

Assignment 8: Integration, Polynomials and Functions

Date Due: April 16

Instructor: Trani

Problem 1

Hickory wood is an important material used in construction. A civil engineer wants to estimate the distribution of the Modulus of Elasticity (MOE) for various specimens of Hickory beams received from a factory. The engineer collects data and finds that the modulus of elasticity is normally distributed (see picture below). The equation of the Probability Density Function (PDF) of the Gaussian (or Normal) distribution is given in any statistics textbook and repeated here for completeness (http://en.wikipedia.org/wiki/Normal_distribution).

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\mu)^2}{2\sigma^2}\right]}$$

where: x is the random variable in question (modulus of elasticity in psi) and μ and σ are the mean and standard deviation (in psi) of the random phenomena modeled. After testing 100 samples, the engineer estimates the mean modulus of elasticity to be $2.162 \text{ e}6$ psi. The standard deviation is found to be $8.4\text{e}4$ psi.

The area under probability density function has a fundamental interpretation in random processes. For example, the area under the curve in Figure 1 can be interpreted as the probability that random variable x will take values between a and infinity.

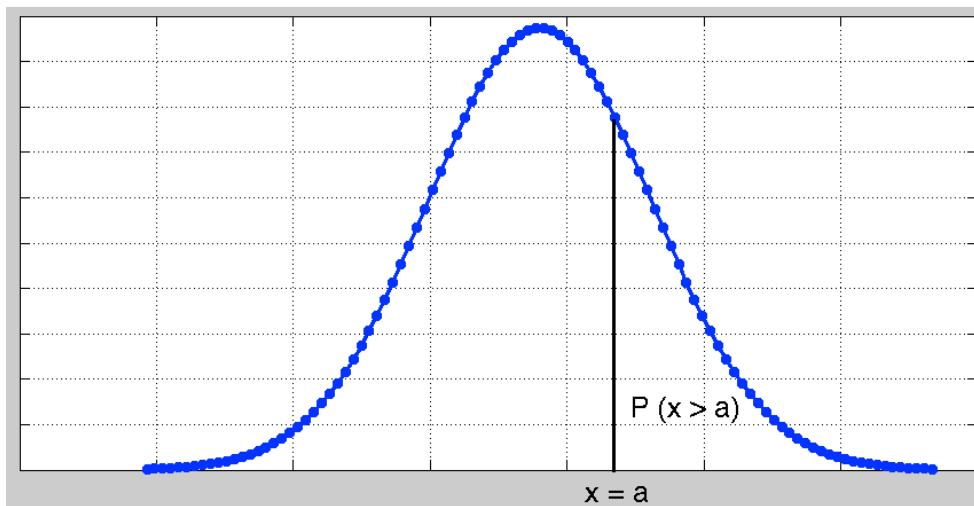


Figure 1. Sample Normal Probability Density Function. Interpretation of the Area Under the Curve.

Task 1

Create a Matlab script to evaluate the function $f(x)$ using the values of μ and σ observed for Hickory beams. Plot the resulting PDF as a function of modulus of elasticity. Label accordingly. Verify that the PDF plot is the well-known bell-shape curve of the Normal distribution.

Task 2

Create a Matlab function to calculate the value of the Normal distribution PDF ($f(x)$). The function needs two take three arguments: x (*the value of the random variable*), μ and σ . The output of the function is $f(x)$.

Task 3

Create another Matlab script to estimate the probably that wood samples MOE exceed $2.00\text{e}6$ psi, the minimum design specification.

Task 4

Find the probability that a lot of 1,000 of random sample beams have MOE values ranging from 2.10×10^6 psi to 2.35×10^6 psi.

Problem 2

In the improvement of waterway and engineer needs to determine the amount of material to be removed in the dredging operation. The Table below shows the waterway depth (column 2) at stations spaced every 5 meters across the waterway (column 1).

Table 1. Waterway Stations and Depth.

Horizontal Coordinate (meters)	Vertical Profile (meters)
0	0.00
5	-2.80
10	-4.50
15	-5.20
20	-5.70
25	-6.90
30	-8.60
35	-9.90
40	-10.20
45	-10.35
50	-10.20
55	-10.10
60	-10.00
65	-9.20
70	-8.20
75	-7.30
80	-5.60
85	-3.90
90	-3.20
95	-2.10

The specification of the cross section of the waterway allowing safe operations is shown below.

