Assignment 6: Linear Programming

Solution Instructor: Trani

Problem 1

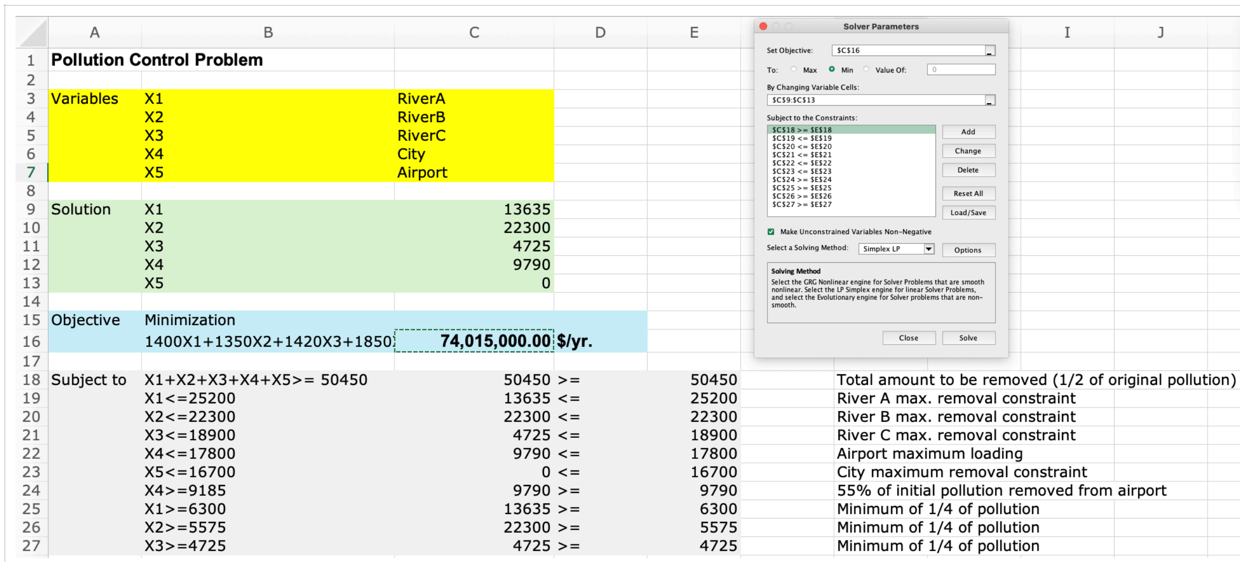
Modify the water management pollution control problem described in the class notes and explained in class. New removal costs are presented in Table 1.

Table 1. Removal Costs and Pollution Values for Water Pollution Control Problem.

Source	Removal Cost ($/kg)	Pollution to Lake (kg)
River A	1,400	25,200
River B	1,350	22,300
River C	1,420	18,900
City	1,850	16,700
Airport	1,760	17,800

Assume that under a new water mandate by EPA we would like to remove at least half of the of the baseline pollution into the lake. Moreover, the airport manager would like to participate in the pollution removal program by removing at least 55% of their baseline pollution allocations per year. In order to be socially responsible to communities near the lake, the pollution processing plants at all three rivers need to remove at least one fourth of their pollutants as a minimum according to a new environmental law.

a) Formulate the problem as a linear programming problem. Solve the new problem using Excel Solver and state the optimal cost.



	A	B	C	D	E			
1	Pollution Control Problem							
2								
3	Variables	X1	RiverA					
4		X2	RiverB					
5		X3	RiverC					
6		X4	City					
7		X5	Airport					
8								
9	Solution	X1	13635					
10		X2	22300					
11		X3	4725					
12		X4	9790					
13		X5	0					
14								
15	Objective	Minimization						
16		1400X1+1350X2+1420X3+1850	74,015,000.00	$/yr.				
17								
18	Subject to	X1+X2+X3+X4+X5>= 50450	50450	>=	50450	Total amount to be removed (1/2 of original pollution)		
19		X1<=25200	13635	<=	25200	River A max. removal constraint		
20		X2<=22300	22300	<=	22300	River B max. removal constraint		
21		X3<=18900	4725	<=	18900	River C max. removal constraint		
22		X4<=17800	9790	<=	17800	Airport maximum loading		
23		X5<=16700	0	<=	16700	City maximum removal constraint		
24		X4>=9185	9790	>=	9790	55% of initial pollution removed from airport		
25		X1>=6300	13635	>=	6300	Minimum of 1/4 of pollution		
26		X2>=5575	22300	>=	5575	Minimum of 1/4 of pollution		
27		X3>=4725	4725	>=	4725	Minimum of 1/4 of pollution		

Solver Solution to the LP Problem.

