitialStation = 5;    % maximum height of highway alignment (m)

% Calculate the area under the polynomial function created either
% interactively or using the polyfit command.

areaUnderPolymonial = quad('elevationCalculation',initialStation,finalStation);

% Calculate the area under the highway alignment

areaUnderHighway = 1/2 * (finalStation - initialStation) * heightHighwayAtInitialStation;

areaUnderHillContour = areaUnderPolymonial - areaUnderHighway;
clc
disp(['The value of the area under the hill contour is (sq. meters)  ', num2str(areaUnderHillCon

% Task (b) - Calculate the volume of material to be removed

volumeOfEarth = areaUnderHillContour * widthOfCut;

disp(['The volume of the cut is (cu. meters)  ', num2str(volumeOfEarth)])
```

Figure 4. Matlab script to calculate the area under the curve.

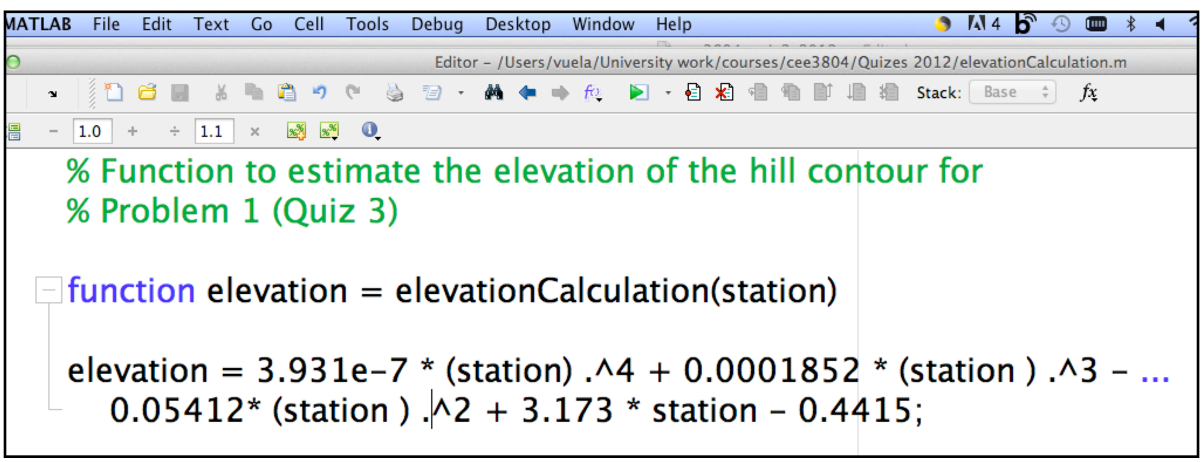


Figure 5. Function to estimate elevation of hill contour. It is very important to include the periods before each exponentiation operand in the equation to calculate the elevation.

Problem 2 (50 Points)

One important problem to civil and environmental engineers is the treatment of water to avoid bacteria growth. A differential equation developed by Monod and modified by Smith to estimate the growth of bacteria in a water medium is given by:

$$\frac{dN}{dt} = rN\left[\frac{\gamma S + N_0 - N}{\gamma a + \gamma S + N_0 - N}\right] \qquad (1)$$

where:

$\frac{dN}{dt}$ = is the rate of change of bacteria concentration in the water medium per unit of time (grams/liter per hour)

N = bacteria concentration in water (grams/liter)

r = bacteria growth rate factor (1/hour)

γ = model constant (dimensionless)

a = half-saturation constant (grams/liter)

