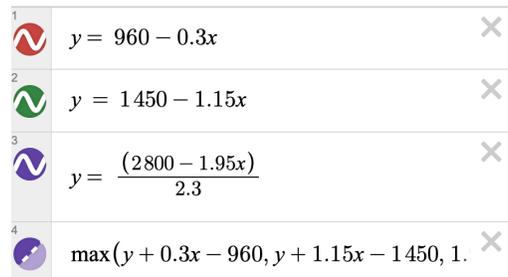


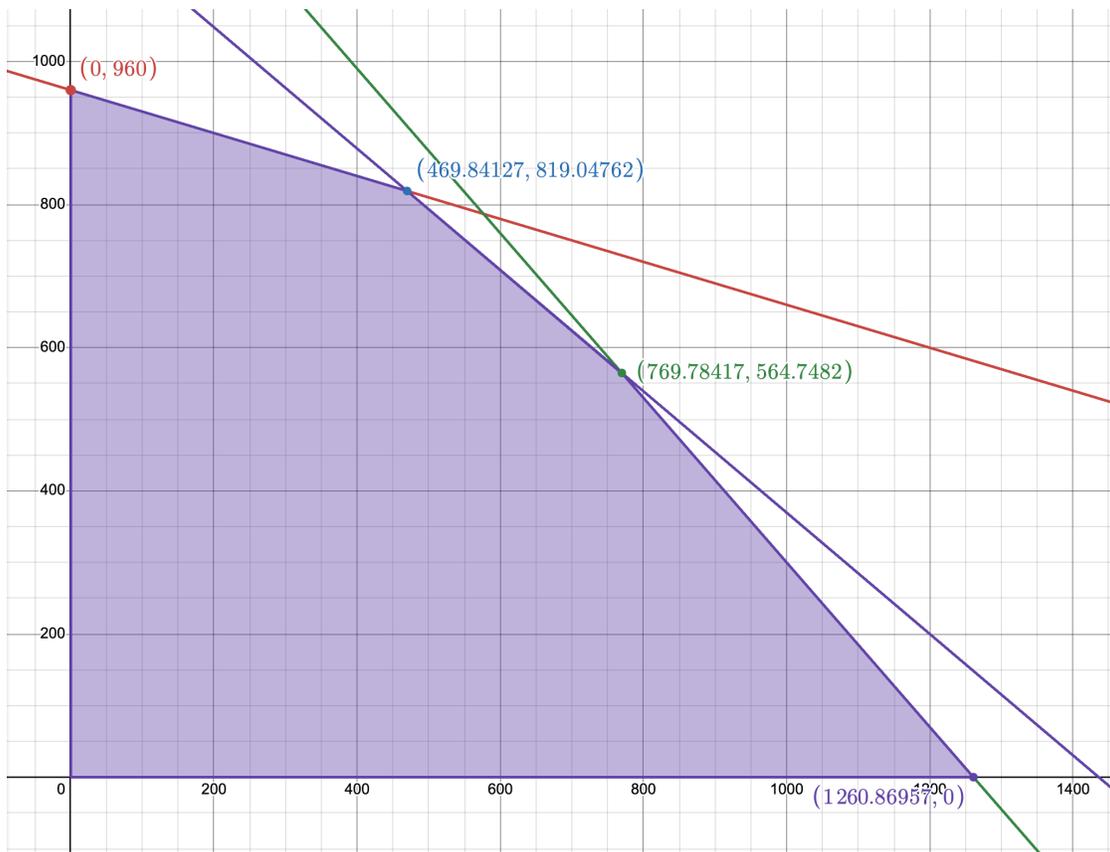
Problem 1

a) Solve the problem graphically. State the optimal solution in your plot. No sketches are acceptable. State the value of the objective function for the optimal solution found. In the graphical solution, label the corner points and state the objective function value at each point.

You can use Desmos, Excel, MATLAB, or another tool to plot the graph, and the optimal solution based on the graph is when $x_1 = 769.78$ and $x_2 = 564.75$.



To draw it, treat x_2 as variable y . And substitute the coordinates of the four corner points into the objective function to identify which one maximizes it.



b) Solve the problem by hand using the Simplex Method explained in class. Show all your steps and tableaus. Indicate the Basic Variables (BV) and the Non-Basic Variables (NBV) in every tableau. Also, highlight the objective function's value in each tableau.

