Assignment 8: Integration and ODEs

Date Due: April 17, 2026 Instructor: Trani

Problem 1

You are task to calculate the volume of earth for a highway project in Virginia. A cross section of the terrain to be removed in the project is shown in Table 1.

Station (meters)	Elevation (meters)
0.0	30.0
4.5	35.0
10.6	42.0
15.8	49.0
19.5	52.0
23.6	56.3
28.6	53.0
32.9	39.0
35.6	30.0
38.5	26.0
41.5	18.0
44.7	15.0
48.3	9.0
55.0	0.0

Task 1

Create a **Matlab script** to read the data. Copy the data into another file (Matlab, Excel, etc.).

Task 2

Add MATLAB code to the script created in Task 1 to interpolate linearly values of elevation from stations 0 to 55 at intervals of 10 centimeters. The goal is to produce a smooth profile of the terrain before calculating the area under the curve in Task 3. Make a plot of the original data and the interpolated data.

Task 3

Estimate the area under the curve projected by the elevation across each station. Use either the Trapezoidal rule. Display the value of the area under the curve in the Command window and also write the value in the title of the plot generated in Task 2.

Task 4

Use the Matlab area command to demonstrate a partial removal of terrain between stations 10.5 and 25.5 meters. Show the area under the partial removal in the plot.

Problem 2

A civil engineer is designing acceleration ramps for a new highway. Figure 1 who's the typical acceleration ramp configuration. The highway is used by a variety of vehicles including cars, light trucks, and heavy trucks. The engineer collected data about three types of vehicles to design the acceleration ramp (L) shown in Figure 1.

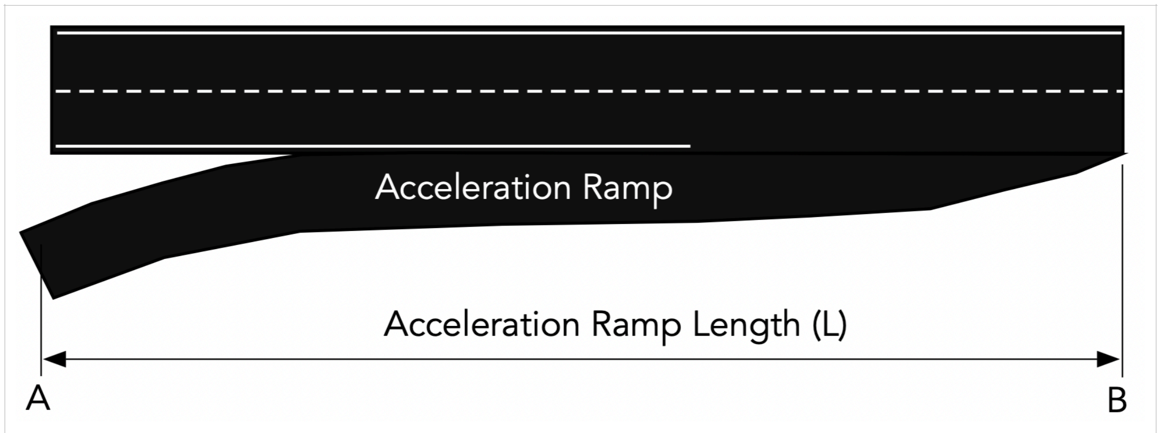

Figure 1. Highway Ramp Scenario for Problem 1.

Using your knowledge Matlab ODE solvers or Simulink (your choice), to solve the differential equations of a vehicle accelerating on the highway ramp. The Simulink model should solve the first-order differential equation of acceleration of the vehicle for a given set of vehicle parameters (k_1 and k_2). Table 1 has the parameters for three types of commonly used highway vehicles. The differential equation is presented below.

$$\frac{dV}{dt} = k_1 - k_2 V^n$$

Where:

$\frac{dV}{dt}$ is the vehicle acceleration in m/s^2

k_1 is the first acceleration constant (m/s^2)

k_2 is the second acceleration constant (in 1/s)

V is the speed of the vehicle (m/s)

n is an empirical factor derived from vehicle testing (dimensionless)

Table 2. Shows shows the acceleration constants for three types of highway vehicles using empirical data.

Table 2. Acceleration Constants for the Three Types of Highway Vehicles.

Vehicle Type	k_1	k_2	n
Mid-size Sedan	2.2	0.045	0.98
Light Truck (loaded)	1.8	0.049	0.87
Heavy Truck (loaded)	1.15	0.031	0.89

Task 1

Create Matlab code (using ODE solver) or a Simulink model to solve the differential equation that estimates vehicle speed as a function of time. You can reuse any of the Matlab code or Simulink models provided in class.

Task 2

Enhance the model of Task 1 to predict the distance travelled by the vehicle accelerating on the ramp.

Task 3

Use the model created in Tasks 1 and 2 to predict the length of the acceleration ramp if a loaded truck is used as the critical vehicle. Assume that the vehicle at location A has an initial speed of 5 m/s (very low speed to simulate a ramp metering light). Compare the acceleration ramp length for a loaded truck and a car.

Problem 3

Figure 2 presents consumption data and emissions is for a small SUV vehicle. The fuel consumption and emissions are presented in columns 3 and 4. The data was collected by the Oak Ridge National Lab.

Speed (m/s)	Fuel Consumption (l/s)	CO2 Emissions (mg/s)
0.00	0.00026	1.89
2.75	0.00035	2.15
4.15	0.00042	3.15
6.00	0.00046	4.78
8.35	0.00057	6.60
11.00	0.00066	9.30
14.20	0.00078	12.80
17.30	0.00092	17.30
19.50	0.00107	21.90
22.25	0.00126	33.60
25.50	0.00145	54.30
28.00	0.00185	97.60

Figure 2. Sport Utility Vehicle Fuel Consumption Data.

Task 1

Create Matlab script to read the data. Use the method of your choice.

Task 2

Add code to the script created in Task 1 to find the best fourth-order polynomial to predict fuel consumption (dependent variable) as a function of speed (independent variable). Display the polynomial in the Command Window and make a screen capture. The polynomial is of the form:

$$FC = A(V^4) + B(V^3) + C(V^2) + DV + E$$

Where:

$A, B, C, D . E$ are the regression coefficients found using the Least-Square Method.

V is the vehicle speed (m/s)

FC is the vehicle instantaneous fuel consumption (liters/second) for a given speed

Task 3

Make a plot (in code) of the fuel consumption data and also show your polynomial fit in the same plot.

Task 4

Add code to find the best high-order polynomial to predict vehicle emissions (mg/s) as a function of speed (m/s). For example, a fifth-order polynomial takes the form:

$$E_{CO2} = A(V^n) + B(V^{n-1}) + C(V^{n-2}) + \ldots D(V^2) + E(V) + F$$

Where:

A, B, C, D, E, F are the regression coefficients found by Matlab using the Least-Square Method.

V is the vehicle speed (m/s)

E_{CO2} is the vehicle instantaneous CO2 emission rate (mg/s)

Task 5

Make a plot (in code) of the emissions data and also show your polynomial fit in the same plot. Comment on the quality of the polynomials to fit the data.