Assignment 6: Linear Programming

Solution

Problem 1

In the construction of a new airport a company requires a minimum of 600,000 of sand and gravel mixture. The final sand/gravel mixture must contain no less than 50.8% (305,000 cu. meters) of sand (fine aggregate) and no more than 53.3% (320,000 cu. meters) of gravel (coarse aggregate).

The gravel and sand materials can be obtained from three sites: 1) Longmont, b) Lyons, and c) Altona. Table 1 shows the proportions of sand and gravel from each site. Note that some unusable material is also included in the excavation process. Because each site is also used in other construction jobs, the maximum amounts of materials excavated from each site are limited to the following: a) 220,000 cu. meters for Longmont, 276,000 cu. meters for Lyons, and 256,000 cu. meters for Altona.

Table 1. Proportions of Sand and Gravel from Three Collection Sites.

Site	Proportion of Sand (%)	Proportion of Gravel (%)	Proportion of Unusable Material (%)
Longmont	46	50	4
Lyons	47	48	5
Altona	46	49	5

The costs of collection and transportation of a cubic meter of material are: a) $103 for Longmont, $110 for Lyons, and $107 for Altona.

Setup the problem as a linear programming problem. The objective is to **minimize the cost of producing the concrete for the airport project**.

Use the Simplex method to setup by hand the first two tableaus of the problem. For each table indicate the Basic Variables, Non-Basic Variables and the value of the objective function (Z).

Initial steps and problem setup (not the first tableau yet)

a) Add slack variables for each **<=** type constraint equation
b) Add a negative slack and an artificial variable for each **>=** constraint
c) Add a large positive number (Big M) to the artificial variables in the objective function.

The artificial variables are identified in boldface in the problem setup.

Let X_1, X_2, and X_3 be the amounts of material to be excavated from each site.

Step 1 – Problem formulation

Objective Function (Minimize)

$-Z+103X_1+110X_2+107X_3+M\mathbf{X}_5+M\mathbf{X}_8=0$

Constraint Equations

$0.46X_1+0.47X_2+0.46X_3-X_4+\mathbf{X}_5=305000$

$0.50X_1+0.48X_2+0.49X_3+X_6=320000$

$0.96X_1+0.95X_2+0.95X_3-X_7+\mathbf{X}_8=600000$

$X_1+X_9=220000$

$X_2+X_{10}=270000$

$X_3+X_{11}=256000$

The last three constraint equations limit the maximum excavation from each site.

Step 2 – Setup the Initial Tableau

Perform row operations in two constraint equations to eliminate the M coefficient from the objective function for two artificial variables $\mathbf{X}_5$ and $\mathbf{X}_8$. After this step the initial tableau is completed (see below).

Initial Tableau

BV	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	RHS
Z	-1	-1.42M+103	-1.42M+110	-1.41M+107	M			M					-905000M
X5		0.46	0.47	0.46	-1	1							305000
X6		0.5	0.48	0.49			1						320000
X8		0.96	0.95	0.95				-1	1				600000
X9		1								1			220000
X10			1								1		270000
X11				1								1	256000

Basic Variables are: X_5, X_6, X8, X_9, X_{10} and X_{11}. Non-basic variables are: X_1, X_2, X_3, X_4, and X_7.

X_1 enters the basis (BV set) and X_9 leaves,

Second Tableau

BV	Z	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	RHS
Z	-1	0	-1.42M+110	-1.41M+107	M			M		1.42M-103			-592600M-222660000
X5		0	0.47	0.46	-1	1				-0.46			203800
X6		0	0.48	0.49			1			-0.5			210000
X8		0	0.95	0.95				-1	1	-0.96			388800
X1		1								1			220000
X10			1								1		270000
X11				1								1	256000

Basic Variables are: X_5, X_6, X_8, X_1, X_{10} and X_{11}. Non-basic variables are: X_9, X_2, X_3, X_4, and X_7.

Find the optimal solution that minimizes the cost using **Excel Solver**. Clearly state the values of the decision variables and the value of the objective function in the optimal solution.