The optimal cost of removal is $74 million dollars per year. The removal from the city is zero (above) because the city has the highest cost of removal ($1850 per kilogram) and does not have a minimum removal constraint such as the airport and the three rivers.

b) The city manager would like to invest in a new processing plant to further reduce pollution into the lake. The new plant is expected to cost $7,600,000 and last for at least 25 years. Using principles of engineering economics and Excel, calculate the yearly payments from the city to a bank to buy the processing plant and pay it off at the end of 25 years. Assume the bank charges 4% yearly over the loan period.

	A	B	C	D
2	**Water Pollution Cost**			
3	**Tasks**			
4	Calculate the monthly payment to pay the cost of a water pollution plant			
5				
6	Loan	7,600,000	Dollar amount of loan	
7	No. of Periods	300	periods in loan	
8	Interest	4%	percent per year	
9				
10	Monthly Payment	($39,982.33)	PMT(interest/month,periods,loan present value,	
11	Yearly Payment	($479,787.91)		
12				
13	Total Payments	($11,994,697.63)		

Equal Uniform Payments Schedule.

The yearly payments are $479,788 as shown in the payment schedule above.

Use the Excel function =PMT(B8/12,B7,B6,0,1)

Problem 2

The construction of a new highway requires a minimum of 1,100,000 cubic meters of sand and gravel mixture. The final sand/gravel mixture must contain no less than 572,000 cu. meters of sand (fine aggregate) and no more than 605,000 cu. meters of gravel (coarse aggregate).

The gravel and sand materials can be obtained from two sites: 1) Miramar and 2) San Diego. Table 1 shows the proportions of sand and gravel from each site. Because each site is also used in other construction jobs, the maximum amounts of materials excavated from each site are limited to the following: a) 540,000 cu. meters for Miramar, and 690,000 cu. meters for San Diego.

Table 1. Proportions of Sand and Gravel from Three Collection Sites.

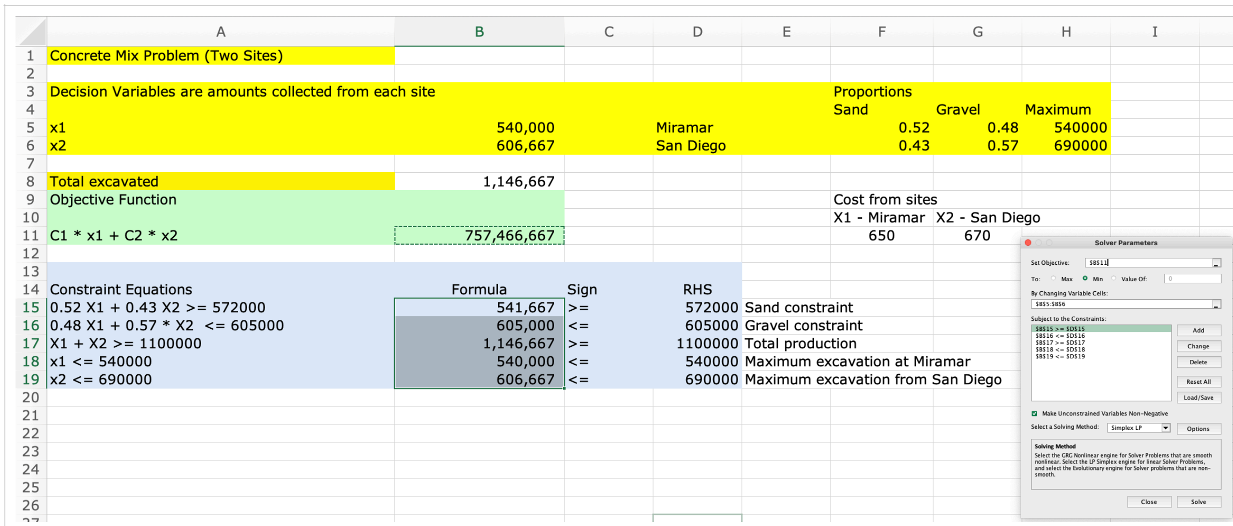
Site	Proportion of Sand (%)	Proportion of Gravel (%)
Miramar	52	48
San Diego	43	57

The costs of collection and transportation of a cubic meter of material are: a) $650 for Miramar, $670 for San Diego.

