

Assignment 8: Integration, Polynomials and Functions

Date Due: April 5, 2021 (midnight)

Instructor: Trani

Problem 1

Asphalt is an important material used in roadway construction. A civil engineer wants to estimate the likelihood that a given mixture of asphalt survives the repeated loads of a busy highway. Using laboratory samples, the engineer collects data and finds that the life expectancy of the asphalt pavement can be approximated using a Gamma distribution according to the equation shown below. The equation of the Probability Density Function (PDF) of the Gamma distribution is:

$$f(x) = \beta^\alpha x^{\alpha-1} e^{-\beta x} / \Gamma(\alpha)$$

where :

x = random variable - bridge pavement life (years)

α = parameter of the distribution

β = parameter of the distribution

$\Gamma(\alpha)$ = Gamma function (in Matlab gamma(a) evaluates the Gamma function for value a)

$f(x)$ = probability that pavement survives x years

$\frac{\alpha}{\beta}$ = mean life expectancy of asphalt pavement (years)

After testing many pavement samples, the engineer estimates the values of α and β to be 6.40 and 0.95 respectively. The area under the probability density function ($f(x)$) has a fundamental interpretation in engineering modeling. For example, the area under the curve to the right of a given value of x in Figure 1 can be interpreted as the probability that the asphalt pavement survives the repeated load surviving conditions. Looking at Figure 1, the probability that the pavement survives less than four years is 0.761. Similarly, the probability of the pavement surviving more than 4 years is 0.239 (1- 0.761).

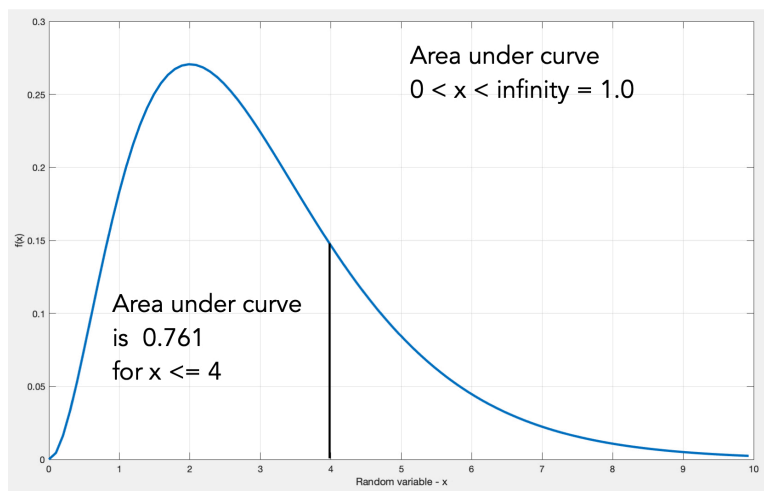


Figure 1. Gamma Probability Distribution with Parameters $\alpha = 3$ and $\beta = 1$. Note: Area Under $f(x)$ when $x < 4$ is 0.716.

Task 1

Create a Matlab script to evaluate the Gamma distribution function $f(x)$ using the parameters of the Gamma distribution. Plot the resulting PDF as a function of pavement life. Label accordingly. Verify that the PDF plot is consistent with the Gamma distribution. See article in Wikipedia (https://en.wikipedia.org/wiki/Gamma_distribution).

Task 2

Create a Matlab function to calculate the value of the Gamma distribution function $f(x)$. The function needs three input arguments: the value of x (*the value of the random variable*) and the values of α and β . The output of the function is $f(x)$.

Task 3

Create another Matlab script to estimate the probably that the asphalt pavement survives more than eight years. This script uses the function created in Task 2 to evaluate the area under the curve $f(x)$. For this problem you can use either the **Quad** or **Trapz** functions in Matlab.

Task 4

Modify the Matlab code to find the probability that the asphalt pavement lasts between eight and eleven years. If you are to communicate with VDOT (your client), what is the median (50th percentile) asphalt pavement life that we can expect from the pavement?

Problem 2

A large cargo ship (called the Ever Given) has blocked the Suez Canal for over five days (see article: https://en.wikipedia.org/wiki/2021_Suez_Canal_obstruction). Figure 1 shows the ship dimensions in meters.

