Assignment 9: Integration and Differential Equations

Solution Instructor: Trani

Problem 1

Improvements to a waterway require estimation of the amount of material to be removed in the dredging operation. Table 1 shows the waterway depth (column 2) at stations spaced every 5 meters across the waterway (column 1). Figure 1 shows the desired dimension sow the waterway after dredging. A minimum depth of 15 meters is required to allow larger contained ships to transit.

Table 1. Waterway Stations and Depth.

Horizontal Coordinate (meters)	Vertical Profile (meters)
0	0.00
5	-2.40
10	-4.10
15	-5.00
20	-6.20
25	-7.15
30	-8.60
35	-9.90
40	-10.10
45	-10.60
50	-10.90
55	-10.50
60	-10.90
65	-10.20
70	-8.50
75	-7.30
80	-5.60
85	-4.90
90	-3.40
95	-2.10

Horizontal Coordinate (meters)	Vertical Profile (meters)
100	0.00

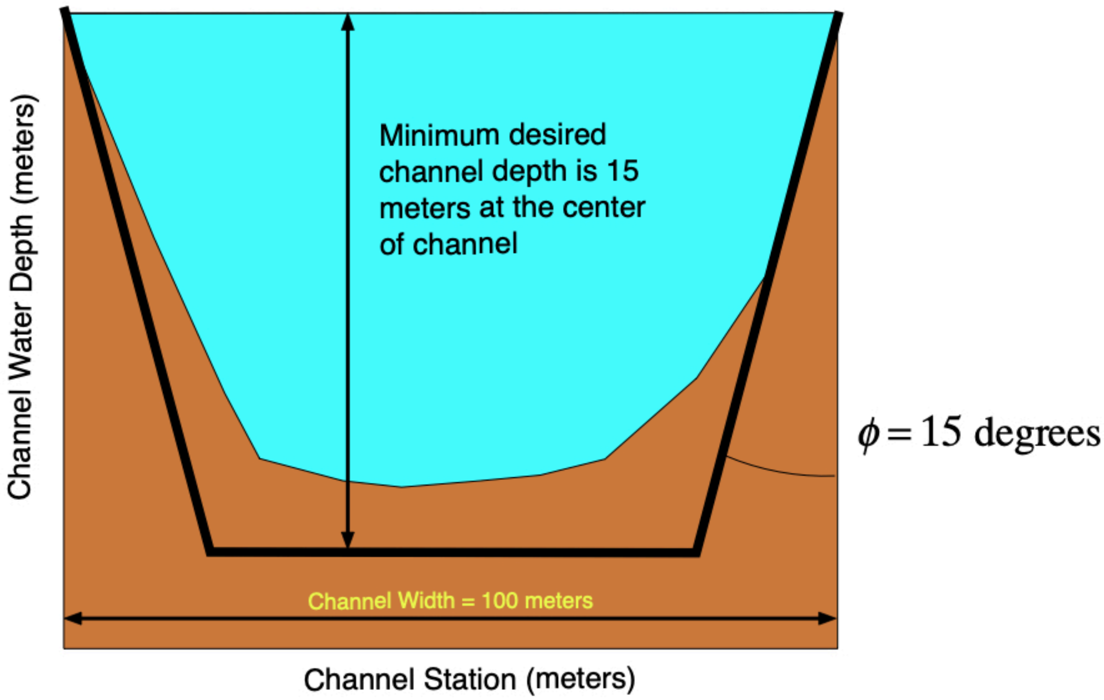


Figure 2. Waterway Cross Section and Desired Dimensions.

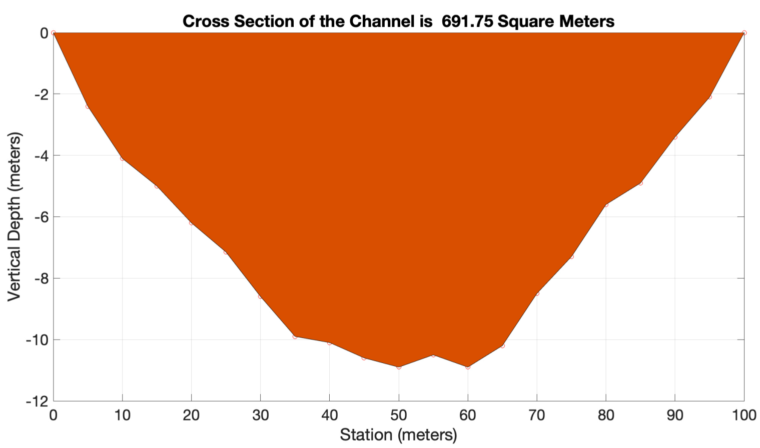
Task 1

Create an Excel file with the station and water depth data of the existing channel. Create Matlab script to read the data provided in the Table 1. Use any importing procedure learned in class.