A) Setup the problem as a linear programming problem. The objective is to **minimize the cost of producing the concrete for the highway project**.

B) Use the Simplex method to setup by hand the first table of the problem. For each table indicate the Basic Variables, Non Basic Variables and the value of the objective function (Z). The first table requires the problem to be in standard form.

C) Find the optimal solution that minimizes the cost using **Excel Solver**. Clearly state the values of the decision variables and the value of the objective function in the optimal solution.

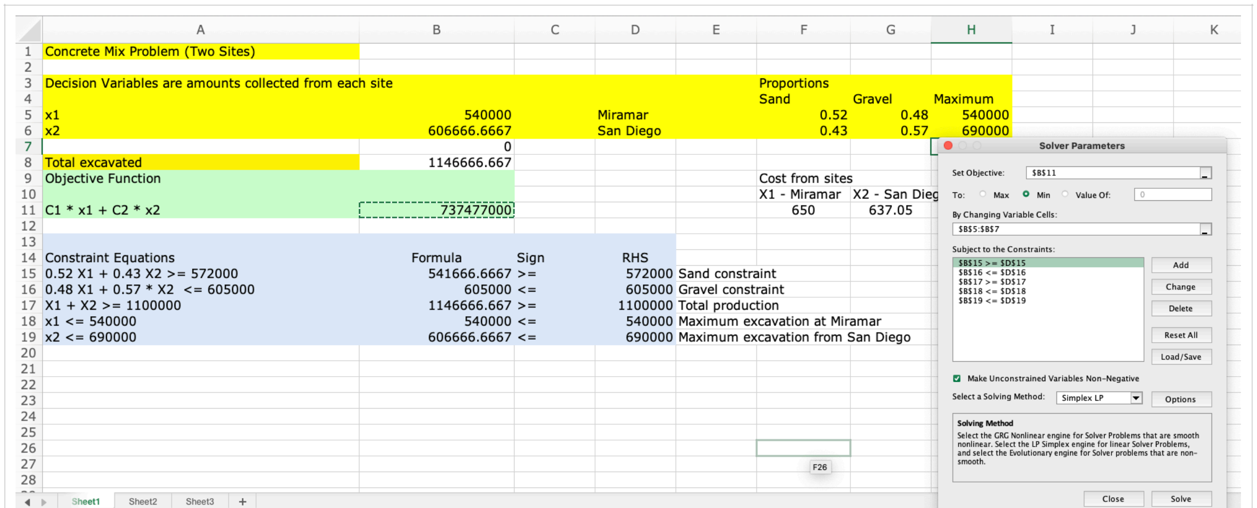


	A	B	C	D	E	F	G	H
1	Concrete Mix Problem (Two Sites)							
2								
3	Decision Variables are amounts collected from each site					Proportions		
4						Sand	Gravel	Maximum
5	x1	540,000		Miramar		0.52	0.48	540000
6	x2	606,667		San Diego		0.43	0.57	690000
7								
8	Total excavated	1,146,667						
9	Objective Function					Cost from sites		
10						X1 - Miramar	X2 - San Diego	
11	C1 * x1 + C2 * x2	757,466,667				650	670	
12								
13								
14	Constraint Equations	Formula	Sign	RHS				
15	0.52 X1 + 0.43 X2 >= 572000	541,667	>=	572000	Sand constraint			
16	0.48 X1 + 0.57 * X2 <= 605000	605,000	<=	605000	Gravel constraint			
17	X1 + X2 >= 1100000	1,146,667	>=	1100000	Total production			
18	x1 <= 540000	540,000	<=	540000	Maximum excavation at Miramar			
19	x2 <= 690000	606,667	<=	690000	Maximum excavation from San Diego			

Best solution. The constraint for sand cannot be met with two sites. The best solution is to excavate 540,000 cu. meters from Miramar and 606,667 cu. meters from San Diego. The sand constraint is not met. Only 541,667 cu. meters of sand can be procured from both sides. The gravel constraint is met.