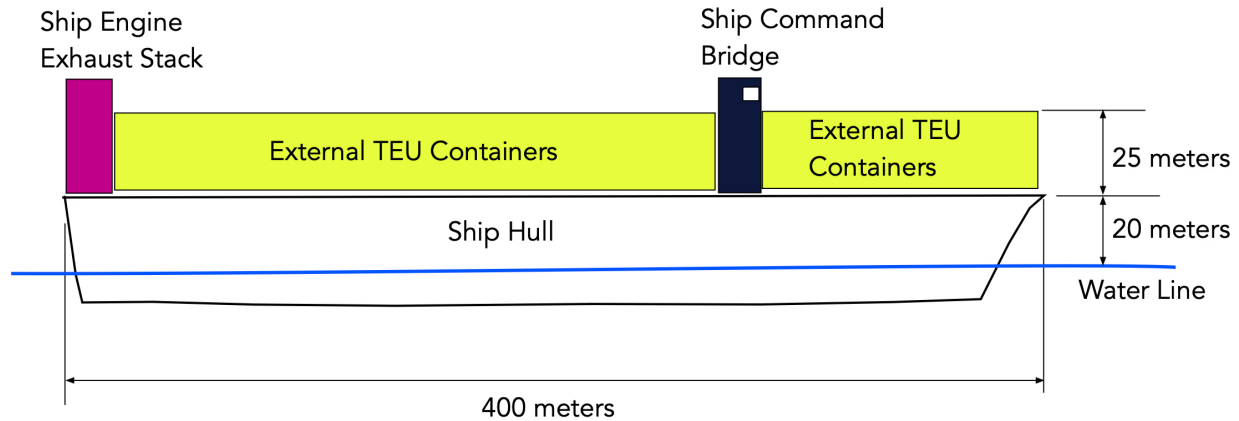


Figure 2. Large Cargo Ship.

The equations to estimate the sail drag (lateral force acting on the ship when subjected to crosswinds) and the power required to overcome the sail drag force are presented below:

$$D_{sail} = \frac{1}{2} \rho V_c^2 S_a C_d$$

D_{sail} = sail drag (Newtons)

V_c = crosswind speed (m/s)

ρ = air density (kg/cu. meter)

S_a = sail area (square meters)

C_d = drag coefficient (dimensionless)

$$P_{sail} = D_{sail} V_c$$

P_{sail} = power to overcome sail drag (Watts)

The Suez canal is at sea level where air density is 1.225 kg/cu.meter. The ship's sides (including the tower of containers exposed to the air) act as flat plates with a C_d value of 1.0. The crosswind that caused the mishap has been estimated at 17 meters/s. The ship's sail area is the area of the ship above the water line exposed to crosswinds. You can estimate the sail area using Figure 1.

Task 1

Create a Matlab function to calculate the ship's sail drag with a crosswind speed of 17 m/s. The function should take the ship's parameters (including the ship's dimensions) and produce two outputs: sail drag (Newtons) and the power to overcome sail drag (Watts).

Task 2

Create a Matlab script to define the ship size and the parameters needed to estimate sail drag and power. In this script call the function created in Task 1 producing the sail drag and power when the crosswind is 17 m/s.

Task 3

The ship has two bow thrusters with a total of 5.0 MW of power. Can the thrusters overcome the sail power?

Task 4

Further improve the script created in Task 2 to estimate the sail drag for crosswind speeds ranging from 10 to 20 m/s. Plot the answers (crosswind speed on the x-axis).

Problem 3

A simple equation to predict the so-called basic resistance of a high-speed rail system (R_{basic}) moving at speed (v) is given by Davis' equation and shown below. This equation includes the resistance due to aerodynamic drag and axle load friction resistance (i.e., resistance between train steel tires and the rail track).

$$R_{basic} = A + Bv + Cv^2$$

where: A , B , and C are train-specific coefficients and v is the train speed in meters per second. The values of A , B , and C are determined using wind tunnel testing and actual testing of the trains in test tracks. The values of A , B , and C for a Japanese High-Speed Train of the series Shinkansen 500 (see <http://en.wikipedia.org/wiki/Shinkansen>) are provided below with their corresponding units.

A = 8.04000; % units are kN
B = 0.12356; % units are kN s/m
C = 0.01099; % units are kN s-s/m-m

Note that the units of A , B and C are consistent to produce a basic resistance of the train in kiloNewtons.

The power required (P) to move the train at speed (v) is the product of Resistance (R) and speed (v).

Task 1

Create a Matlab function to estimate the basic resistance (R) (in Newtons) and the power required (P) (in Watts if R is estimated in Newtons) to move for the high-speed train for a given speed (v) in m/s. The function needs to produce two outputs (R and P).

Task 2

Create a Matlab script that calls the function created in Task 1. Using the script, estimate the train resistance and power required for speeds ranging from 20 to 85 m/s at intervals of 0.1 m/s. Make a plot of the resistance vs. speed and power vs speed using the subplot command.