Task 2

```
%% Set up the Import Options and import the data
opts = spreadsheetImportOptions("NumVariables", 2);

% Specify sheet and range
opts.Sheet = "Sheet1";
opts.DataRange = "A2:B22";

% Specify column names and types
opts.VariableNames = ["HorizontalCoordinatemeters", "VerticalProfilemete
opts.VariableTypes = ["double", "double"];

% Import the data
tbl = readtable("/Users/atrani/Courses/CEE 3804/CEE 3804 Spring 2022 C

%% Convert to output type
HorizontalCoordinatemeters = tbl.HorizontalCoordinatemeters;
VerticalProfilemeters = tbl.VerticalProfilemeters;
```

```
% Calculate area under the curve

areaOfChannel = - trapz(HorizontalCoordinatemeters,VerticalProfilemeters);

clc
disp(['Area of the Channel is  (sq.m.) ',num2str(areaOfChannel) ])

% Plot the station versus water depth

plot(HorizontalCoordinatemeters,VerticalProfilemeters,'o--r')
xlabel('Station (meters)')
ylabel('Vertical Depth (meters)')
title(['Cross Section of the Existing Channel is  ',num2str(areaOfChannel), ' Square
grid
```

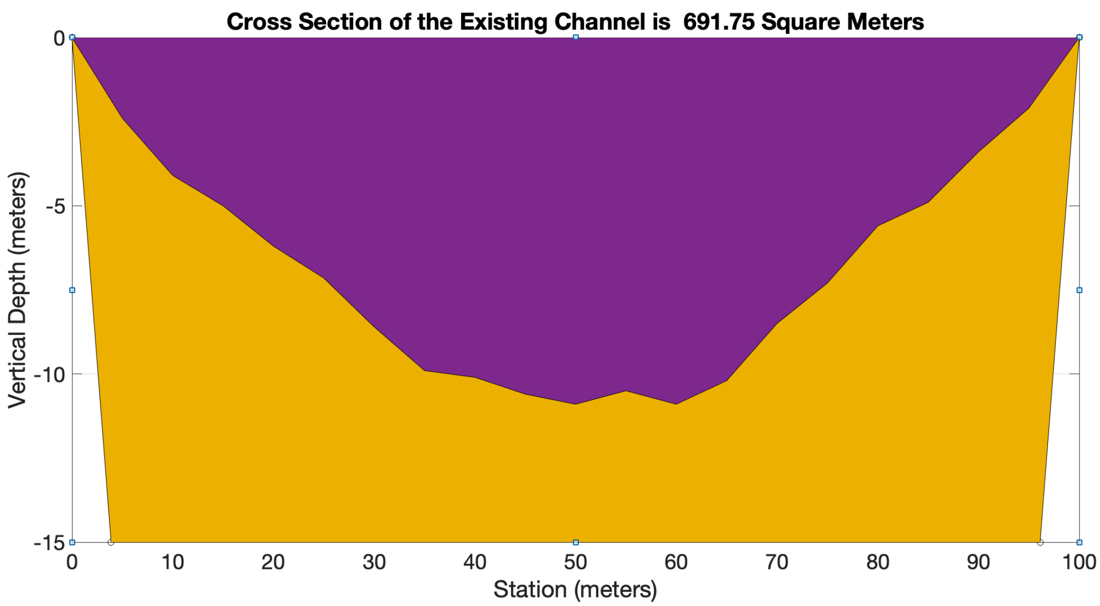



```
% Calculate the cross section area of proposed trapezoidal channel

channelSlope_deg = 17;        % channel slope in degrees
channelDepth_m = 20;          % meters

% Calculate stations of corner points at the bottom of channel

deg2radian_conversion = 1/57.29;
stationOffset_meters = channelDepth_m * sin(channelSlope_deg*deg2radian_conve

cornerPointLeft_meters = stationOffset_meters;
cornerPointRight_meters = 100-stationOffset_meters;

areaOfNewChannel = (100+ (100 - 2*stationOffset_meters))/2 * channelDepth_m;
disp(['Area of Proposed Channel (sq.m.) ',num2str(areaOfNewChannel) ])
disp(['Cross Section of Area to be Excavated (sq.m.)',num2str(areaOfNewChannel-ar
```

Task 3
Further improve the script in Task 3 to estimate the area needed for dredging. Your code should be able to estimate the coordinates of the new profile (with a flat bottom) as a function of two parameters: channel depth and slope angle (phi). Run the code with channel depth of 15 meters and phi=15 degrees. Display the answer in the command window with appropriate labels and units.