Objective

$$\text{Maximize } Z = 1650x_1 + 1530x_2$$

Subject to

$$0.3x_1 + x_2 \leq 960$$

$$1.15x_1 + x_2 \leq 1450$$

$$1.95x_1 + 2.3x_2 \leq 2800$$

$$x_1, x_2 \geq 0$$

Step 1) Add a Slack variable to make \leq to $=$ constraints. Therefore, the standard formula will be:

$$Z - 1650x_1 - 1530x_2 = 0$$

$$0.3x_1 + x_2 + x_3 = 960$$

$$1.15x_1 + x_2 + x_4 = 1450$$

$$1.95x_1 + 2.3x_2 + x_5 = 2800$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Step 2) Make an initial tableau using the standard form and identify the NBV and BV

BV	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	-1650	-1530	0	0	0	0
x_3	0	0.3	1	1	0	0	960
x_4	0	1.15	1	0	1	0	1450
x_5	0	1.95	2.3	0	0	1	2800

BV = x_3, x_4, x_5 and NBV = x_1, x_2

Step 3) Find the pivot column/row. For the pivot column, we select the lowest value, and for the pivot row, we apply the ratio test and use the row with the lowest ratio.

- Pivot column

BV	Z	x ₁	x ₂	x ₃	x ₄	x ₅	RHS
Z	1	-1650	-1530	0	0	0	0
x ₃	0	0.3	1	1	0	0	960
x ₄	0	1.15	1	0	1	0	1450
x ₅	0	1.95	2.3	0	0	1	2800

- Pivot row

BV	Z	x ₁	x ₂	x ₃	x ₄	x ₅	RHS	Ratio (RHS/pivot column value)
Z	1	-1650	-1530	0	0	0	0	
x ₃	0	0.3	1	1	0	0	960	3200 (960/0.3)
x ₄	0	1.15	1	0	1	0	1450	1260.87 (1450/1.15)
x ₅	0	1.95	2.3	0	0	1	2800	1435.90 (2800/1.95)

Step 4) Since the value of the pivot column/row intersection is 1.15, we need to divide the pivot row by 1.15 to make it 1.

BV	Z	x ₁	x ₂	x ₃	x ₄	x ₅	RHS
Z	1	-1650	-1530	0	0	0	0
x ₃	0	0.3	1	1	0	0	960
x ₄	0	1	0.87 (1/1.15)	0	0.87 (1/1.15)	0	1260.87
x ₅	0	1.95	2.3	0	0	1	2800

Step 5) Do row operations to all rows except for the pivot row.

For Z row = x_4 row * 1650 + z

Z	1	0	-94.5 (0.87*1650-1530)	0	1435.5 (0.87*1650-0)	0	2,080,435.5
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After the row operation for Z

BV	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	0	-94.5	0	1435.5	0	2,080,435.5
x_3	0	0.3	1	1	0	0	960
x_4	0	1	0.87	0	0.87	0	1260.87
x_5	0	1.95	2.3	0	0	1	2800

Step 5-1) Do row operations to all rows except for the pivot row.

For x_5 row = x_4 row * (-1.95) + x_5

x_5	0	0	0.60	0	-1.70	1	341.30
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After the row operation for x_5

BV	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	0	-94.5	0	1435.5	0	2,080,435.5
x_3	0	0.3	1	1	0	0	960
x_4	0	1	0.87 (1/1.15)	0	0.87 (1/1.15)	0	1260.87
x_5	0	0	0.60	0	-1.70	1	341.30

Step 5-2) Do row operations to all rows except for the pivot row.

For x_3 row = x_4 row * (-0.3) + x_3

x_3	0	0	0.74	1	-0.26	0	581.74
-------	---	---	------	---	-------	---	--------

After the row operation for x_3

BV	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	0	-94.5	0	1435.5	0	2,080,435.5
x_3	0	0	0.74	1	-0.26	0	581.74
x_4	0	1	0.87 (1/1.15)	0	0.87 (1/1.15)	0	1260.87
x_5	0	0	0.60	0	-1.70	1	341.30

Step 6) Since we did a row operation for the pivot column, we need to check if it is an optimal solution or not. Since the coefficient of x_2 is still a negative value, we need another iteration of it.

Step 7) Make a second tableau and find the pivot column/row. For the pivot column, we select the lowest value, and for the pivot row, we apply the ratio test and use the row with the lowest ratio.