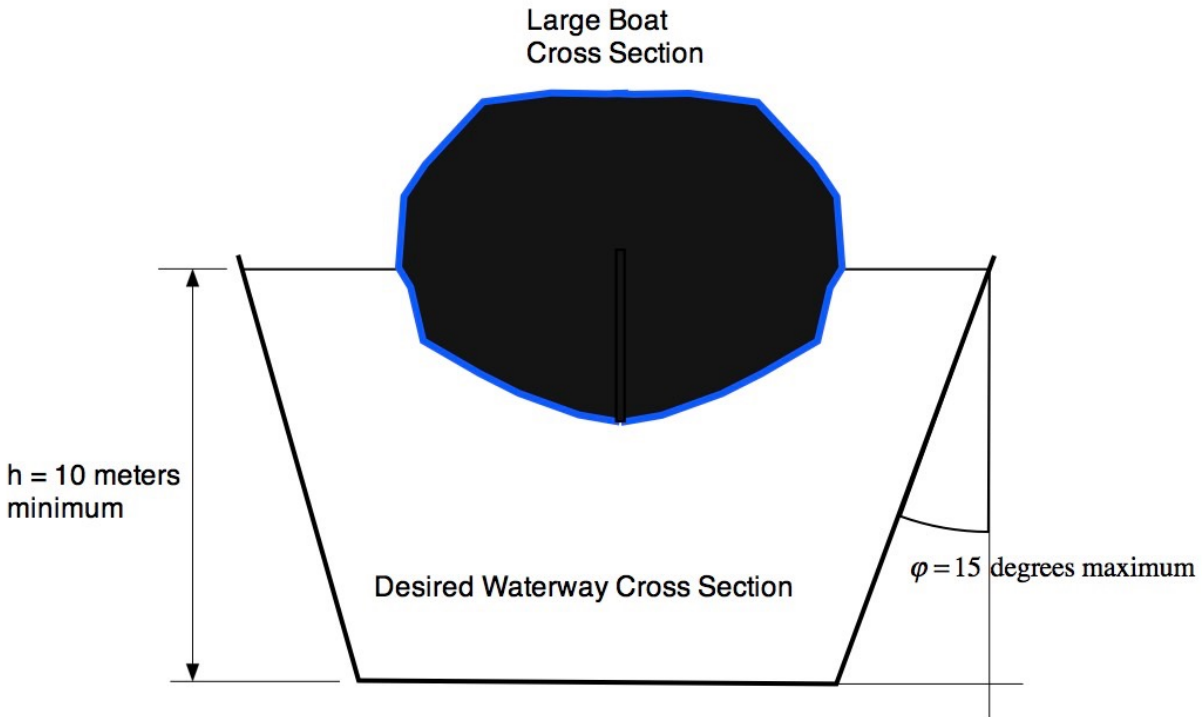


Figure 2. Specification of the Waterway Cross Section.

Task 1

Create a Matlab script to import or read the data provided in the Table 1. Use any importing procedure learned in class **other than** using the import Wizard in Matlab.

Task 2

Improve the script created in Task 1 to estimate the area under the cross section area of the existing waterway profile. Use the Newton-Cotes panel numerical integration algorithms learned in class.

Task 3

Repeat Task 2 but using the Trapezoidal numerical integration algorithm.

Task 4

Further improve the script in Task 3 to estimate the area needed for dredging.

Problem 3

Use the values of Table 1 in Problem 2 to approximate a polynomial to the data.

Task 1

Create a Matlab script to estimate the best polynomial (using least-square estimation) to fit the data provided in Table 1. In your selection of the best polynomial estimate the Sum of the Squared Errors (SSE) and decide which polynomial to use after trying various options (i.e., the solution with the minimum value of SSE). Tabulate the values of SSE vs the order of the polynomial used.

Task 2

Improve the script in Task 1 and plot the original data, the polynomial fit and the values of the local errors. Use a two window viewport to display the results as performed in class. In your solution clearly state the polynomial used and the values of the polynomial coefficients found. Plot the best solution found in Task 1.

Problem 4

Use the Car data numeric file (see syllabus week 11) to solve the problem. A sample of the data file is shown in the following lines.

```
% Car data file

% Contains the following information:
%
% Column 1 = Car weight (lb)
% Column 2 = Turning Circle (ft)
% Column 3 = Car engine displacement (cu. inches)
% Column 4 = Horsepower (HP)
% Column 5 = Gas Tank Size (gallons)

2700  37    112   130   13.2
3265  42    163   160   18
2935  39    141   130   21.1
2670  35    121   108   15.9
```

Task 1

Create a Matlab script to estimate the read the car data. Plot the vehicle weight (x-axis) vs the engine displacement (y-axis). Also plot the vehicle weight vs. engine horsepower (y-axis).

Task 2

Improve the Matlab script to estimate the best polynomial (using least-square estimation) to fit the vehicle mass and engine displacement. In your selection of the best polynomial estimate the Sum of the Squared Errors (SSE) and decide which polynomial to use after trying various options (i.e., the solution with the minimum value of SSE). Repeat the procedure to find the best fit that relates vehicle mass and engine horsepower. Save both sets of coefficients for Task 3.

Task 3

Create a new Matlab function that estimates the engine displacement and the horsepower as a function of vehicle mass. In other words, the function produces two outputs (engine displacement and horsepower) for one one input (mass). Test the function with the following mass vector.

```
mass = [2300 2760 3400 2230 3450];
```

Plot the resulting values with the original dat as background.