Assignment 3: Optimization and Excel Solver

Date Due: Solution

Instructor: Trani

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Show all your work including code and results of your computation in the spreadsheet as screen captures.

Problem 1

A company develops the following Linear Programming problem to minimize the cost of producing two types of steel pins commonly used the construction industry. The objective function is the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.

Objective Maximize $Z = 60X_1 + 50 X_2$

Subject to

 $\begin{array}{l} X_2 + X_1 <= 220 \\ 0.1 \ X_1 - 0.12 \ X_2 >= 0 \\ X_1 - X_2 <= 120 \\ X_1, \ X_2 >= 0 \quad \mbox{ (non-negativity conditions)} \end{array}$

Task 2

Solve the problem using Excel Solver. State the exact solution found by Excel for the two decision variables. State the value of the objective function for the optimal solution found.

| Maximization Problem | | | | |
|-----------------------|---------|----|-------------|--|
| | | | | Solver Parameters |
| Decision Variables | | | | Set Objective: \$8\$10 |
| | | | | To: • Max · Min · Value Of: 0 |
| X1 | 170 | | Steel Pin 1 | By Changing Variable Cells: \$8\$5:\$8\$6 |
| X2 | 50 | | Steel Pin 2 | Subject to the Constraints: \$8\$14 <= \$D\$14 |
| | | | | S8515 >= 30515 S8516 <= 50516 S8517 >= 50517 S8518 >= 50518 |
| Objective Function | | | | Delete |
| | | | | Reset All Load/Save |
| 60 X1 + 50 X2 | 12700 | | | Make Unconstrained Variables Non-Negative |
| | | | | Select a Solving Method: Simplex LP Options |
| Constraint Equations | | | | Solving Method Select the CRC Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non- |
| | Formula | | | and select the evolutionary engine for solver problems that are non- smooth. |
| X1 + X2 <= 220 | 220 | <= | 220 | Close Solve |
| 0.1 X1 - 0.12 X2 >= 0 | 11 | >= | 0 | 1 |
| X1 - X2 <= 120 | 120 | <= | 120 | |
| x1 >= 0 | 170 | >= | 0 | |
| x2 >= 0 | 50 | >= | 0 | |

Task 3

Since number of pins to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.

The solution is integer for X1 and X2. However, in Solver you can **force an integer solution** by adding another constraint equation in the Solver panel that forces X1 and X2 to be integer (B5 and B6 in my solution). Also, in the "Options" panel select Integer Optimality (%) to zero. The solution shown I changed the first constraint to be <= 220.5 instead of 220 in order to make the optimal solution non-integer. Adding the additional constraint equation to the Solver panel produces the same integer solution as the original problem.

| Maximization Problem | | | | |
|-------------------------------------|---------|----|-------------|--|
| Decision Variables | | | | |
| | | | | |
| X1 | 170 | | Steel Pin 1 | |
| X2 | 50 | | Steel Pin 2 | |
| | | | | Solver Parameters |
| Objective Function | | | | Set Objective: \$B\$10 |
| | | | | By Changing Variable Cells: \$8\$5:\$8\$6 |
| 60 X1 + 50 X2 | 12700 | | | Subject to the Constraints: |
| | | | | \$8\$14 <= \$D\$14 Add \$8\$15 >= \$D\$15 \$8\$16 <= \$D\$16 |
| Constraint Equations | | | | \$8\$17 >= \$D\$17 Change \$8\$18 >= \$D\$18 Delete |
| | Formula | | | Reset All |
| X1 + X2 <= 220 | 220 | <= | 220.5 | Make Unconstrained Variables Non-Negative |
| $0.1 \times 1 - 0.12 \times 2 >= 0$ | 11 | >= | 0 | Select a Solving Method: Simplex LP Options |
| X1 - X2 <= 120 | 120 | <= | 120 | Solving Method Select the GRG Nonlinear engine for Solver Problems that are smooth |
| x1 >= 0 | 170 | >= | 0 | nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non- smooth. |
| x2 >= 0 | 50 | >= | 0 | Close Solve |

Problem 2

You are in charge of a civil engineering pavement company that makes concrete for various highway projects in the State of Virginia. Your company has various sites across the state to take sand and gravel materials necessary to make a concrete mix used in pavement projects. For a construction job near Roanoke, Virginia there are two sites to extract sand and gravel raw materials: a) Starkey and b) Laymantown. Due to variations in the soil properties at each site, the raw material from Starkey produces 43% sand and 57% gravel. Material from Laymantown produces 55% sand and 45% gravel.