- Pivot column

BV	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	0	-94.5	0	1435.5	0	2,080,435.5
x_3	0	0	0.74	1	-0.26	0	581.74
x_1	0	1	0.87	0	0.87	0	1260.87
x_5	0	0	0.60	0	-1.70	1	341.30

BV = x_3, x_1, x_5 and NBV = x_2, x_4

- Pivot row

BV	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	0	-94.5	0	1435.5	0	2,080,435.5
x_3	0	0	0.74	1	-0.26	0	581.74
x_1	0	1	0.87	0	0.87	0	1260.87
x_5	0	0	0.60	0	-1.70	1	341.30

Step 8) Since the value of the pivot column/row intersection is 0.6, we need to divide the pivot row by 0.6 to make it 1.

BV	Z	x ₁	x ₂	x ₃	x ₄	x ₅	RHS
Z	1	0	-94.5	0	1435.5	0	2,080,435.5
x ₃	0	0	0.74	1	-0.26	0	581.74
x ₁	0	1	0.87	0	0.87	0	1260.87
x ₅	0	0	1	0	-2.83	1.67	568.83

Step 9) Do row operations to all rows except for the pivot row.

For Z row = x₅ row * 94.5 + z

Z	1	0	0	0	1168.07	157.82	2134189.94
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After the row operation for Z

BV	Z	x ₁	x ₂	x ₃	x ₄	x ₅	RHS
Z	1	0	0	0	1168.07	157.82	2134189.94
x ₃	0	0	0.74	1	-0.26	0	581.74
x ₁	0	1	0.87	0	0.87	0	1260.87
x ₅	0	0	1	0	-2.83	1.67	568.83

Step 9-1) Do row operations to all rows except for the pivot row.

For X₁ row = x₅ row * (-0.87) + x₁

x ₁	0	1	0	0	3.33	-1.45	765.99
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After the row operation for x₁

BV	Z	x ₁	x ₂	x ₃	x ₄	x ₅	RHS
Z	1	0	0	0	1167.03	157.50	2,134,190.25
x ₃	0	0	0.74	1	-0.26	0	581.74
x ₁	0	1	0	0	3.33	-1.45	765.99
x ₅	0	0	1	0	-2.83	1.67	568.83

Step 9-2) Do row operations to all rows except for the pivot row.

For X3 row = x5 row * (-0.74) + x3

x3	0	0	0	1	1.83	-1.25	160.81
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After the row operation for x3

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	0	0	0	1167.03	157.50	2,134,190.25
x3	0	0	0	1	1.83	-1.25	160.81
x1	0	1	0	0	3.33	-1.45	765.99
x2	0	0	1	0	-2.83	1.67	568.83

The BV: x3, x1, x2 and NVB: x4, x5

Step 10) Since we did a row operation for the pivot column, we need to check if the current solution is optimal solution or not. **Since there are no negative values for the Z row, we can say that it is an optimal solution.** The optimal solution is $x1 = 765.99$ and $x2 = 568.83$. When comparing it to the value obtained from the graphic, $x1 = 769.78$ and $x2 = 564.75$. They are slightly offset, but the values are quite close.

C) Solve the problem using Excel Solver. State the solution found by Excel for the two decision variables. State the value of the objective function for the optimal solution found. Compare the Excel Solver solution with the solution obtained manually in parts (A-B).

	A	B	C	D
1				
2		Decision Variable		
3	x1	769.78		
4	x2	564.75		
5				
6		Objective Fuction		
7	Max	2134208.63		
8				
9		Constraint Equations		
10	$0.3x_1 + x_2 \leq 960$	795.68 <=		960
11	$1.15x_1 + x_2 \leq 1450$	1450.00 <=		1450
12	$1.95x_1 + 2.3x_2 \leq 2800$	2800.00 <=		2800