D) If the San Diego site offers a 7% discount in purchases of more than 500,000 cu. meters of material, would you consider their offer and re-allocate differently the procurement of sand and gravel from both sites? Explain and solve using Excel Solver to support your answer.

Since the best solution presented involves extracting 590,000 cu. Meters from San Diego, the 7% offer will be applicable. The new cost of extracting material from San Diego is $637.50.



	A	B	C	D	E	F	G	H
1	Concrete Mix Problem (Two Sites)							
2								
3	Decision Variables are amounts collected from each site					Proportions		
4						Sand	Gravel	Maximum
5	x1	540000		Miramar		0.52	0.48	540000
6	x2	606666.6667		San Diego		0.43	0.57	690000
7		0						
8	Total excavated	1146666.667						
9	Objective Function					Cost from sites		
10						X1 - Miramar	X2 - San Dieg	
11	C1 * x1 + C2 * x2	737477000				650	637.05	
12								
13								
14	Constraint Equations	Formula	Sign	RHS				
15	0.52 X1 + 0.43 X2 >= 572000	541666.6667	>=	572000	Sand constraint			
16	0.48 X1 + 0.57 * X2 <= 605000	605000	<=	605000	Gravel constraint			
17	X1 + X2 >= 1100000	1146666.667	>=	1100000	Total production			
18	x1 <= 540000	540000	<=	540000	Maximum excavation at Miramar			
19	x2 <= 690000	606666.6667	<=	690000	Maximum excavation from San Diego			

Best solution with the discounted cost of extracting material from San Diego. The sand constraint cannot be met. The best solution is to excavate 540,000 cu. meters from Miramar and 606,667 cu. meters from San Diego. The sand constraint is not met. Only 541,667 cu. meters of sand can be procured from both sides. The gravel constraint is met.

Problem 3

A company develops a sketch in two dimensions of a Linear Programming problem to minimize the cost of producing two types of commonly used steel rebars (called X_1 and X_2 in the follow up equations) used in the construction industry. The objective of the problem is to maximize the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.