Concrete Mix Problem (Three Sites)							
Decision Variables are amounts collected from each site					Proportions		
					Sand	Gravel	Maximum
x1	124160		Longmont		0.46	0.5	220000
x2	276000		Lyons		0.47	0.48	276000
x3	256000		Altona		0.46	0.49	256000
Total excavated	656160						
Objective Function					Totals needed		
					Sand	Gravel	
103 * x1 + 110 * x2 + 107 * x3	70540480						
Constraint Equations	Formula						
0.46 X1 + 0.47 X2 + 0.46 X3 >= 305000	304593.6	>=	305000	Sand constraint			
0.5 X1 + 0.48 * X2 + 0.49 * X3 <= 320000	320000	<=	320000	Gravel constraint			
0.95 X1 + 0.96 * X2 + 0.95 * X3 >= 600000	624593.6	>=	600000	Total production (accounts for material not used at e			
x1 < 220000	124160	<=	220000	Maximum excavation from Longmont			
x2 < 276000	276000	<=	276000	Maximum excavation from Lyons			
x3 < 256000	256000	<=	256000	Maximum excavation from Altona			

Figure 1. Optimal solution found by Excel Solver. Note that the optimal solution shows a small shortage (deficit of 6 tons) compared to the Desired 305,000 cubic meters of sand material.

Problem 2

A company develops the following Linear Programming problem to minimize the cost of producing two types of commonly used doubler plates used in the construction industry. The objective of the problem is to maximize the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.

Objective Maximize $Z = 105 X_1 + 120 X_2$

Subject to

$X_2 + 1.7 X_1 <= 1300$

$-1.5 X_1 + X_2 <= 305$

$3 X_1 + X_2 <= 1800$

$X_1, X_2 >= 0$ (non-negativity conditions)

For each task below, use screen captures to show your work. Show the formulas of the cells to make out task simpler in grading. Also, show the Solver panel to help in grading.

Task 1

Solve the **problem graphically**. State the solution found for the two decision variables. State the value of the objective function for the optimal solution found. In the graphical solution, label the corner points and state the value of the objective function at each corner point.

Figure 2 shows a graphical solution to the problem.

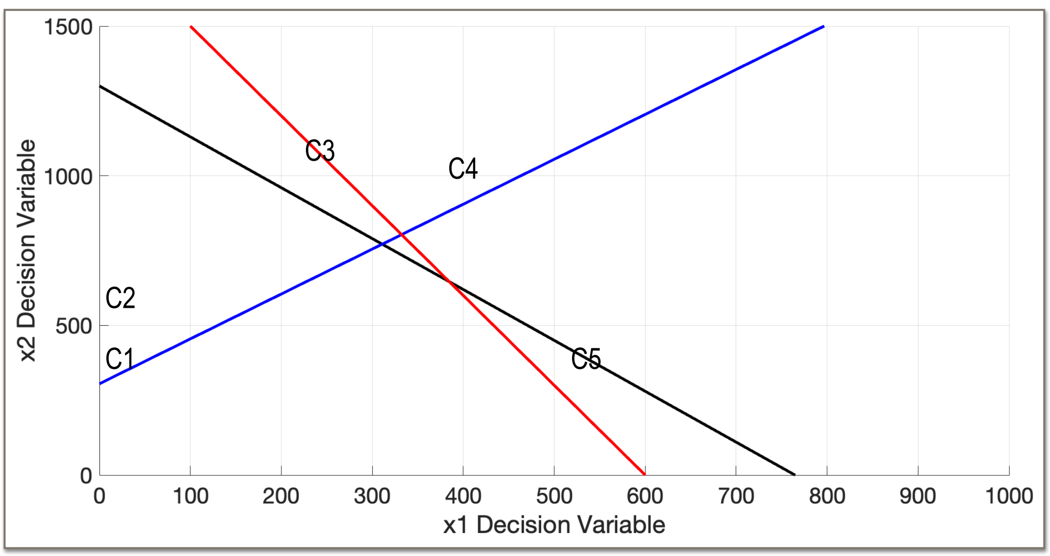


*Figure 2. Graphical solution to the problem. **Optimal solution** is: **x1 =310.9 and x2=771.4**. The value of the objective function at the optimal point is $125,217.2 (point C3 in the Figure).*