S = concentration of bacteria nutrient (grams/liter)

N_0 = initial bacteria concentration in the water (grams/liter)

Note that all units in equation (1) are consistent (no need to change units). Also note that N_0 is the initial condition for bacteria concentration in water and also a constant.

a) Your task is to develop a **Simulink model** to estimate the growth of bacteria for the first 6 hours after the water is contaminated with some initial bacteria concentration (N_0). Use the following initial conditions for the problem:

$S = 0.05$ grams/liter - concentration of bacteria nutrient

$N_0 = 0.001$ grams/liter - initial bacteria concentration

In your model use the following constants:

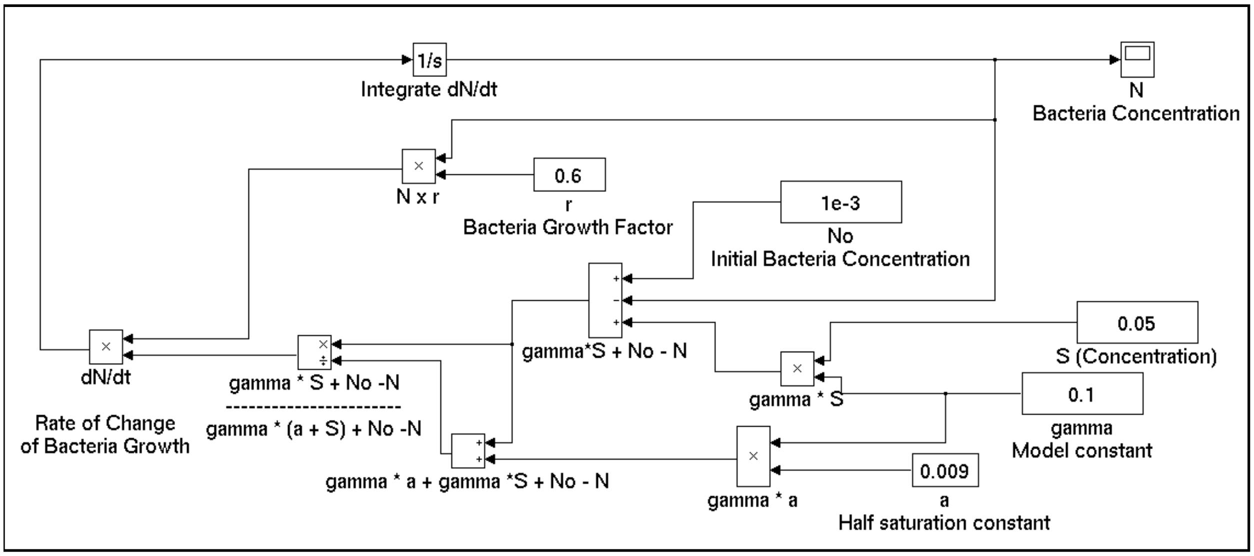
$\gamma = 0.1$ dimensionless

$a = 0.009$ grams/liter

$r = 0.6$ per hour

Solution:

Figure 6 shows the Simulink solution to the problem. Only one integrator is needed to obtain the solution for $\frac{dN}{dt} = rN\left[\frac{\gamma S + N_0 - N}{\gamma a + \gamma S + N_0 - N}\right]$.



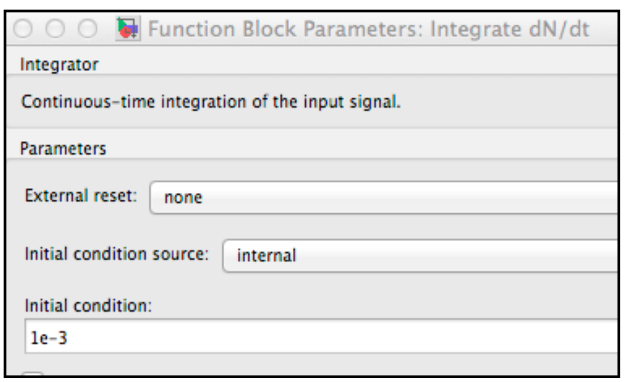


Figure 6. Simulink Model of Bacteria Growth. The Initial Conditions of the Integrator Block are also Shown .

b) **Export the results of the Simulink model** (i.e., time and bacteria concentration N) to Matlab and make a plot showing the growth of bacteria concentration (N) over 6 hours. Estimate (using the plot) the bacteria concentration after 5 hours. Label the plot appropriately.

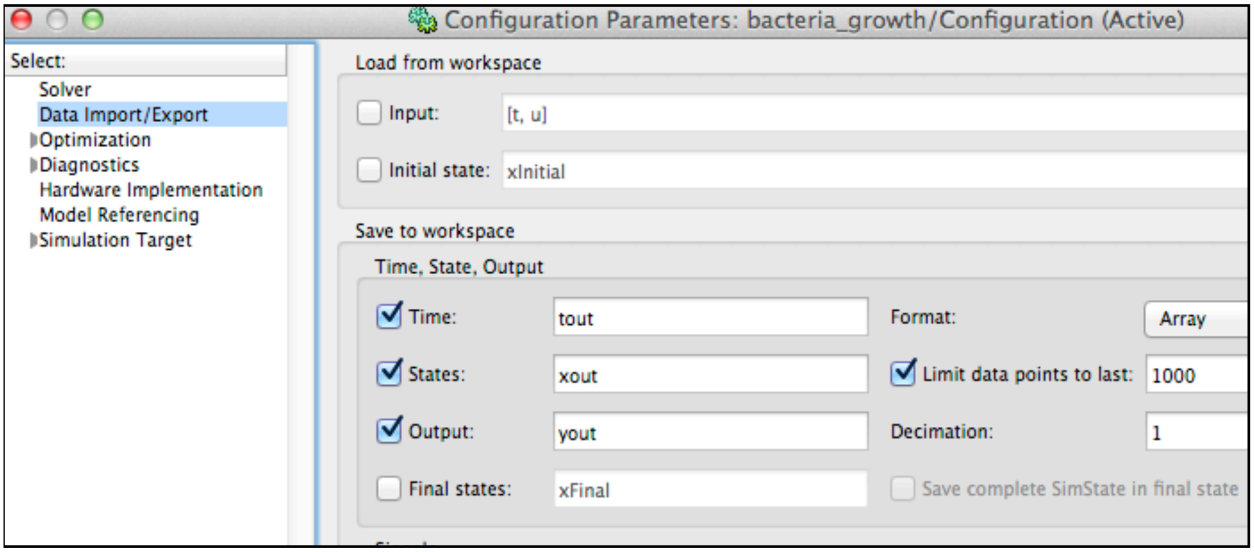


Figure 7. Export the Results to Matlab Workspace. Select States (xout) in the Configuration Parameters.

To make a plot in Matlab we use the following commands at the Command Window. The results are shown in Figure 8.

