Assignment 9: Dynamic Systems: Differential Equations

Date Due: April 23   Instructor: Trani

Problem 1

A civil engineer needs to model the dynamics of a building subject to seismic loads. The building and its foundation is modeled as series of spring and dampers as shown in Figure 1.
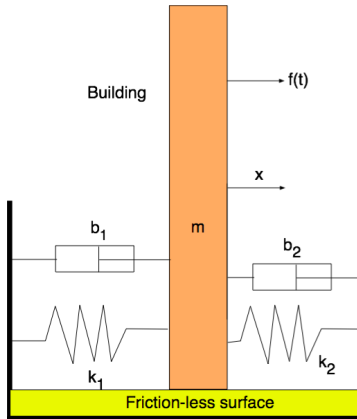

Figure 1. Building and Foundation System for Problem 1.

The fundamental equation of motion for the acceleration of the building at the foundation is:

$$\ddot{x} = \frac{f(t)}{m} + \left(\frac{b_2 - b_1}{m} \right)\dot{x} + \left(\frac{k_2 - k_1}{m} \right)x \qquad \text{(Eq. 1)}$$

where:

$\ddot{x} =$ lateral acceleration of the building system (m/s^2)

$\dot{x} =$ lateral speed of building system (m/s)

$x =$ lateral displacement of building system (m)

Also, m is the mass of the building (kg), f(t) is a forcing function (units are Newtons) that is generated by seismic waves. k1 and k2 are spring constants (N/m) and b1 and b2 are damping constants (N/m/s). The values of the constants are:

m = 4,000,000;   % mass (kg)

b1 = 400,000;   % damper constant (N / (m/s))

b2 = 200,000;   % damper constant (N / (m/s))

k1 = 7e6;   % spring constant (N/m)

k2 = 5e6;   % spring constant (N/m)

Task 1

Using Simulink, construct a block model of the building system and solve the differential equations of motion of the building system to find the lateral speed and the lateral displacement responses as a function of time.

Task 2

Solve the dynamic response of the building system when is subject to an initial instantaneous displacement of 15 cm caused by a seismic shock. Model the seismic shock using the initial condition for the state variable (x - lateral displacement) and for this analysis assume the forcing function f (t) is zero as you explore the natural dynamics of the system. Export the lateral speed response and the lateral displacement (both state variables) to the workspace. Plot the state variables as a function of time using the subplot command. Study the output of the model and comment on the trends observed.

Task 3

Model the dynamic response of the building subject to a small earthquake by introducing the forcing function f(t) in equation (1). The forcing function f(t) can be modeled using a **Repeated Sequence Interpolated Block** (found in the Sources library in Simulink) shown below. This block requires two inputs: a) a time vector and b) the output signal of the forcing function f(t). An example of the block parameters are shown in the figure below.
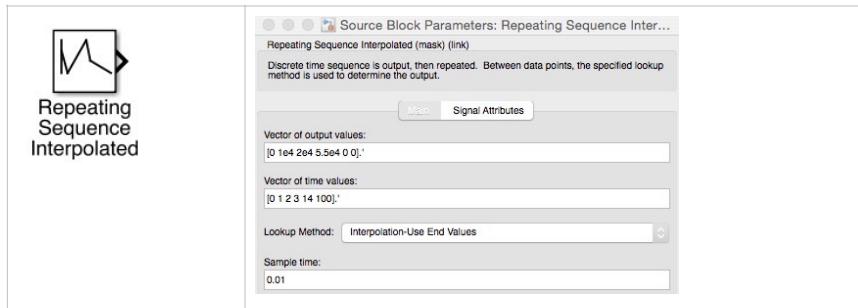
The actual values of time and the forcing function f(t) are shown in Table 1. Simulate the response of the building to the earthquake for 30 seconds or until the building oscillation subsides. Use the Runge-Kutta 5th Order integration algorithm to solve this problem. Export the lateral speed response and the lateral displacement (both state variables) to the workspace. Plot the state variables as a function of time using the subplot command. Study the output of the model and comment on the trends observed.

Table 1. Forcing Function Values of Simulated Earthquake Event. The last time in the table is made artificially large so that the sequence is not repeated in the 30 second simulation period of this event.

Time of Event (seconds)	Force (N)
0.0	0.0
1.6	140000.0
2.3	380000.0
3.5	435000.0
4.4	450000.0
5.2	200000.0
6.8	480000.0
8.3	200000.0
9.8	30000.0
10.5	0.0
100.0	0.0

Problem 2

You are tasked to reconstruct a car accident. The data from a data logger connected to the OBD diagnostic system is provided to your team. The car acceleration data recorded is presented in Table 2.

Table 2. Car OBD II System Data.

Time (s)	Acceleration (m/s2)
0	0.00
1.5	-1.50
2.5	-2.30
3	-2.80
3.5	-2.65
4	-2.00
4.5	-2.20
5	-2.40
5.5	-6.20
6	-12.30

Task 1

Create a Simulink model to estimate the velocity and distance traveled by the car during the accident. It is recommended that you use a **Repeated Sequence Interpolated** block. Alternatively, you could use a **Table Lookup Block** (in the Table Lookup library in Simulink). Based on initial data provided by the data logger, the vehicle was traveling at 30 m/s when the accident sequence (Table 2) started. Your model should limit the speed of the vehicle to be positive. Use the limit values in the integration block.

Task 2

Using the model created in Task 2, estimate the distance traveled by the vehicle during the accident. Was this a bad accident? Explain by looking at the accelerations recorded.

Task 3

Read about OBD systems in the Wikipedia article (http://en.wikipedia.org/wiki/On-board_diagnostics) and briefly answer the following:

   a) How many protocols are available for OBD systems?

   b) In the SAE J1850 PWM protocol, how many bytes constitute the message length used?

   c) What types of data can be collected by OBD data loggers provided by car insurance companies?

   d) Explain the term vehicle telematics.

   e) Are vehicle OBD embedded systems secure? Explain.

Problem 3

Task 1

Repeat Problem 1 but use Matlab ODE solver ODE45 (with Matlab code) to execute the problem.

Task 2

Solve the dynamic response of the building system when is subject to an initial instantaneous displacement of 15 cm caused by a seismic shock. Model the seismic shock using the initial condition for the state variable (x - lateral displacement) and for this analysis assume the forcing function f (t) is zero as you explore the natural dynamics of the system. Plot the state variables as a function of time using the subplot command. Study the output of the model and comment on the trends observed.

Task 3

Model the dynamic response of the building subject to a small earthquake by introducing the forcing function f(t) in equation (1). The forcing function f(t) can be modeled in the Matlab code using the interpolation function (INTERP1) with two vectors: time and force with given values (see Table 1). The INTERP1 function will be used inside the function that calculates the rates of change of velocity and displacement of the building.