

## Assignment 5: Matlab Basics

Date Due: March 15, 2015

Instructor: Trani

### Problem 1

This problem deals with a beam supported at both ends and with a single load  $W$  at a known location. The beam is shown in Figure 1.

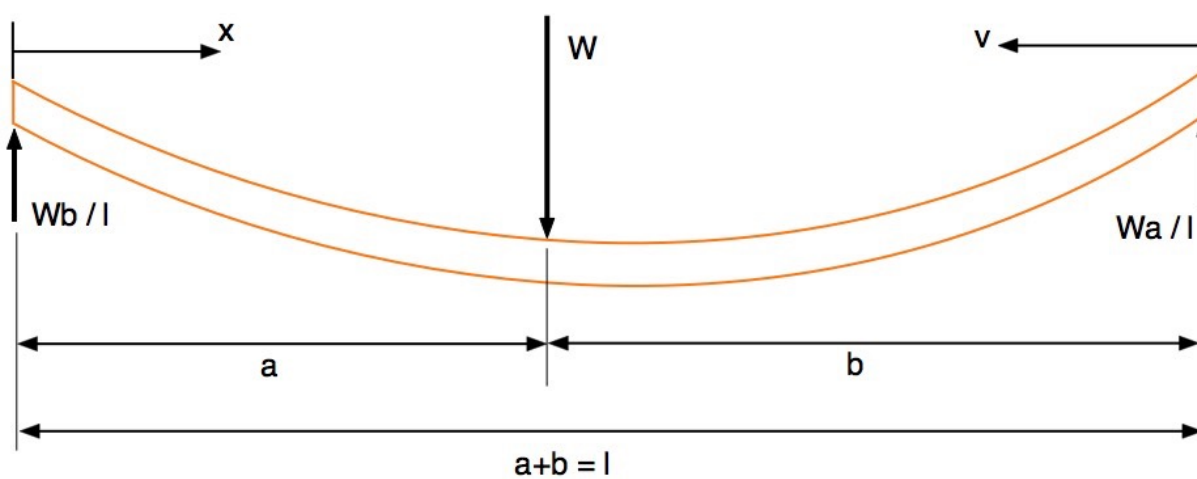


Figure 1. A simple beam supported at both ends with a load  $W$  at some known location. Adapted from: [http://www.engineersedge.com/beam\\_bending/beam\\_bending3.htm](http://www.engineersedge.com/beam_bending/beam_bending3.htm).

Nomenclature for beam deflection and stress calculation equations.

$W$  = load (N)

$E$  = Modulus of elasticity ( $N/m^2$ )

$I$  = Moment of inertia ( $m^4$ )

$x$  = Distance from left side of the beam to the loading point (m)

$v$  = Distance from right hand side beam end point to the loading point (m)

$l$  = beam length (m)

$a, b$  = distances from each beam end point towards the loading point (m)

Let:

$$y_a = \frac{-Wbx}{6EI}(l^2 - x^2 - b^2)$$

$$y_b = \frac{-Wav}{6EI}(l^2 - v^2 - a^2)$$

where:

$y_a$  = deflection of the beam from left datum point (section a) in meters

$y_b$  = deflection of the beam from right hand side datum point (section b) in meters

### Task 1:

Create a Matlab Script to estimate the deflections of the beam ( $y_a$  and  $y_b$ ) as a function of known quantities  $W$ ,  $E$ ,  $I$ ,  $l$ ,  $a$ , and  $b$ . Your script should take as inputs the following beam parameters:  $W$ ,  $E$ ,  $I$ ,  $l$ ,  $a$  and  $b$ . These values will be included in the script directly.

The output of the script consists of vectors with the values of beam deflections ( $y_a$  and  $y_b$ ) and their stations (values of  $x$  and  $v$ ). The values of ( $y_a$  and  $y_b$ ) should be calculated every 0.1 meter along the beam using incremental values of  $x$  and  $v$ . Your script should also plot the deflections ( $y_a$  and  $y_b$ ) as a function of distances ( $x$  and  $v$ ).

### Task 2:

Test your Matlab Script using the following values for the beam model parameters. Assume a steel beam is 8 meters long and has the following parameters.

$W = 6000$  N

$E = 200e9$  N/m-m

$I = 0.001$  m<sup>4</sup>

$l = 8$  meters

$a = 3.5$  meters

$b = 4.5$  meters

Plot the results of the beam deflections as a continuous distance (from the left datum of the beam) and verify that the deflections make sense. Note that I added a negative sign to the deflection equations to show the deflections downwards.

### Task 3:

Add to the script created in Task 1 to estimate the reactions on both ends of the beam. Write the results back to the command window with labels.

Show all your screen captures of the output and the Matlab scripts.

## Problem 2

A civil engineer is designing an acceleration ramp on a highway using data provided by a truck manufacturer. The truck manufacturer provides solutions to the equations of motion to predict the velocity and distance profiles of the truck as a function of time. The equations are shown below.