The construction job in Roanoke requires a minimum of 85,000 cubic meters of sand and gravel mix. The pavement design engineer requires a minimum of 25,000 cubic meters of sand and no more than 38,000 cubic meters of gravel in making the concrete mix for this highway job. The unit delivery costs (includes the cost of raw materials and the hauling costs) are \$120 and \$130 per cubic meter from Starkey and Laymantown, respectively.

For each task and subtask below, use screen captures to show me how is that the analysis is done.

Task 1:

Formulate this problem as a linear programming problem. Clearly state the objective function and the constraint equations of the problem.

Task 2:

Solve the problem graphically. Plot the lines of constant values of the objective function.

Task 3:

| Mixing Problem | | | |
|---------------------------------------|---------------|---|--|
| Decision Variables | | | |
| X1 X2 | - 84,444.4 | | Starkey .aymantown |
| Objective Eurotion | | Solver Resu | |
| Objective Function 120 X1 + 130 X2 | 10,977,777.8 | Keep Solver Solution Restore Original Values | Reports Feasibility Feasibility-Bound: |
| Constraint Equations | | Return to Solver Parameters Dialo Save Scenario Can | |
| | Formula | | |
| X1 + X2 >= 85000 | 84,444.4 | >= | 85000 |
| 0.43 X1 + 0.55 X2 >= 25000 | 46,444.4 | >= | 25000 |
| 0.57 X1 + 0.45 X2 <= 38000 | 38,000.0 | <= | 38000 |
| x1 >= 0 | - | >= | 0 |
| x2 >= 0 | 84,444.4 | >= | 0 |

Solver cannot find a feasible solution (you can show that graphically as well). However, it offers the closest solution by allocating all the production to the Laymantown site. Note that the solution offered falls short by 556 cubic meters of material.

By relaxing the total material constraint from >=85,000 to say >=83,000 we can find an optimal solution that involves hauling material from both side. The new solution is how below.

| Mixing Problem | | | | | |
|----------------------------|--------------|----|------------|---|-------------------------|
| | | | | | |
| Decision Variables | | | | Solver Parameters | |
| | | | | Set Objective: \$8\$10 | _ |
| V1 | E 416 7 | | Starkov | To: Max • Min Value Of: 0 | |
| X1 | 5,416.7 | | Starkey | \$B\$5:\$B\$6 | _ |
| X2 | 77,583.3 | | Laymantown | Subject to the Constraints: \$B\$14 >= \$D\$14 | |
| | | | | \$8\$15 >= \$D\$15 \$8\$16 <= \$D\$16 | Add Change |
| Objective Function | | | | \$8\$17 >= \$D\$17 \$8\$18 >= \$D\$18 | Delete |
| | | | | | Reset All |
| 120 X1 + 130 X2 | 10,735,833.3 | | | | Load/Save |
| | 20,700,000 | | | Make Unconstrained Variables Non-Negative Select a Solving Method: Simplex LP | |
| | | | | | Options |
| Constraint Equations | | | | Solving Method Select the GRG Nonlinear engine for Solver Problems that a | are smooth |
| | Formula | | | nonlinear. Select the LP Simplex engine for linear Solver Pr and select the Evolutionary engine for Solver problems tha smooth. | roblems, it are non- |
| X1 + X2 >= 85000 | 83,000.0 | >= | 83000 | Close | Solve |
| 0.43 X1 + 0.55 X2 >= 25000 | 45,000.0 | >= | 25000 | | |
| 0.57 X1 + 0.45 X2 <= 38000 | 38,000.0 | <= | 38000 | | |
| x1 >= 0 | 5,416.7 | >= | 0 | | |
| x2 >= 0 | 77,583.3 | | 0 | | |

Task 4:

Suppose that the engineer decides to change the specification of the concrete mix to achieve higher durability against repeated vehicle load cycles. A minimum of 23,000 cubic meters of sand are needed for the job and no less than 51,000 cubic meters of gravel. The solution is shown below. Note that all the allocation is made to site 1 (Starkey).

