

## Assignment 9/Quiz 3: Dynamic Systems: Differential

**Equations** Date Due: April 28, 2017

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### Problem 1

A civil engineer needs to model the dynamics of a building subject to seismic loads. The building and its foundation is modeled as series of spring and dampers as shown in Figure 1.

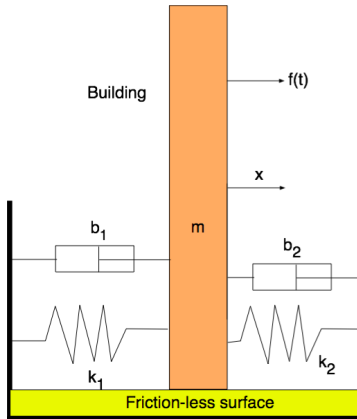


Figure 1. Building and Foundation System for Problem 1.

The fundamental equation of motion for the acceleration of the building at the foundation is:

$$\ddot{x} = \frac{f(t)}{m} + \left( \frac{b_2 - b_1}{m} \right) \dot{x} + \left( \frac{k_2 - k_1}{m} \right) x \quad (\text{Eq. 1})$$

where:

$\ddot{x}$  = lateral acceleration of the building system ( $\text{m/s}^2$ )

$\dot{x}$  = lateral speed of building system ( $\text{m/s}$ )

$x$  = lateral displacement of building system ( $\text{m}$ )

Also,  $m$  is the mass of the building ( $\text{kg}$ ),  $f(t)$  is a forcing function (units are Newtons) that is generated by seismic waves.  $k_1$  and  $k_2$  are spring constants ( $\text{N/m}$ ) and  $b_1$  and  $b_2$  are damper constants ( $\text{N/m/s}$ ). The values of the constants are:

$m = 3,300,000;$	% mass ( $\text{kg}$ )
$b_1 = 500,000;$	% damper constant ( $\text{N} / (\text{m/s})$ )
$b_2 = 450,000;$	% damper constant ( $\text{N} / (\text{m/s})$ )
$k_1 = 8\text{e}6;$	% spring constant ( $\text{N/m}$ )
$k_2 = 7.6\text{e}6;$	% spring constant ( $\text{N/m}$ )

## Task 1

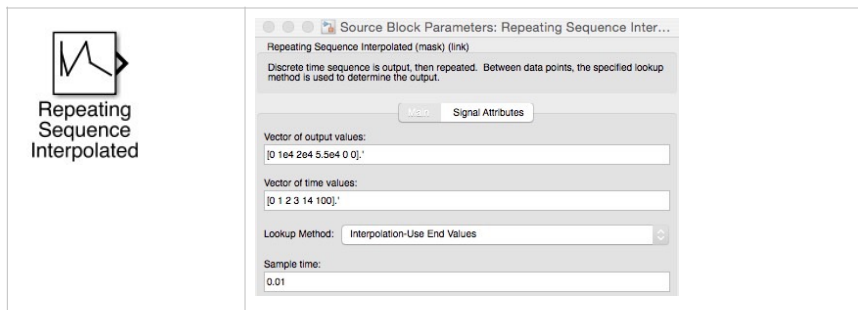
Using Simulink, construct a block model of the building system and solve the differential equations of motion of the building system to find the lateral speed and the lateral displacement responses as a function of time.

## Task 2

Solve the dynamic response of the building system when is subject to an initial instantaneous displacement of 7 cm caused by a seismic shock. Model the seismic shock using the initial condition for the state variable ( $x$  - lateral displacement) and for this analysis assume the forcing function  $f(t)$  is zero as you explore the natural dynamics of the system. Export the lateral speed response and the lateral displacement (both state variables) to the Matlab workspace. Plot the state variables as a function of time using the subplot command. Study the output of the model and comment on the trends observed.

## Task 3

Model the dynamic response of the building subject to a small earthquake by introducing the forcing function  $f(t)$  in equation (1). The forcing function  $f(t)$  can be modeled using a **Repeated Sequence Interpolated Block** (found in the Sources library in Simulink) shown below. This block requires two inputs: a) a time vector and b) the output signal of the forcing function  $f(t)$ . An example of the block parameters are shown in the figure below.



The actual values of time and the forcing function  $f(t)$  are shown in Table 1. Simulate the response of the building to the earthquake for 20 seconds or until the building oscillation subsides. Use the Runge-Kutta 5th Order integration algorithm to solve this problem. Export the lateral speed response and the lateral displacement (both state variables) to the workspace. Plot the state variables as a function of time using the subplot command. Study the output of the model and comment on the trends observed. Find the time for the oscillations to dampen to 5% of the peak displacement.

Table 1. Forcing Function Values of Simulated Earthquake Event. The last time in the table is made artificially large so that the sequence is not repeated in the 20 second simulation period of this event.

