## Assignment 9: Integration, Polynomials, and Interpolation

Date Due: April 17,2023
Instructor: Trani

## Problem 1

Data collected using a car dynamometer is included in the data file called fuelConsumption_CO2_emissionsData.xlsx. The data includes instantaneous fuel consumption and $\mathrm{CO}_{2}$ emissions.

| Speed (km/hr) | Fuel Consumption (l/s) |  | CO2 Emissions ( $\mathrm{mg} / \mathrm{s}$ ) |
| ---: | ---: | ---: | ---: |
|  | 0 | 0.00021 | 1.675 |
| 10 | 0.00028 | 1.948 |  |
|  | 15 | 0.00032 | 2.43 |
|  | 20 | 0.00036 | 3.32 |

Figure 1. Medium Car Fuel Consumption and Co2 Emissions Data.

## Task 1

Create a Matlab script to import the car fuel consumption and emissions data. The script converts speed to units of meters/second and plots the speed ( $\mathrm{m} / \mathrm{s}$ ) against the fuel consumption and $\mathrm{CO}_{2}$ emissions. Observe the trends in the data.

## Task 2

Add to the script created in Task 1. Create a polynomial of 3rd order polynomial fit to the fuel consumption data. Speed is the independent variable and fuel consumption is the dependent variable. Plot the fitted 3rd order polynomial with the original data (same plot) to judge the goodness of fit graphically. Use interpolation to find solutions of fuel consumption when the car travels at 12.5 and $18 \mathrm{~m} / \mathrm{s}$.

## Task 3

Add to the script created in Task 1. Create a polynomial of 5th order polynomial fit to the $\mathrm{CO}_{2}$ data. Speed is the independent variable and $\mathrm{CO}_{2}$ is the dependent variable. Plot the fitted 5th order polynomial with the original data (same plot) to judge the goodness of fit graphically.

## Task 4

Create a Matlab function to estimate fuel consumption and $\mathrm{CO}_{2}$ emissions using the two polynomial fits found in Tasks 2 and 3. The function should produce two outputs: fuel consumption and $\mathrm{CO}_{2}$ emissions. The function uses one input (speed in meters per second).

## Problem 2

Consider the GPS car data file obtained while driving a medium car on a road network according to the US06 driving cycle (file name is us06_drivingCycle.m). The US06 driving cycle is a driving cycle used by the Environmental Protection Agency (EPA) to evaluate the emissions produced by cars and light trucks. More information can be found at: https://www.epa.gov/ vehicle-and-fuel-emissions-testing/dynamometer-drive-schedules.

| \% Data for the US06 driving cycle |  |
| :--- | :--- |
| \% Source: EPA |  |
| \% Column1 $=$ time (seconds) |  |
| \% Column 2 = speed ( $\mathrm{m} / \mathrm{s}$ ) |  |
| 0 | 0 |
| 1.0000 | 0 |
| 2.0000 | 0 |
| 3.0000 | 0 |
| Figure 2. USO6 Driving Cycle Data. |  |

## Task 1

Create a Matlab script to: a) Read the data in Matlab (your choice of method to read the data), and b) plot the speed (in meters per second) in the y-axis versus time (in the x-axis). In the script create separate vectors for speed and time. Observe that the speed data is consistent with the motion of a car (i.e., no negative speed values or unreasonable acceleration values).

## Task 2

Use the two vectors created in Task 1 and use the Matlab Trapezoidal numerical integration methods to estimate the area under the curve of the speed profile. What is the integral of speed? State the units of the area under the curve.

## Task 3

Improve the script created in Task 1 of Problem 2 and use the function created in Task 4 of Problem 1 to estimate the instantaneous fuel consumption and $\mathrm{CO}_{2}$ emissions produced by the car during the driving cycle.

## Problem 3

Use the rainfall data file available in the Syllabus (Week 6) to solve the problem.

## Task 1

Read the file in Matlab and make a histogram with 15 bins. The histogram shows the rainfall amounts (in inches) for the month of July in the x-axis and the frequency of observations in the $y$-axis. Create a basic script to read the data and show me the first 105 lines of the script in your homework solution.

## Task 2

Calculate the mean and the standard deviation of the rainfall amounts for the month of July. Use the Mean and Std functions in Matlab.

## Task 3

Realizing that rainfall is random from year to year, an engineer wants to model the rainfall amount data using a Gaussian distribution (see histogram). The Probability Density Function (PDF) of the Gaussian (or Normal) distribution is given in any statistics textbook and repeated here for completeness (http://en.wikipedia.org/wiki/Normal_distribution).
$f(x)=\frac{1}{\sigma \sqrt{2 \pi}} e^{\left.-\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right\rfloor}$
where: x is the random variable in question (rainfall amount in inches) and $\mu$ and $\sigma$ are the mean and standard deviation (in inches) of the rainfall amount estimated in Task 2.

## Task 4

The area under probability density function has a fundamental interpretation in random processes. For example, the area under the curve in Figure 3 can be interpreted as the probability that random variable $x$ will take values between a and infinity.


Figure 3. Sample Normal Probability Density Function. Interpretation of the Area Under the Curve.

Using Matlab trapezoidal integration rule, find the probability that rainfall amounts exceed 6.5 inches.

Hint: Since numerical integration to infinity is not possible, use a large number for the random variable.

## Task 5

Using Matlab trapezoidal integration rule and find the probability that rainfall amounts range between 2.5 and 7.5 inches in the month of July. Use the Matlab AREA function demonstrated in class to show graphically the area used in the numerical integration procedure.