The corner points (points to be investigated) are as follows:

C1: Z= 0

C2: Z= 105*0 + 120*305=36,600

C3: Z= 105*310.9+120*771.4=125,217.2

C4: Z= 105*384.6+120*646.2=117,927

C5: Z= 105*300+120*0=31,500

Corner point 3 offers the highest value of Z (maximizes the value).

Task 2

Solve the **problem manually using the Simplex Method explained in class**. Show all your steps and tableaus. Indicate the Basic Variables (BV) and the Non-Basic Variables (NBV) in every tableau. Also highlight the value of the objective function in every tableau.

	A	B	C	D	E	F	G	H
1	BV	Z	X1	X2	X3	X4	X5	RHS
2	Z	1	-105	-120				0
3	X3		1.7	1	1			1300
4	X4		-1.5	1		1		305
5	X5		3	1			1	1800
6								
7	BV	Z	X1	X2	X3	X4	X5	RHS
8	Z	1	-285	0		120		36600
9	X3		3.2	0	1	-1		995
10	X2		-1.5	1		1		305
11	X5		4.5	0		-1	1	1495
12								
13	BV	Z	X1	X2	X3	X4	X5	RHS
14	Z	1	0	0	89.0625	30.9375		125217.188
15	X1		1	0	0.3125	-0.3125		310.9375
16	X2		0	1	0.46875	0.53125		771.40625
17	X5		0	0	-1.40625	0.40625	1	95.78125
18								

First tableau: BV: X3 X4 X5 NBV: X1 X2

Second tableau: X4 leaves BV New BV: X2

Third tableau: X3 leaves BV New BV: X1

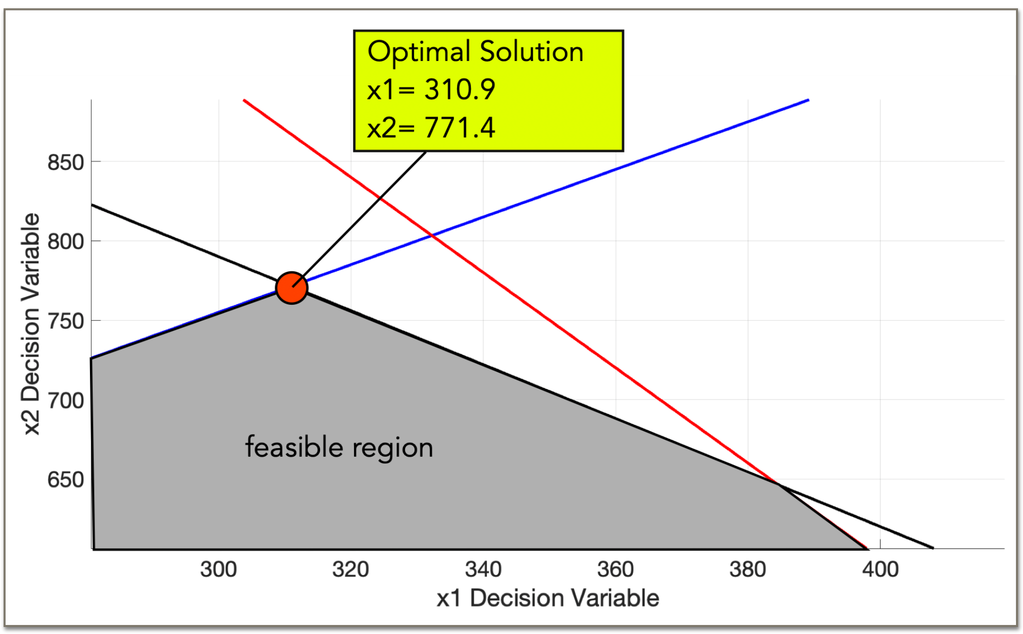
Task 3

Solve the problem using Excel Solver. State the solution found by Excel for the two decision variables.