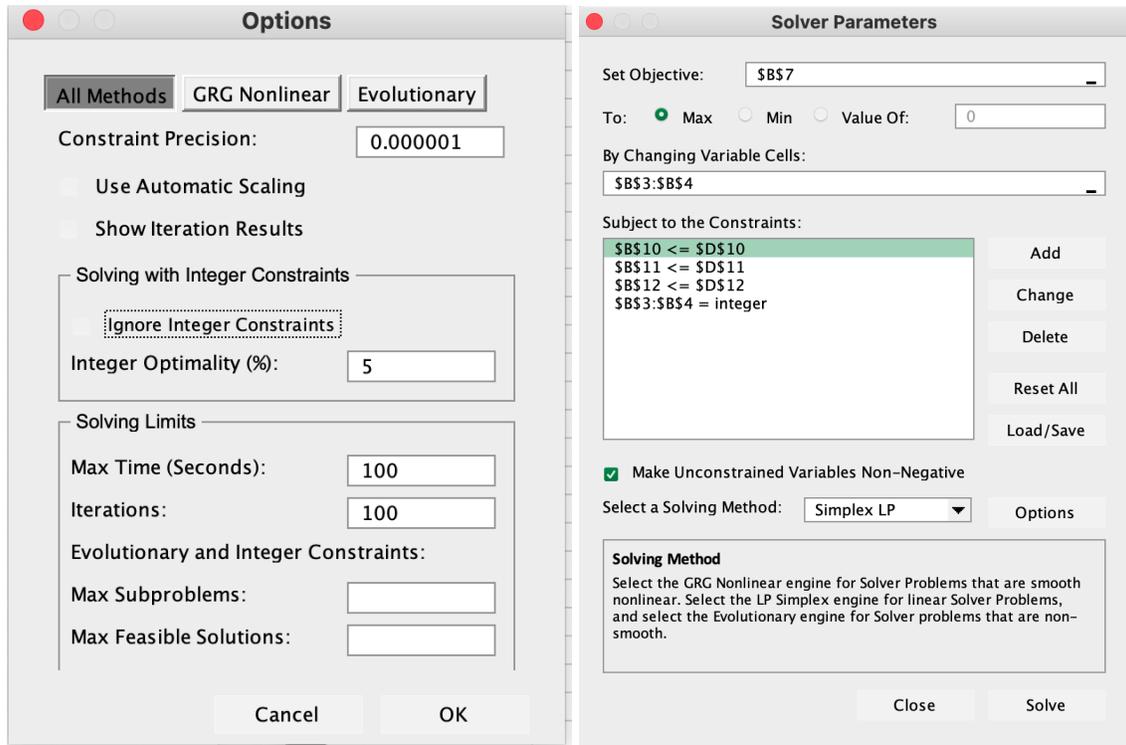
Method	x1	x2	Max
Graph	769.78	564.75	2134208.63
Hand calculation	765.99	568.83	2,134,190.25
Excel	769.78	564.75	2,134,208.63

The optimal values between the Graph/Excel are off by 18.38 (close enough in engineering calculations). The hand calculation considered two decimal places.

d) Since the number of beams to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.

The optimal value of the objective function with an integer solution is when $x_1 = 770$ and $x_2 = 564$.

Note: Please **uncheck** the 'Ignore integer constraints' in the options of the solver.



	A	B	C	D
1				
2		Decision Variable		
3	x1		770	
4	x2		564	
5				
6		Objective Fuction		
7	Max		2133420	
8				
9		Constraint Equations		
10	$0.3x_1 + x_2 \leq 960$		795 <=	960
11	$1.15x_1 + x_2 \leq 1450$		1450 <=	1450
12	$1.95x_1 + 2.3x_2 \leq 2800$		2799 <=	2800
13				

Problem 2

a) Set up the problem as a linear programming problem. The objective is to minimize excavation and transportation costs when procuring concrete for the airport project.

Objective

$$\text{Minimize } Z = 600x_1 + 565x_2 + 580x_3 + 375x_1 + 400x_2 + 390x_3 \text{ or}$$

$$\text{Minimize } Z = 975x_1 + 965x_2 + 970x_3$$

Subject to

$$x_1 + x_2 + x_3 \geq 565,000$$

$$0.42x_1 + 0.43x_2 + 0.54x_3 \geq 275,000$$

$$0.58x_1 + 0.57x_2 + 0.46x_3 \leq 300,000$$

$$x_1 \leq 223,000$$

$$x_2 \leq 256,000$$

$$x_3 \leq 243,000$$

$$x_1, x_2, x_3 \geq 0$$

b) Use the Simplex method to solve the first two tableaus of the problem by hand. For each table, indicate the Basic Variables, Non-Basic Variables, and the value of the objective function (Z).