Area of the Existing Channel is (sq.m.) 691.75
Area of Proposed Channel (sq.m.) 1441.76
Cross Section of Area to be Excavated (sq.m.) 750.01

Task 4
Improve the script created in Task 3 and plot the existing channel profile and the new profile after dredging. improve the script in Task 3 to estimate the area needed for dredging.



Task 5
Run the script created in Task 3 with channel depth of 20 meters and phi= 17 degrees.

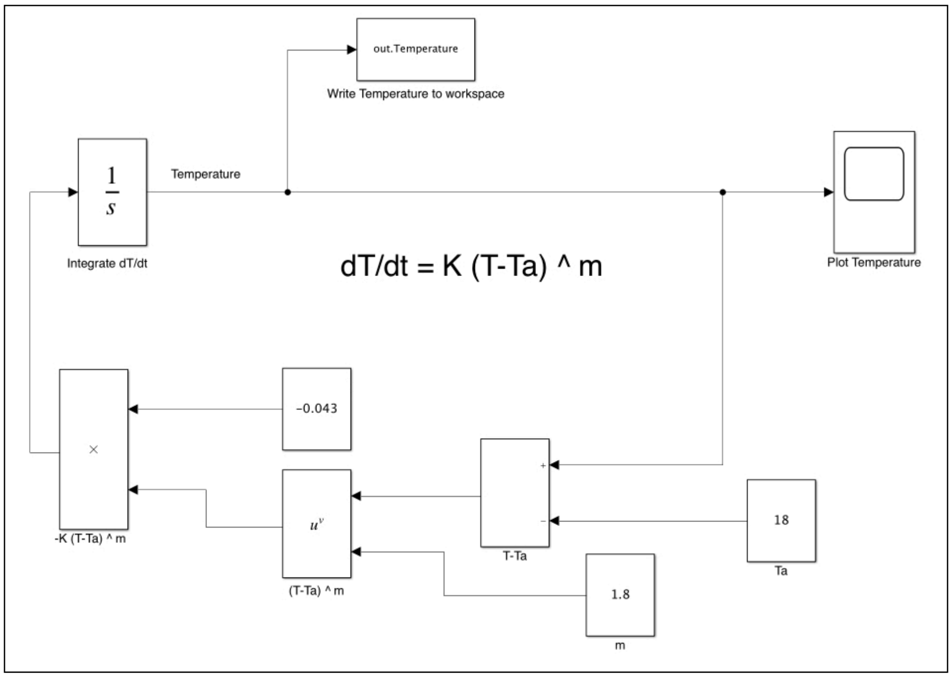
Area of the Channel is (sq.m.) 691.75
Area of Proposed Channel (sq.m.) 1883.04
Cross Section of Area to be Excavated (sq.m.) 1191.29

Problem 2

Asphalt is a very important material to civil engineers. The fabrication of asphalt requires the material to be heated to allow better mixing. The first order differential equation (dT/dt) that estimates the rate of change of temperature of concrete as a function of time is:

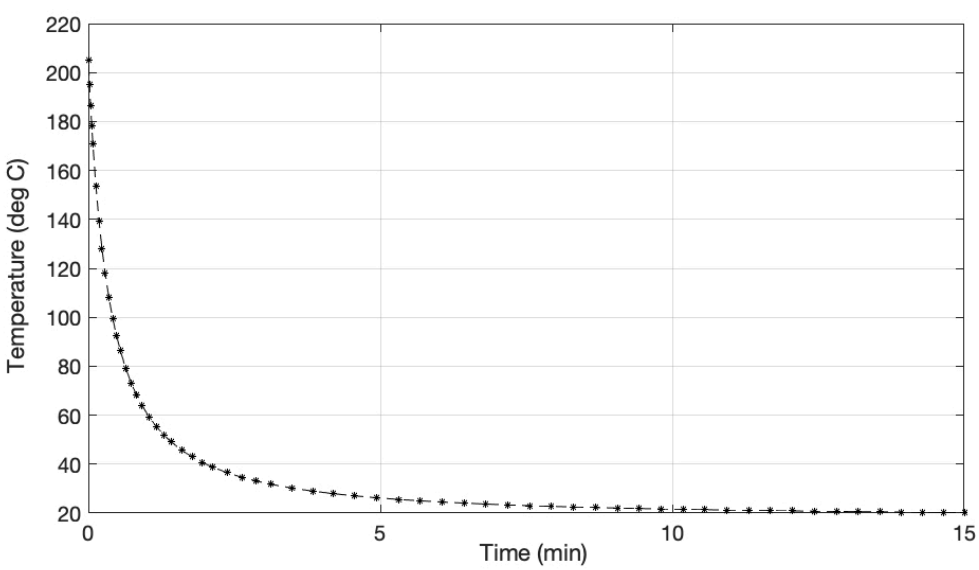
Task 1

Create a Simulink model (block diagram) to solve the problem.