```
plot(tout,xout,'o-r')
xlabel('Time (hrs)')
ylabel('Bacteria Concentration (gm/l)')
grid
```

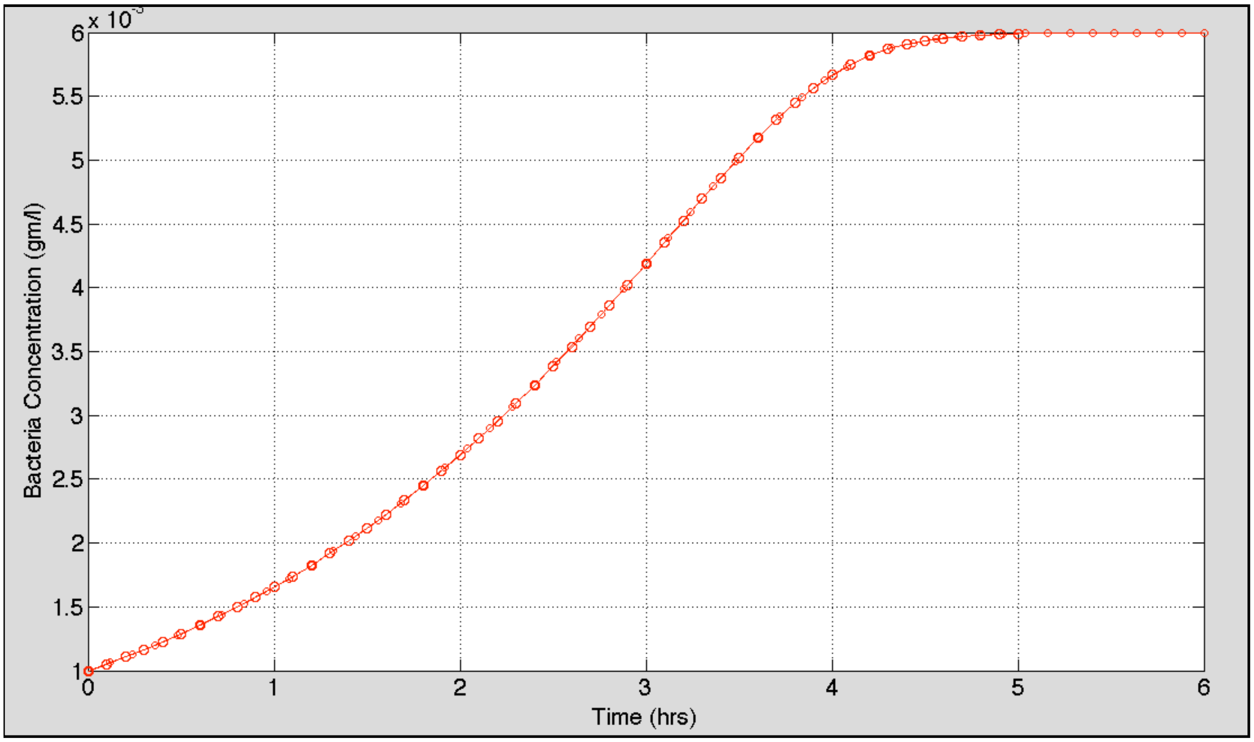


Figure 8. Bacteria Growth as a Function of Time.

Using simple linear interpolation using Matalb, the value of N at time t=5 hours is found to be:

valueOfN_at_5_hours = interp1(tout,xout,5) which produces:

valueOfN_at_5_hours = 0.0060   grams/liter