| Mixing Problem | | | | | |
|--|--------------|----|------------|--|---------------|
| Decision Variables | | | | | |
| | | | | | |
| X1 | 85,000.0 | | Starkey | | |
| X2 | - | | Laymantown | Solver Parameters | |
| | | | | Set Objective: \$8\$10 | _ |
| Objective Function | | | | To: Max • Min Value Of: By Changing Variable Cells: | |
| | | | | SB\$5:\$B\$6 | - |
| 120 X1 + 130 X2 | 10,200,000.0 | | | Subject to the Constraints: \$B\$14 >= \$D\$14 | Add |
| 120 XI + 130 X2 | 10,200,000.0 | | | \$8515 >= \$D\$15 \$8516 <= \$D\$16 \$8517 >= \$D\$17 | Change |
| | | | | SB\$17 >= SD\$17 SB\$18 >= SD\$18 | Delete |
| Constraint Equations | | | | | Reset All |
| | Formula | | | | Load/Save |
| X1 + X2 >= 85000 | 85,000.0 | >= | 85000 | Make Unconstrained Variables Non-Negative Select a Solving Method: Simplex LP | |
| 0.43 X1 + 0.55 X2 >= 23000 | 36,550.0 | | 23000 | Select a Solving Method: Simplex LP | Options |
| $0.57 \times 1 + 0.45 \times 2 <= 51000$ | 48,450.0 | | 51000 | Select the GRG Nonlinear engine for Solver Problems th nonlinear. Select the LP Simplex engine for linear Solve and select the Evolutionary engine for Solver problems | er Problems, |
| $x_1 >= 0$ | 85,000.0 | | 0 | and select the Evolutionary engine for solver problems smooth. | that are non- |
| | 35,000.0 | | 0 | Close | Solve |
| x2 >= 0 | | >= | | | |

Problem 3

Solve the Osaka Bay problem described in class with the following modifications:

- a) Fuji ships carry 700 metric tons of cargo and require a crew of 2.
- b) Haneda ships carry 1000 metric tons of cargo and require a crew of 3

Task 4:

Solve the problem using Excel Solver. Comment on the results obtained in Tasks 2 and 3.

The solution to the revised problem is shown below. Note that the solution, while optimal, does not produce integer values for X1 and X2. Therefore, force the integer solution by adding an additional constraint equation. The integer solution is also presented below.

| 40.00 33.33 | | | | |
|-------------------------|---|---------------------------|---|--|
| (1222.22) | | | Solver Parameters Set Objective: 58510 To: Max Min Value Of: 0 By Changing Variable Cells: 58553856 | - |
| 01333.33 | | | Subject to the Constraints: \$8514 <= \$D\$14 \$8515 <= \$D\$15 \$8516 <= \$D\$16 \$8517 >= \$D\$16 \$8517 >= \$D\$17 | Add Change |
| Formula | | | \$8\$18 >= \$D\$18 | Delete Reset All |
| 40.00 33.33 40.00 | <= <= >= | 180 40 60 0 0 | nonlinear. Select the LP Simplex engine for linear Solve | er Problems. |
| | 33.33 61333.33 Formula 180.00 40.00 33.33 40.00 | 33.33 61333.33 | 33.33 Number of Si 61333.33 | 33.33 Number of Ships Type 2 33.33 Solver Parameter 61333.33 Solver Parameter 600 Solver Parameter 90 Solver Parameter 90 Solver Parameter 91 Solver Parameter 92 Solver Parameter 93 Solver Parameter 90 Solver Parameter 91 Solver Parameter 92 Solver Parameter |

Optimal solution with integer values for X1 and X2 (shown below).

| Revised Problem for O | saka Bay | | | | |
|--|-----------------|----|-------------|--|-------------------------------|
| Decision Variables | | | | | |
| x1 | 39.00 | | Number of S | | |
| x2 | 34.00 | | Number of S | hips Type 2 Solver Parameters | |
| Objective Function | | | | Set Objective: \$8\$10 To: • Max • Min • Value Of: • By Changing Variable Cells: | |
| 700 x1 + 1000 x2 | 61300.00 | | | \$855:5856 Subject to the Constraints: \$8514 <= \$D\$14 \$8515 <= \$D\$15 | Add |
| Constraint Equations | Formula | | | 58516 <= 5D516 58517 >= 5D517 58518 >= 5D518 5855-5856 = integer | Change Delete Reset All |
| 2 x1 + 3 x2 <= 180 | Formula | | 190 | | Load/Save |
| $2 \times 1 + 3 \times 2 <= 180$ $\times 1 <= 40$ | 180.00 39.00 | | 180 40 | Make Unconstrained Variables Non-Negative Select a Solving Method: Simplex LP | Options |
| x2 <= 60 | 34.00 | | 60 | Solving Method Select the GRG Nonlinear engine for Solver Problems t nonlinear. Select the LP Simplex engine for linear Solv | er Problems, |
| x1 >= 0 | 39.00 | | 0 | and select the Evolutionary engine for Solver problem smooth. | s that are non- |
| x2 >= 0 | 34.00 | >= | 0 | Close | Solve |