Step 1) Add a Slack and Artificial variable to make \leq to $=$ constraints (called standard form). For the \geq constraint, we need to subtract the slack variable and add the artificial variable. For the \leq constraint, we only need to add the slack variable. Therefore, the formula will be:

Objective

$$\text{Maximize } -Z = -975x_1 - 965x_2 - 970x_3 \text{ or}$$

$$\text{Maximize } -Z + 975x_1 + 965x_2 + 970x_3 + M(a_1 + a_2) = 0$$

Subject to

$$x_1 + x_2 + x_3 - s_1 + a_1 = 565,000$$

$$0.42x_1 + 0.43x_2 + 0.54x_3 - s_2 + a_2 = 275,000$$

$$0.58x_1 + 0.57x_2 + 0.46x_3 + s_3 = 300,000$$

$$x_1 + s_4 = 223,000$$

$$x_2 + s_5 = 256,000$$

$$x_3 + s_6 = 243,000$$

$$x_1, x_2, x_3 \geq 0$$

Where s_n are Slack Variables and a_n are Artificial Variables

Step 2) Make an initial tableau with the Big M method

BV	z	x1	x2	x3	a1	a2	s1	s2	s3	s4	s5	s6	RHS
z	-1	975	965	970	M	M	0	0	0	0	0	0	0
-M	a1	0	1	1	1	1	0	-1	0	0	0	0	565000
-M	a2	0	0.42	0.43	0.54	0	1	0	-1	0	0	0	275000
s3	0	0.58	0.57	0.46	0	0	0	0	0	1	0	0	300000
s4	0	1	0	0	0	0	0	0	0	0	1	0	223000
s5	0	0	1	0	0	0	0	0	0	0	0	1	256000
s6	0	0	0	1	0	0	0	0	0	0	0	1	243000

Step 3) Perform row operations for z row to eliminate M from the artificial data.

BV	z	x1	x2	x3	a1	a2	s1	s2	s3	s4	s5	s6	RHS
z	-1	975	965	970	M	M	0	0	0	0	0	0	0
a1	0	1	1	1	1	1	0	-1	0	0	0	0	565000
a2	0	0.42	0.43	0.54	0	1	0	-1	0	0	0	0	275000

new z row = z - M * (a1) - M *(a2)

BV	z	x1	x2	x3	a1	a2	s1	s2	s3	s4	s5	s6	RHS
z	-1	(-1.42M+975)	(-1.43M+965)	(-1.54M+970)	0	0	*M*	*M*	0	0	0	0	*-840000M*
a1	0	1	1	1	1	1	0	-1	0	0	0	0	565000
a2	0	0.42	0.43	0.53	0	1	0	-1	0	0	0	0	275000
s3	0	0.58	0.57	0.46	0	0	0	0	0	1	0	0	300000
s4	0	1	0	0	0	0	0	0	0	0	1	0	223000
s5	0	0	1	0	0	0	0	0	0	0	0	1	256000
s6	0	0	0	1	0	0	0	0	0	0	0	1	243000

BV	z	x1	x2	x3	a1	a2	s1	s2	s3	s4	s5	s6	RHS	Ratio
z	-1	(-1.42M+975)	(-1.43M+965)	(-1.54M+970)	0	0	*M*	*M*	0	0	0	0	*-840000M*	
a1	0	1	1	1	1	1	0	-1	0	0	0	0	565000	565000
a2	0	0.42	0.43	0.53	0	1	0	-1	0	0	0	0	275000	518867.9245
s3	0	0.58	0.57	0.46	0	0	0	0	0	1	0	0	300000	652173.913
s4	0	1	0	0	0	0	0	0	0	0	1	0	223000	Inf
s5	0	0	1	0	0	0	0	0	0	0	0	1	256000	Inf
s6	0	0	0	1	0	0	0	0	0	0	0	1	243000	243000

Step 4) Do row operations to all rows except for the pivot row. We expect to see the row operation for each row in this step.

For Z row = $s_6 * (-1.54M + 970) + z$
For a1 row = $s_6 * (-1) + z$
For a2 row = $s_6 * (-0.54) + z$
For s3 row = $s_6 * (-0.46) + z$
For s4 row = $s_6 * (0) + z$
For s5 row = $s_6 * (0) + z$
For s6 row = $s_6 * (-1) + z$

- **Second Tableau**

BV	z	x1	x2	x3	a1	a2	s1	s2	s3	s4	s5	s6	RHS	
z	-1	(-1.42M+975)	(-1.43M+965)	0	0	0	"M"	"M"	0	0	0	0	-1.54M+970	-465,780M - 235,710,000
a1	0	1	0	0	1	0	0	-1	0	0	0	0	-1	322,000
a2	0	0	0.42	0.43	0	0	1	0	-1	0	0	0	-0.54	143,780
s3	0	0	0.58	0.57	0	0	0	0	0	1	0	0	-0.46	188,220
s4	0	0	1	0	0	0	0	0	0	0	1	0	0	223,000
s5	0	0	0	1	0	0	0	0	0	0	0	1	0	256,000
x3	0	0	0	0	1	0	0	0	0	0	0	0	1	243,000

c) Find the optimal solution that minimizes the total transportation and excavation cost using Excel Solver. Clearly state the values of the decision variables and the optimal objective function value.