Time of Event (seconds)	Force (N)
0.0	0.0
3.0	260000.0
4.5	420000.0
6.0	435000.0
9.0	200000.0
12.0	30000.0
15.0	0.0
100.0	0.0

## Problem 2

### Task 1

Solve task 1 in problem 1 using the Matlab Runge-Kutta 5th ODE order solver. IN your solution, define two variables representing the state variable of the problem:

$x_1$  = displacement of building (m)

$x_2$  = lateral speed of the building (m/s)

Write the fundamental equation for building lateral acceleration as:

$$\ddot{x} = \frac{f(t)}{m} + \left(\frac{b_2 - b_1}{m}\right)\dot{x} + \left(\frac{k_2 - k_1}{m}\right)x$$

$$\ddot{x} = \dot{x}_2$$

and

$$\dot{x}_1 = x_2$$

Note that this implies that in the original problem:

$$\ddot{x} = \dot{x}_2$$

$$\dot{x} = x_2$$

$$x = x_1$$

This definition of variables allows us to write the second order differential equation as two first-order equations as needed in Matlab.

$$\dot{x}_2 = \frac{f(t)}{m} + \left(\frac{b_2 - b_1}{m}\right)x_2 + \left(\frac{k_2 - k_1}{m}\right)x_1$$

and

$$\dot{x}_1 = x_2$$

The last two equations are to be solved by Matlab numerically using the Runge-Kutta algorithm.

### Task 2

Plot the dynamic response of the building subject to the initial displacement of 7 cm. Comment on the results obtained.

## Problem 3

Download the two Excel files named: a) export\_airport\_runways.xlsx and b) export\_NC\_airports.xls provided in the syllabus web site. Samples of the files are shown below for reference.

Table. Airport database (for North Carolina).

Location_id	Airport_ID	State	City	Airport_Type	Land Area (acres)	Lat(seconds)	Lon(seconds)	Lat (deg) format	Lon (deg) format	Lat (deg)
16494.*A	6NC8	NORTH CAROLINA	ADVANCE	PR	3	129377.4930N	289384.1950W	12	12	35.94
16494.1*A	30NC	NORTH CAROLINA	ADVANCE	PR		129775.0000N	289645.0000W	12	12	36.05
16499.3*A	ASJ	NORTH CAROLINA	AHOSKIE	PU	210	130671.0930N	277815.0800W	12	12	36.30
16499.31*A	15NC	NORTH CAROLINA	AHOSKIE	PR	40	130463.0000N	277106.0000W	12	12	36.24
16503.*A	VUJ	NORTH CAROLINA	ALBEMARLE	PU	300	127500.1010N	288542.8640W	12	12	35.42
16503.01*H	9NC1	NORTH CAROLINA	ALBEMARLE	PR		127308.0000N	288701.0000W	12	12	35.36
16506.*A	RHP	NORTH CAROLINA	ANDREWS	PU	206	126702.8270N	301906.9390W	12	12	35.20
16506.52*A	87NC	NORTH CAROLINA	ANGIER	PR		127690.0000N	283156.0000W	12	12	35.47
16506.53*A	NC44	NORTH CAROLINA	ANGIER	PR		127681.0000N	283262.0000W	12	12	35.47
16510.*A	NC11	NORTH CAROLINA	APEX	PR	7	128669.5430N	284129.0310W	12	12	35.74
16510.01*A	NC81	NORTH CAROLINA	APEX	PR		128706.5440N	283895.0260W	12	12	35.75
16515.12*A	1NC0	NORTH CAROLINA	ASHEBORO	PR	100	128599.4910N	287789.1410W	12	12	35.72
16515.21*A	7NC9	NORTH CAROLINA	ASHEBORO	PR		128618.4910N	287680.1350W	12	12	35.73

Table. Sample File runways\_aircraft\_airports.xls.

Location_id	RunwayName	Length (ft)	Width (ft)	RunwayMaterial
16494.*A	40717	2500	60	TURF
16494.*A	40717	2500	60	TURF
16494.1*A	16/34	2700	50	TURF
16494.1*A	16/34	2700	50	TURF
16499.3*A	40561	4502	75	ASPH-G
16499.3*A	40561	4502	75	ASPH-G
16499.31*A	14/32	1800	50	TURF
16499.31*A	14/32	1800	50	TURF
16503.*A	04L/22R	3500	75	ASPH-G
16503.*A	04L/22R	3500	75	ASPH-G
16503.*A	04R/22L	5500	100	ASPH-G
16503.*A	04R/22L	5500	100	ASPH-G

### Task 1:

Import both files into Access creating a new database. Define **Location\_id** as a text datatype in both tables. The **Location\_id** field, serves to connect both files in a database. Establish a one-to-many relationship between the North Carolina airports and the runways\_NC\_airports files using the field **Location\_id**. Verify that the relationship works. Explain how do you know the relationship works.

### Task 2:

Create an MS Access query to find all the airports in North Carolina with at least a runway longer than 3,900 feet. In your query, include the number of airports and show the resulting Access table. In your solution table include the following fields:

- Airport ID
- Runway label
- Runway length
- Runway width

### Task 3:

Create another MS Access query to find the **public airports** in North Carolina with **runways longer than 5,700 feet** and with **Asphalt (ASPH) runways**. State the number of airports and show the resulting Access table showing the following fields:

- Airport ID
- Runway label (s)
- Runway length

### Task 4:

Create an MS Access query to find all the runways at airports in North Carolina with **land areas greater than 30 acres**. The query should produce a table with the airport ID, runway label, runway length, airport latitude and longitude.