Objective Maximize $Z = 340 X_1 + 326 X_2$

Subject to

$X_2 + 1.1 X_1 <= 270$

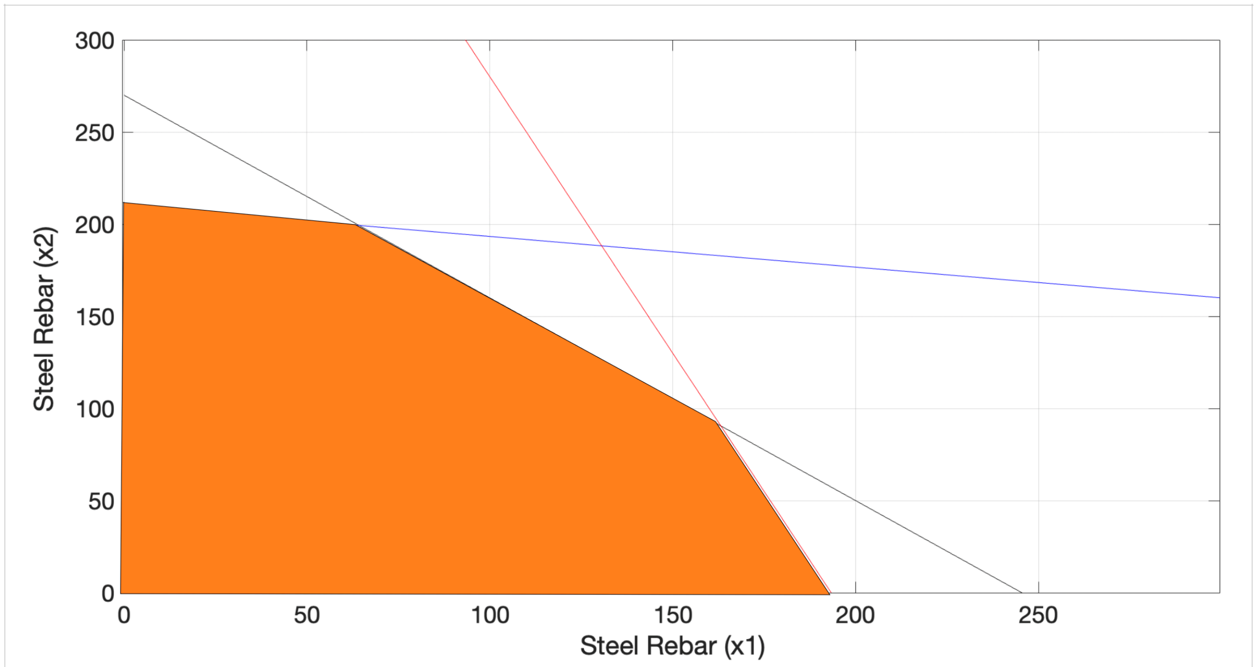
$X_1 + 6X_2 <= 1260$

$3X_1 + X_2 <= 580$

$X_1, X_2 >= 0$ (non-negativity conditions)

For each task below, use screen captures to show your work. Show the formulas of the cells to make out task simpler in grading. Also, show the Solver panel to help in grading.

a) Solve the **problem graphically**. State the optimal solution found for the two decision variables. State the value of the objective function for the optimal solution found. In the graphical solution, label the corner points and state the value of the objective function at each corner point.



Graphical Solution to LP Problem.

Table Corner Point Solutions to the Problem.

X1	X2	Z	Remark
0	0	0	
0	210	68,460	
64.3	199.3	86,833.8	Optimal Solution
163	81	81,826	
191	0	64,940	

b) Solve the **problem manually using the Simplex Method explained in class**. Show all your steps and tableaus. Indicate the Basic Variables (BV) and the Non Basic Variables (NBV) in every tableau. Also highlight the value of the objective function in every tableau.

Add three slack variables since there are three constraint equations with <= type constraints.

Table. Initial Tableau to Solve the Problem.

BV	Z	X1	x2	X3	X4	X5	RHS
Z	1	-340	-326	0	0	0	0
X3	0	1.1	1	1	0	0	270
X4	0	1	6	0	1	0	1260
X5	0	3	1	0	0	1	570

The basic variables are x3, x4 and x5. Non-basic variables are X1 and X2. Current solution for Z = 0.

BV	Z	X1	x2	X3	X4	X5	RHS	Ratio
Z	1	-340	-326	0	0	0	0	
X3	0	1.1	1	1	0	0	270	245.5
X4	0	1	6	0	1	0	1260	1260.0
X5	0	3	1	0	0	1	570	190.0

Step 2: Select pivot column (column with coefficient for X1 - in yellow).

Step 3: Select the pivot row as the one with the smallest ratio of RHS and picot column coefficients (row in yellow)

Step 4: Perform row operations to zero all elements of pivot column.

Table. Row operations in row X5.

BV	Z	X1	x2	X3	X4	X5	RHS
Z	1	-340	-326	0	0	0	0
X3	0	1.1	1	1	0	0	270
X4	0	1	6	0	1	0	1260
X5	0	1	1/3	0	0	1/3	190

Table Second Tableau of the Solution.

BV	Z	X1	x2	X3	X4	X5	RHS
Z	1	0	-212.67	0	0	113.33	64600
X3	0	0	0.63	1	0	-0.37	61
X4	0	0	5.67	0	1	-0.34	1070
X1	0	1	0.37	0	0	0.37	190

The basic variables are x3, x4 and x1. Non-basic variables are X5 and X2. Current solution for Z = 64,600. X1 = 190, x2 = 0, x3 = 61, x4=1070, and x5=0.

New pivot column is x2. New pivot row is x3.

BV	Z	X1	x2	X3	X4	X5	RHS	Ratio
Z	1	0	-212.67	0	0	113.33	64600	
X3	0	0	0.63	1	0	-0.37	61	96.8
X4	0	0	5.67	0	1	-0.34	1070	188.7
X1	0	1	0.37	0	0	0.37	190	513.5

Perform row operations on row x3.

BV	Z	X1	x2	X3	X4	X5	RHS
Z	1	0	-212.67	0	0	113.33	64600
X3	0	0	1	1.5873	0	-0.5873	96.8254
X4	0	0	5.67	0	1	-0.34	1070
X1	0	1	0.37	0	0	0.37	190

Table. Third Tableau of the Solution.