Optimization Problem for company that produces doublers				
Decision Variables				
x1	310.9		Doublers of type A	
x2	771.4		Doublers of type B	
Objective Function				
105 x1 + 120 x2	125217.19			
Constraint Equations				
	Formula			
x2 + 1.7 x1 < 1300	1300	<=	1300	
(-1.5) x1 + x2 < 305	305	<=	305	
3 x1 + x2 < 1800	1704.2	<=	1800	

State the value of the objective function for the optimal solution found. Compare the Excel Solver solution

Optimal solution is: x1 =310.9 and x2=771.4. The value of the objective function at the optimal point is $125,217.2.



Task 3

Since number of doublers to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.

	A	B	C	D	E
1	Optimization Problem for company that produces doublers				
2	x1	310		Doublers of type A	
3	x2	770		Doublers of type B	
4					
5	Objective Function				
6					
7	105 x1 + 120 x2	124950			
8					
9	Constraint Equations				
10		Formula			
11	1.7 x1 + x2 < 1300	1297	<=	1300	
12	(-1.5) x1 + x2 < 305	305	<=	305	
13	3 x1 + x2 < 1800	1700	<=	1800	
14					

Figure 3. Integer solution to the problem. X1 = 310 and X2 = 770 units.

Problem 3

Solve the lake pollution control problem described in class with the following attributes:

Pollution Source	Loading (kg/year)	Unit Cost of Removal ($/kg)	Minimum Removal
River A	18,700	32	8,000
River B	19,400	34	7,500
River C	23,500	33	1/2 of the quantity removed from River B
Airport	25,600	48	1/2 of the quantity removed from River A
City	34,300	110 without treatment plant 35 with treatment plant	1/2 of City's original loading
Totals	121,500		

Task 1:

Formulate the problem as a linear programming problem to minimize the cost of pollution removal.

Task 2:

Solve the water pollution control problem if the total desired pollution removal is 60,000 kg. In solving the new problem, assume the city invested in new pollution treatment plant at a cost of $30,000,000. Find out the total cost of pollution removal for this task.

	A	B	C
1			
2	RiverA: x1	8000	
3	RiverB: x2	7500	
4	RiverC: x3	23350	
5	Airport: x4	4000	
6	City: x5	17150	
7			
8	32x1+34x2+33x3+48x4+110x5	3360050	
9			
10	x1>=8000	8000	8000
11	x2>=7500	7500	7500
12	x3>=0.5*x2	23350	3750
13	x4>=0.5*x1	4000	4000
14	x5>=0.5*34300	17150	17150
15	x1+x2+x3+x4+x5=60000	60000	60000
16	x1<=18700	8000	18700
17	x2<=19400	7500	19400
18	x3<=23500	23350	23500
19	x4<=25600	4000	25600
20	x5<=34300	17150	34300

	A	B	C
1			
2	RiverA: x1	8000	
3	RiverB: x2	7500	
4	RiverC: x3	23350	
5	Airport: x4	4000	
6	City: x5	17150	
7			
8	32x1+34x2+33x3+48x4+35x5	2073800	
9			
10	x1>=8000	8000	8000
11	x2>=7500	7500	7500
12	x3>=0.5*x2	23350	3750
13	x4>=0.5*x1	4000	4000
14	x5>=0.5*34300	17150	17150
15	x1+x2+x3+x4+x5=60000	60000	60000
16	x1<=18700	8000	18700
17	x2<=19400	7500	19400
18	x3<=23500	23350	23500
19	x4<=25600	4000	25600
20	x5<=34300	17150	34300

Task 3:

Assume the treatment plant life is 50 years. Estimate if the construction of such a facility is justified by comparing the solution of removal costs over the 50-year life cycle.

In fifty years we can save: 50*(3360050-273800)= \$64,312,500

Investment of treatment plant: \$30,000,000 < \$64,312,500

The construction of such a facility is justified.