Problem 4

| Pollution Source | Loading (kg/year) | Unit Cost of Removal (\$/kg) | Minimum Removal |
|------------------|-------------------|--|--------------------------------|
| River A | 17,400 | 36 | 7,000 |
| River B | 16,700 | 38 | 8,000 |
| River C | 34,500 | 32 | 1/2 of River A removal |
| Airport | 25,600 | 56 | 1/2 of River B removal |
| City | 16,500 | 105 without treatment plant 30 with treatment plant | 1/2 of City's original loading |
| Totals | 110,700 | | |

Solve the lake pollution control problem described in class with the following attributes:

Task 2:

Solve the water pollution control problem if the total desired pollution removal is 45,000 kg. In solving the new problem, assume the city invested in new pollution treatment plant at a cost of \$30,000,000. Find out the total cost of pollution removal for this task. Task 3:

| ‡ × √ fx | | 14 · · · · | - | | Solver Parame | iters |
|-------------|---------------------------------------|-----------------|----------------|--|---|--|
| A | В | С | D | Set Objectiv | | |
| Revised Pol | lution Control Problem | | | те: и | Aax ○ Min ○ Value C g Variable Cells: | H. 0 |
| | | | | SC\$9.5C5 | | - |
| | (1 | RiverA | | Subject to t | he Constraints: | |
| | (2 | RiverB | | SC\$18 >= SC\$19 <= | \$E\$18 \$E\$19 | Add |
| | (3 | RiverC | | SCS20 <- SCS21 <- | \$E\$20 \$E\$21 | Change |
| | (4 (5 | Airport | | SCS22 <= SCS23 <= SCS24 >= | 5E523 5E524 | Delete |
| ^ | .5 | City | | SCS18 >> SCS19 <> SCS20 <> SCS21 <> SCS22 <> SCS23 <> SCS24 >> SCS25 >> SCS26 >> SCS26 >> SCS27 >> | \$E\$25 \$E\$26 | Reset All |
| Decision X | (1 | 7000 | | SC\$27 >= SC\$28 >= | - \$E\$27 - \$E\$28 | Load/Save |
| Variables X | (2 | 8000 | | 🖸 Make U | nconstrained Variables Non- | Negative |
| | (3 | 17750 | | Select a Sol | ving Method: Simplex LP | Options |
| | (4 | 4000 | | Solving Me | thed | |
| × | (5 | 8250 | | Select the incollinear. | thed IRG Nonlinear engine for Solve Select the LP Simplex engine for the Evolutionary engine for Sol | r Problems that are smooth ir linear Solver Problems, |
| Objective N | linimization | | - | and select smooth. | the Evolutionary engine for sor | ver problems that are non- |
| | 6X1+38X2+32X3+56X4+105X5 | 2,214,250.00 | \$/vr | | | |
| - | 0.1+30.2+32.3+30.4+103.5 | 2,214,200.00 | <i>\\</i> /y1. | | | Close Solve |
| Subject to | (1+X2+X3+X4+X5 | 45000 | >= | | 45000 | |
| × | (1<= 17400 | 7000 | | | 17400 | |
| | 2<=16700 | 8000 | | | 16700 | |
| | (3<=34500 | 17750 | | | 34500 | |
| | 4<=25600 | 4000 | | | 25600 | |
| | (5<= 16500 (1>= 7000 | 8250 7000 | | | 16500 7000 | |
| | (2>=8000 | 8000 | | | 8000 | |
| | $(3 - 1/2 \times 1) = 0$ | 14250 | | | 0 | |
| | $(4 - 1/2 \times 2) = 0$ | | >= | | 0 | |
| > | (5>= 16500/2 | 8250 | >= | | 8250 | |
| Variables | ollution Control Problem | RiverA | | | | |
| Names | X2 | RiverB | | | | |
| Names | X3 | RiverC | | | | |
| | X3 X4 | | | | | |
| | X5 | Airport City | | | | |
| | ×3 | City | | | | |
| Decision | X1 | | | 7000 | | |
| Variables | X2 | | | 8000 | | |
| Variables | X3 | | | 9500 | | |
| | X4 | | | 4000 | | |
| | X5 | | | .6500 | | |
| | <u>,,,</u> | | - 1 | | | |
| Objective | Minimization | | | | | |
| 2 Sjeen e | 36X1+38X2+32X3+56X4 | +3072 4 | ,579,0 | 00.00 | \$/wr | |
| | 3071730727327373084 | +30A3 I | ,373,0 | 00.00 | ψ/ γ1. | |
| Subject to | X1+X2+X3+X4+X5 | | | 5000 | ~- | 45000 |
| Subject to | X1 + X2 + X3 + X4 + X5 X1 <= 17400 | | 4 | 7000 | | 17400 |
| | X1<= 17400 X2<=16700 | | | | | |
| | | | | 8000 | | 16700 |
| | X3<=34500 | | | 9500 | | 34500 |
| | X4<=25600 | | | 4000 | | 25600 |
| | X5<= 16500 | | 1 | .6500 | | 16500 |
| | X1>= 7000 | | | 7000 | | 7000 |
| | X2>=8000 | | | 8000 | | 8000 |
| | X3 - 1/2 X1 >= 0 | | | 6000 | | 0 |
| | X4 - 1/2 X2 >= 0 | | | 0 | >= | 0 |
| | X5>= 16500/2 | | | 6500 | | 8250 |