The values of the decision variables are:

$$X_1 = 68,333, X_2 = 256,000, X_3 = 243,000$$

The optimal objective function value is 549,375,000

	A	B	C	D	E	F	G	H	I	J	K
1											
2	Variables	X1	Cave Spring								
3		X2	Penn Forest								
4		X3	Poeges Mill								
5											
6	Optimal	X1	68333								
7		X2	256000								
8		X3	243000								
9											
10		Minimization									
11	Objective	600*C6+565*C7+580*C8+375*C6+400*C7+390*C8		549,375,000	\$/yr						
12											
13											
14	Subject to	x1+x2+x3 >= 565,000		567333.333	>=	565000					
15		0.42x1+0.43x2+0.54x3 >= 275,000		270000	>=	270000					
16		0.58x1+0.57x2+0.46x3 <= 300,000		297333.333	<=	300000					
17		x1 <= 223,000		68333.3333	<=	223000					
18		x2 <= 256,000		256000	<=	256000					
19		x3 <= 243,000		243000	<=	243000					
20											
21											
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29											
30											
31											
32											
33											
34											

Solver Parameters

Set Objective:

To: Max Min Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- \$C\$14 >= \$E\$14
- \$C\$15 >= \$E\$15
- \$C\$16 <= \$E\$16
- \$C\$17 <= \$E\$17
- \$C\$18 <= \$E\$18
- \$C\$19 <= \$E\$19

Make Unconstrained Variables Non-Negative

Select a Solving Method: Options

Solving Method
 Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

	A	B	C	D	E
1					
2	Variables	X1	Cave Spring		
3		X2	Penn Forest		
4		X3	Poeges Mill		
5					
6	Optimal	X1	68333		
7		X2	256000		
8		X3	243000		
9					
10		Minimization			
11	Objective	600*C6+565*C7+580*C8+375*C6+400*C7+390*C8		549,375,000	\$/yr
12					
13					
14	Subject to	x1+x2+x3 >= 565,000		567333.333	>= 565000
15		0.42x1+0.43x2+0.54x3 >= 275,000		270000	>= 270000
16		0.58x1+0.57x2+0.46x3 <= 300,000		297333.333	<= 300000
17		x1 <= 223,000		68333.3333	<= 223000
18		x2 <= 256,000		256000	<= 256000
19		x3 <= 243,000		243000	<= 243000
20					

Problem 3

a) Formulate the problem as a linear programming problem to minimize the cost of pollution removal.

Objective

$$\text{Maximize } Z = 305x_1 + 320x_2 + 310x_3 + 405x_4 + 630x_5 + 245x_6$$

Subject to

$$x_1 \geq \frac{1}{3} \times 22,400$$

$$x_2 \geq \frac{1}{3} \times 32,400$$

$$x_3 \geq \frac{1}{3} \times 23,500$$

$$x_4 \geq \frac{1}{2} \times 24,600$$

$$x_5 \geq \frac{1}{2} \times 38,300 \times (1 - y)$$

$$x_6 \geq \frac{1}{2} \times 38,300 \times y$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 70,000 \text{ (or } = 70,000)$$

$$x_1 \leq 22,400$$

$$x_2 \leq 32,400$$

$$x_3 \leq 23,500$$

$$x_4 \leq 24,600$$

$$x_5 \leq 38,300 \times (1 - y)$$

$$x_6 \leq 38,300 \times y$$

$$y \in \{0, 1\} - \text{optional}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

b) Set up the problem in standard form, including slack and artificial variables and the penalty in the objective function. Do not solve.