Task 2

Test the Simulink model created in Task 1 using using values for K for asphalt is 0.043 (1/minute), m = 1.8 and ambient temperature is 18 deg. C.Assume the initial temperature of the asphalt is 205 degrees.



Task 3
Plot the results of Task 2 in Matlab by exporting the answers to the Matlab workspace and making a plot of asphalt temperature versus time.

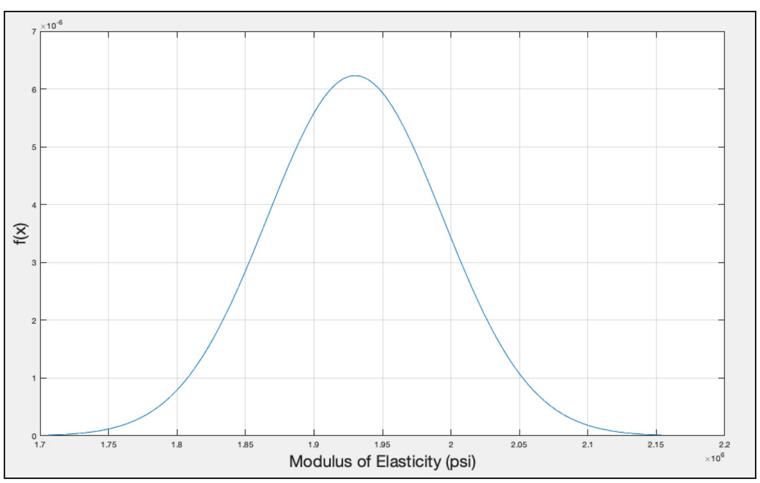
Task 4
Compare the numerical solutions of the Simulink model using the Runge-Kutta 3rd order and the Runge-Kutta 5th oder solutions.

Runge-Kutta 5th order provides more accuracy and more data points in the solution.

Problem 3

Task 1
Create a Matlab script to evaluate the function f(x) using the values of μ and σ observed for Hickory beams. Plot the resulting PDF as a function of modulus of elasticity. Label accordingly. Verify that the PDF plot is the well-known bell-shape curve of the Normal distribution.

```
% Purpose: shows a normally distributed random variable
% integrates numerically the a normal distribution between two
% intervals a and b

% Define the parameter of the distribution (MU) and standard deviation (

mu = 1.93e6;
sigma = 6.4e4;

% Define upper and lower bounds to get a nice plot of the Gaussian PDF

npoints = 100;
parameter = 3.5;                % 3.5 standard deviations from mean
low = mu - parameter*sigma;       % low value to make a plot
high = mu + parameter* sigma;      % high value to make a plot
interval = (high - low) / npoints;  % interval between data points

% Define random variable x

x=low:interval:high;


% Define the function of the random variable x (PDF function)
% cehck your statistics book in case you forgot the PDF of a
% normal distribution

fx = 1/(sigma * sqrt(2*pi)) * exp(-1/2 * ((x-mu)/sigma).^2 );

% Plot the random variable x versus the PDF function

plot(x,fx)
xlabel('Modulus of Elasticity (psi)','fontsize',20)
ylabel('f(x)','fontsize',20)
```



Plot of MOD for the problem.

Task 2

Create a Matlab function to calculate the value of the Normal distribution PDF (f(x)). The function needs two take three arguments: *x (the value of the random variable)*, μ and σ. The output of the function is *f(x).*

```
% Function to evaluate the normal distribution

% Passes back a single argument containing f(x) for the normal PDF function

function [fx] = normalFunction(x,mu,sigma)
% global MU SIGMA

fx = 1./(sigma .* sqrt(2*pi)) .* exp(-1/2 .* ((x-mu)./sigma).^2 );
```

Task 3

Probability MOD > 2.0e6 = 0.137

Task 4

Find the probability that a lot of 1,000 of random sample beams have MOE values ranging from 2.00e6 psi to 2.20e6 psi.

Probability 2.0e6 <MOD < 2.2e6 = 0.137 (the same because the value of fx goes to zero as x increases (see plot below).

Use the same script tom fo Task 3. Just replace the value of b in the code for 3e6 or something really high.