Solution with City cost with no treatment plant

Solution with City cost with treatment plant

Note that we save 635,000 per year. The treatment plant is \$30 million. The payback period is 47 years (assuming no demand increase).

Task 3

Using the solution on Task 2, suppose a new (stricter) environmental law takes effect and It is desired to reduce the total pollution discharge to the lake to 55,000 kg/yr instead. Estimate the cost of removal and the amounts to be removed from each pollution source. Contrast the removal cost in Tasks 2 and 3. Comment.

| \$ × ✓ J | fx | | | OOO Solve | r Parameters | |
|------------|----------------------------------|--------------|--|--|--------------------|-----------|
| А | В | С | | Set Objective: \$C\$16 | | _ |
| Revised P | ollution Control Problem | | | To: 🔿 Max 😐 Min | Value Of: 0 | |
| | | | | By Changing Variable Cells: | | |
| Variables | X1 | RiverA | | \$C\$9:\$C\$13 | | _ |
| | | | | Subject to the Constraints: | | |
| Names | X2 | RiverB | | \$C\$18 >= \$E\$18 \$C\$19 <= \$E\$19 | | Add |
| | X3 | RiverC | | \$C\$20 <= \$E\$20 | | Change |
| | X4 | Airport | | \$C\$21 <= \$E\$21 \$C\$22 <= \$E\$22 | | |
| | X5 | City | \$C\$23 <= \$E\$23 \$C\$24 >= \$E\$24 | | | Delete |
| | | | | \$C\$25 >= \$E\$25 \$C\$26 >= \$E\$26 | | Reset All |
| Decision | X1 | 7000 | | \$C\$27 >= \$E\$27 | | |
| Variables | X2 | 8000 | | \$C\$28 >= \$E\$28 | | Load/Save |
| variables | X3 | 27750 | | 🗹 Make Unconstrained Varia | ables Non-Negative | |
| | X4 | 4000 | | Select a Solving Method: S | implex LP 💌 | Options |
| | | | | Solving Method | | |
| | X5 | 8250 | | Select the GRG Nonlinear engin | | |
| | | | | nonlinear. Select the LP Simple and select the Evolutionary eng | | |
| Objective | Minimization | | | smooth. | | |
| | 36X1+38X2+32X3+56X4+105X5 | 2,534,250.00 | \$/yr. | | | |
| | | ·· | | | Close | Solve |
| Subject to | X1+X2+X3+X4+X5 | 55000 | >= | 5500 | 00 | |
| | X1<= 17400 | 7000 | <= | 1740 | 00 | |
| | X2<=16700 | 8000 | <= | 1670 | 00 | |
| | X3<=34500 | 27750 | <= | 3450 | 00 | |
| | X4<=25600 | 4000 | <= | 2560 | 00 | |
| | X5<= 16500 | 8250 | <= | 1650 | 00 | |
| | X1>= 7000 | 7000 | >= | 700 | 00 | |
| | X2>=8000 | 8000 | | 800 | | |
| | X3 - 1/2 X1 >= 0 | 24250 | | | 0 | |
| | X4 - 1/2 X2 >= 0 | | >= | | 0 | |
| | X4 - 1/2 X2 >= 0 X5>= 16500/2 | 8250 | | 825 | - | |
| | AJ/- 10300/2 | 8230 | /- | 82. | | |