Objective

$$\text{Maximize } -Z = -305x_1 - 320x_2 - 310x_3 - 405x_4 - 630x_5 - 245x_6 \text{ or}$$

$$\text{Maximize } -Z + 305x_1 + 320x_2 + 310x_3 + 405x_4 + 630x_5 + 245x_6 + M(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7) = 0$$

Subject to

$$x_1 - s_1 + a_1 = \frac{1}{3} \times 22,400$$

$$x_2 - s_2 + a_2 = \frac{1}{3} \times 32,400$$

$$x_3 - s_3 + a_3 = \frac{1}{3} \times 23,500$$

$$x_4 - s_4 + a_4 = \frac{1}{2} \times 24,600$$

$$x_5 - s_5 + a_5 = \frac{1}{2} \times 38,300 \times (1 - y)$$

$$x_6 - s_6 + a_6 = \frac{1}{2} \times 38,300 \times y$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 - s_7 + a_7 = 70,000$$

$$x_1 + s_8 = 22,400$$

$$x_2 + s_9 = 32,400$$

$$x_3 + s_{10} = 23,500$$

$$x_4 + s_{11} = 24,600$$

$$x_5 + s_{12} - 38,300 \times (1 - y) = 0$$

$$x_6 + s_{11} - 38,300 \times y = 0$$

$$y \in \{0, 1\}$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Where s_n are Slack Variables and a_n are Artificial Variables

C) Solve the water pollution control problem if the total desired pollution removal is now 70,000 kg. Find out the total cost of pollution removal per year without a water treatment plant.

The cost of pollution removal will be \$29,004,917.

We can adjust the amount of total desired pollution removal by changing the constraint below.

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70000$	70000 =	70000
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	A	B	C	D	E	F	G	H	I	J	K	L
1												
2	Variables	X1	River A									
3		X2	River B									
4		X3	River C									
5		X4	Airport									
6		X5	City w/o treatment plan									
7		X6	City w treatment plan									
8												
9	Optimal	X1		19917								
10		X2		10800								
11		X3		7833.33333								
12		X4		12300								
13		X5		19150								
14		X6		0								
15												
16												
17		Minimization										
18	Objective	305*C9+320*C10+310*C11+405*C12+630*C13+245*C14		29,004,917 \$/yr								
19												
20												
21	Subject to	x1 >= 22400*1/3		19917 >=	7466.66667							
22		x2 >= 32400*1/3		10800 >=	10800							
23		x3 >= 23500*1/3		7833.33333 >=	7833.33333							
24		x4 >= 24600*1/3		12300 >=	12300							
25		x5 >= 38300*1/2		19150 >=	19150							
26		x6 >= 38300*1/2		0 >=	0							
27		x1+x2+x3+x4+x5+x6 = 70000		70000 =	70000							
28		x1 <= 22400		19917 <=	22400							
29		x2 <= 32400		10800 <=	32400							
30		x3 <= 23500		7833.33333 <=	23500							
31		x4 <= 24600		12300 <=	24600							
32		x5 <= 38300		19150 <=	38300							
33		x6 <= 38300		0 <=	0							
34												
35												
36												
37		y (y=1 when city installs treatment plant, y=0 otherwise)		0								
38												

Solver Parameters

Set Objective:

To: Max Min Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- \$C\$21 >= \$E\$21 Add
- \$C\$22 >= \$E\$22 Change
- \$C\$23 >= \$E\$23 Delete
- \$C\$24 >= \$E\$24 Reset All
- \$C\$25 >= \$E\$25 Load/Save
- \$C\$27 >= \$E\$27
- \$C\$28 <= \$E\$28
- \$C\$29 <= \$E\$29
- \$C\$30 <= \$E\$30
- \$C\$31 <= \$E\$31
- \$C\$32 <= \$E\$32