The velocity profile is,

$$V_t = \frac{k_1}{k_2}(1 - e^{-k_2 t}) + V_0 e^{-k_2 t}$$

The distance traveled (S) is,

$$S_t = \frac{k_1}{k_2} t - \frac{k_1}{k_2^2}(1 - e^{-k_2 t}) + \frac{V_0}{k_2}(1 - e^{-k_2 t})$$

where:

$V_t$  = is the velocity (m/s) of the truck as a function of time (t)

$V_0$  = is the initial velocity of the truck (m/s)

$k_1$  = is an acceleration constant (m/s<sup>2</sup>)

$k_2$  = is another acceleration constant (1/s)

$S_t$  = is the distance traveled by the truck (m)

$t$  = is the time (seconds)

### Task 1:

Create a Matlab script to estimate the velocity and distance profiles ( $V_t$  and  $S_t$ ) for any time ( $t$ ), initial velocity, and truck acceleration constants ( $k_1$  and  $k_2$ ). Test your script with the following parameters:

$$V_0 = 0 \text{ m/s}$$

$$k_1 = 1.0 \text{ (m/s}^2\text{)}$$

$$k_2 = 0.028 \text{ (1/s)}$$

$$t = 60 \text{ (seconds)}$$

### Task 2:

Use the Matlab script created in Task 1 to examine the sensitivity of  $V_t$  and  $S_t$  with time ( $t$ ). Create solutions of velocity and distance traveled as a function of time ( $t$ ) starting at time = 0 and ending at 90 seconds (steps of 1 second). Use these values to estimate the velocity and distance traveled every second. In your script record and output (to the command window) the speed and distance traveled by the truck after 90 seconds.

### Task 3:

Plot the velocity and distance profiles of the truck (in Matlab) as the results of the analysis in Task 2.

### Task 4:

Find the length of the acceleration ramp needed for this vehicle if the design speed of the highway is 70 mph. Comment on the process to select such value.

### Problem 3

In hydraulics, one of the fundamental problems is to determine the reaction force ( $F$ ) generated by a leaking tank is shown in the figure below. The following formulas apply to the problem using basic hydrostatic analysis:  $v$  is the velocity of the leaking water flow (m/s),  $d$  is the horizontal distance traveled by the leaking water (meters),  $Q$  is the volumetric flow rate (m<sup>3</sup>/s), and  $F$  is the reaction force acting on the tank (Newtons).

$$v = \phi \sqrt{2gh_1}$$

$$d = 2\sqrt{h_1 h_2}$$

$$Q = \phi \mu A \sqrt{2gh_1}$$

$$F = 2\gamma g A h_1$$

where:  $g$  is the gravity constant (9.81 m/s<sup>2</sup>),  $h_1$  is the water depth to the leaking point (meters),  $h_2$  is the distance from the bottom of the tank to the leaking point (meters),  $A$  is the area of the leaking orifice (m<sup>2</sup>),  $\mu$  is the contraction coefficient (0.62 for thin-walled tanks, 0.97 otherwise),  $\phi$  is the equivalent friction (0.97 for water) and  $\gamma$  is the specific weight of the water (1000 kg/m<sup>3</sup>).

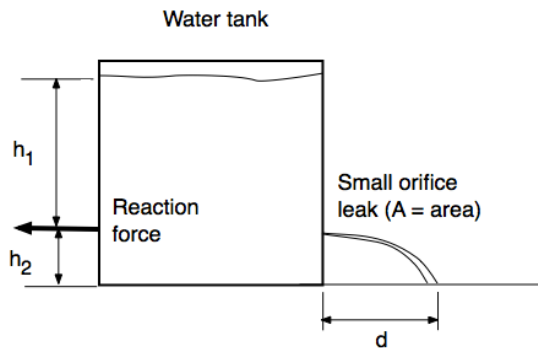


Figure 1. Leaking Water Tank Geometry.

#### Task 1:

Create a Matlab script to estimate the four parameters of the leaking water tank problem ( $v, d, Q, F$ ).

The inputs to the script should be the values of  $h_1$ ,  $h_2$ ,  $A$ ,  $\phi$ , and  $\gamma$ . The value of  $\mu$  should be calculated in the script based on the wall properties of the tank (i.e., thin or thick). It is required that the key word "thin" or "thick" be an input to the script to determine the appropriate value of  $\mu$ . Write the output of the script to the command line and label accordingly.

#### Task 2:

Test your script with the following values:  $h_1 = 18$ ,  $h_2 = 4$ ,  $A = 0.05$ ,  $\phi = 0.97$ ,  $\gamma = 1000$  for both thin and thick wall tanks. Comment on the results obtained.

#### Task 3:

Use the script developed in Task 1 to examine the sensitivity of  $d$  with water tank depth. Create solutions for  $Q$  when  $h_1$  varies from 10 meters to 18 meters at steps of 0.1 meters. Make a plot of  $h_1$  versus  $Q$ .