BV	Z	X1	x2	X3	X4	X5	RHS
Z	1	0	0	337.56	0	-11.57	85192
X2	0	0	1	1.5873	0	-0.5873	96.8254
X4	0	0	0	-9	1	3.0	521
X1	0	1	0	-0.59	0	0.59	154.17

The basic variables are x2, x4 and x1. Non-basic variables are X5 and X3. Current solution for Z = 85,192.

X1 = 154.17, x2 = 96.82, x3 = 0, x4=521, and x5=0.

The solution is not optimal because the value of coefficient for X5 in Z-row is negative. Need one more tableau. New pivot column is x5. New pivot row is x4 (lowest Nono-negative).

BV	Z	X1	x2	X3	X4	X5	RHS	Ratio
Z	1	0	0	337.56	0	-11.57	85192	
X2	0	0	1	1.5873	0	-0.5873	96.8254	-164.86531
X4	0	0	0	-9	1	3.0	521	173.666666
X1	0	1	0	-0.59	0	0.59	154.17	261.305084

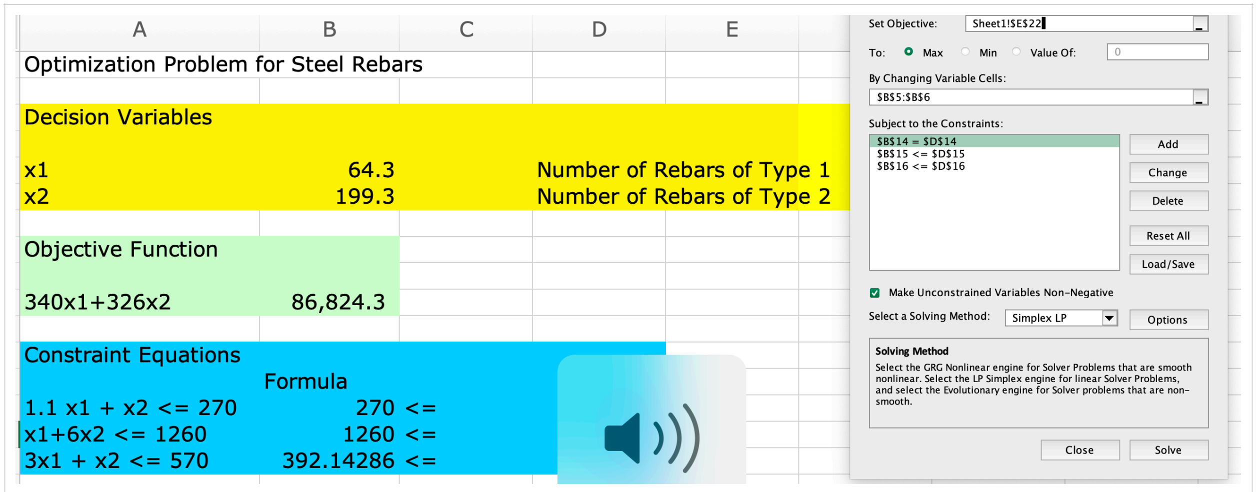
Perform row operations on row X4.

BV	Z	X1	x2	X3	X4	X5	RHS
Z	1	0	0	337.56	0	-11.57	85192
X2	0	0	1	1.5873	0	-0.5873	96.8254
X4	0	0	0	-3	0.334	1	173.67
X1	0	1	0	-0.59	0	0.59	154.17

BV	Z	X1	x2	X3	X4	X5	RHS
Z	1	0	0	302.85	3.8567	0	87201
X2	0	0	1	-0.1746	0.1958	0	198.81
X5	0	0	0	-3	0.334	1	173.67
X1	0	1	0	1.18	-0.1967	0	51.71

The basic variables are x2, x5 and x1. Non-basic variables are X4 and X3. Current solution for Z = 87,201. X1 = 51.71, x2 = 198.81, x3 = 0, x4=0, and x5=173.67.

The solution is optimal. The solution obtained by hand differs slightly from the Solver solution because I carried two significant figures in the calculations.



Excel Solution to LP Problem. The Excel Solver Panel is Shown on the Right.

d) Since number of steel rebars to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.