Make Unconstrained Variables Non-Negative

Select a Solving Method: Options

Solving Method
 Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

	A	B	C	D	E
1					
2	Variables	X1	River A		
3		X2	River B		
4		X3	River C		
5		X4	Airport		
6		X5	City wo treatment plan		
7		X6	City w treatment plan		
8					
9	Optimal	X1	19917		
10		X2	10800		
11		X3	7833.33333		
12		X4	12300		
13		X5	19150		
14		X6	0		
15					
16					
17		Minimization			
18	Objective	$305 * C9 + 320 * C10 + 310 * C11 + 405 * C12 + 630 * C13 + 245 * C14$	29,004,917	\$/yr	
19					
20					
21	Subject to	$x1 \geq 22400 * 1/3$	19917	\geq	7466.66667
22		$x2 \geq 32400 * 1/3$	10800	\geq	10800
23		$x3 \geq 23500 * 1/3$	7833.33333	\geq	7833.33333
24		$x4 \geq 24600 * 1/3$	12300	\geq	12300
25		$x5 \geq 38300 * 1/2$	19150	\geq	19150
26		$x6 \geq 38300 * 1/2$	0	\geq	0
27		$x1 + x2 + x3 + x4 + x5 + x6 = 70000$	70000	$=$	70000
28		$x1 \leq 22400$	19917	\leq	22400
29		$x2 \leq 32400$	10800	\leq	32400
30		$x3 \leq 23500$	7833.33333	\leq	23500
31		$x4 \leq 24600$	12300	\leq	24600
32		$x5 \leq 38300$	19150	\leq	38300
33		$x6 \leq 38300$	0	\leq	0
34					
35					
36					
37		y (y=1 when city installs treatment plant, y=0 otherwise)	0		
38					

D) Solve the water pollution control problem if the total desired pollution removal is now 70,000 kg. In solving the new problem, assume the city invested \$45,000,000 in a new pollution treatment plant. Find out the total cost of pollution removal (per year) for this task.

$\$65,885,167 = \$20,885,167$ (minimum cost when operating new pollution treatment plant) + $\$45,000,000$ (cost to build a new pollution treatment plant)

We can choose to install the treatment plant or not, using the binary constraint below (optional)

y (y=1 when city installs treatment plant, y=0 otherwise)	1
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	A	B	C	D	E
1					
2	Variables	X1	River A		
3		X2	River B		
4		X3	River C		
5		X4	Airport		
6		X5	City wo treatment plan		
7		X6	City w treatment plan		
8					
9	Optimal	X1	7467		
10		X2	10800		
11		X3	7833.33333		
12		X4	12300		
13		X5	0		
14		X6	31600		
15					
16					
17		Minimization			
18	Objective	$305 * C9 + 320 * C10 + 310 * C11 + 405 * C12 + 630 * C13 + 245 * C14$	20,885,167 \$/yr		
19					
20					
21	Subject to	$x1 \geq 22400 * 1/3$	7467 \geq		7466.66667
22		$x2 \geq 32400 * 1/3$	10800 \geq		10800
23		$x3 \geq 23500 * 1/3$	7833.33333 \geq		7833.33333
24		$x4 \geq 24600 * 1/3$	12300 \geq		12300
25		$x5 \geq 38300 * 1/2$	0 \geq		0
26		$x6 \geq 38300 * 1/2$	31600 \geq		19150
27		$x1 + x2 + x3 + x4 + x5 + x6 = 70000$	70000 =		70000
28		$x1 \leq 22400$	7467 \leq		22400
29		$x2 \leq 32400$	10800 \leq		32400
30		$x3 \leq 23500$	7833.33333 \leq		23500
31		$x4 \leq 24600$	12300 \leq		24600
32		$x5 \leq 38300$	0 \leq		0
33		$x6 \leq 38300$	31600 \leq		38300
34					
35					
36					
37		y (y=1 when city installs treatment plant, y=0 otherwise)	1		

e) Assume the treatment plant's life is 60 years. Estimate if the construction of such a facility is justified by comparing the solution of removal costs over the 60-year life cycle

Cost when the treatment plant is not installed - Cost when the city installs the treatment plant) * 60

$$= (29,004,917 - 20,885,167) * 60 = \$487,185,000$$

\$ 45,000,000 (installation cost) < \$487,185,000

Yes, it is justified because the saving cost is way higher than the installation cost.

Problem 4

A) How did Dr. Dantzig get involved in the development of the Simplex Method?

Dantzig solved the Simplex method after arriving late to a graduate school class. "With the outbreak of World War II, Dantzig took a leave of absence from the doctoral program at Berkeley to work as a civilian for the United States Army Air Forces. Dantzig was asked to work out a method the Air Force could use to improve their planning process."

B) Which movie used the urban legend of George Dantzig?

- Good Will Hunting

C) Name three universities that employed George Dantzig.

-Stanford University, University of California, Berkeley, University of Michigan, University of Maryland

D) State three examples used in industry that use George Dantzig's methods.

1. An airline industry for crew scheduling and fleet assignments,
2. An oil industry for raw product
3. manufacturing, revenue management, telecommunications, advertising, architecture, circuit design, and countless other areas.