

## Assignment 5: Linear Programming and Optimization

Date Due: March 5, 2026.

Instructor: Trani

### Problem 1

A company develops the following Linear Programming problem to estimate the cost of producing two types of concrete beams commonly used in the construction industry. The objective of the problem is to maximize the company's profit (in dollars per production batch). The company would like to maximize profit in solving this problem.

**Objective**            Maximize  $Z = 1650x_1 + 1530x_2$

**Subject to**

$$0.3x_1 + x_2 \leq 960$$

$$1.15x_1 + x_2 \leq 1450$$

$$1.95x_1 + 2.3x_2 \leq 2800$$

$$x_1 \geq 0 \text{ and } x_2 \geq 0 \quad (\text{non-negativity conditions})$$

For each item below, use screen captures to show your work. Show the formulas for the cells to make grading easier. Also, show the Solver panel to help in grading.

**A)** Solve the **problem graphically**. State the optimal solution in your plot. **No sketches are acceptable**. State the value of the objective function for the optimal solution found. In the graphical solution, label the corner points and state the objective function value at each point.

**B)** Solve the **problem by hand using the Simplex Method explained in class**. Show all your steps and tableaus. Indicate the Basic Variables (BV) and the Non-Basic Variables (NBV) in every tableau. Also, highlight the objective function's value in each tableau.

**C)** Solve the problem using Excel Solver. State the solution found by Excel for the two decision variables. State the value of the objective function for the optimal solution found. Compare the Excel Solver solution with the solution obtained manually in parts (A-B).

**D)** Since the number of beams to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.

### Problem 2

In constructing a new concrete runway at an airport, a company requires a minimum of 565,000 cubic meters of sand-and-gravel mixture. The final sand/gravel mixture must contain no more than 300,000 cu. meters of gravel and no less than 275,000 cu. meters of sand.

The gravel and sand materials can be obtained from three sites: 1) Cave Spring, 2) Penn Forest, and 3) Poeges Mill. Table 1 shows the average proportions of sand and gravel extracted at each site. Because each site is also used for other construction jobs, the maximum amount of material excavated from each site is limited to the following: a) 223,000 cu. meters for Cave Spring, 256,000 cu. meters for Penn Forest,

and 243,000 cu. meters for Poeges Mill. The costs of excavation and transportation of the material to the airport construction site are shown in Table 1.

Table 1. Proportions of Sand and Gravel from Three Collection Sites. The Last Two Columns Show the Excavation and Transportation Costs for Each Site to the Airport Construction Site.

Site	Proportion of Sand (%)	Proportion of Gravel (%)	Excavation Cost (\$/cu.meter)	Transportation Cost (\$/cu.meter)
Cave Spring	42	58	600	375
Penn Forest	43	57	565	400
Poeges Mill	54	46	580	390

- Set up the problem as a linear programming problem. The objective is to minimize excavation and transportation costs when procuring concrete for the airport project.
- Use the Simplex method to solve the **first two tableaus of the problem by hand**. For each table, indicate the Basic Variables, Non-Basic Variables, and the value of the objective function (Z).
- Find the optimal solution that minimizes the total transportation and excavation cost using **Excel Solver**. Clearly state the values of the decision variables and the optimal objective function value.

### Problem 3

Solve the lake pollution control problem described in class with the following attributes:

Pollution Source	Pollution Loading (kg/year)	Unit Cost of Removal (\$/kg)	Minimum Removal
River A	22,400	305	1/3 of the annual pollution loading
River B	32,400	320	1/3 of the annual pollution loading
River C	23,500	310	1/3 of the annual pollution loading
Airport	24,600	405	1/2 of the annual loading
City	38,300	630 without a treatment plant 245 with treatment plant	1/2 of the city's original pollution loading
Totals	141,200		

- Formulate the problem as a linear programming problem to minimize the cost of pollution removal.
- Set up the problem in **standard form, including slack and artificial variables and the penalty in the objective function**. Do not solve.
- Solve the water pollution control problem if the total desired pollution removal is now 70,000 kg. Find out the total cost of pollution removal per year without a water treatment plant.

D) Solve the water pollution control problem if the total desired pollution removal is now 70,000 kg. In solving the new problem, assume the city invested \$45,000,000 in a new pollution treatment plant. Find out the total cost of pollution removal (per year) for this task.

E) Assume the treatment plant's life is 60 years. Estimate if the construction of such a facility is justified by comparing the solution of removal costs over the 60-year life cycle.

## **Problem 4**

Read the article about George Dantzig. [https://en.wikipedia.org/wiki/George\\_Dantzig](https://en.wikipedia.org/wiki/George_Dantzig) and answer the following questions:

- A) How did Dr. Dantzig get involved in the development of the Simplex Method?
- B) Which movie used the urban legend of George Dantzig?
- C) Name three universities that employed George Dantzig.
- D) State three examples used in industry that use George Dantzig's methods.