

Assignment 3: Optimization with Excel

Date Due: February 12, 2017

Instructor: Trani

Show all your work including code and results of your computation in the spreadsheet as screen captures.

Problem 1

A company that makes concrete has two products in the market. Product A is a premium concrete mix that sells for \$765 per ton. Product B is a standard concrete mix that is easier to make and sells for \$650 per ton.

With the concrete mixing hardware available, the company can produce up to 600 tons of premium concrete per day or up to 720 tons of the standard product. Because the concrete mixes are produced using the same machinery, linear combinations of both products not exceeding their maximum individual productions can be produced in one day. For example, the company may produce 300 tons of premium concrete in the first 12 hours of the day and then produce another 360 tons of standard concrete in the remaining 12 hours of the day (assume 24 hour operation). The company employs special trucks to deliver the concrete to various clients in the region. Because the specific weight of both products is not the same, the delivery trucks can haul up to 675 tons of premium concrete per day or up to 775 tons per day of the standard concrete. Linear combinations of both products not exceeding their maximum individual hauling rates can be delivered in one day.

Task 1:

Formulate the problem as a linear programming problem. The idea is to maximize the revenue to the company.

Let :

x_1 = amount of standard concrete produced

x_2 = amount of premium concrete produced

$$\text{Max } Z = 650x_1 + 765x_2$$

Develop the bounds of constraint equations

$$x_2 = 600 - \frac{600}{720}x_1$$

$$x_2 = 675 - \frac{675}{775}x_1$$

$$x_1, x_2 \geq 0$$

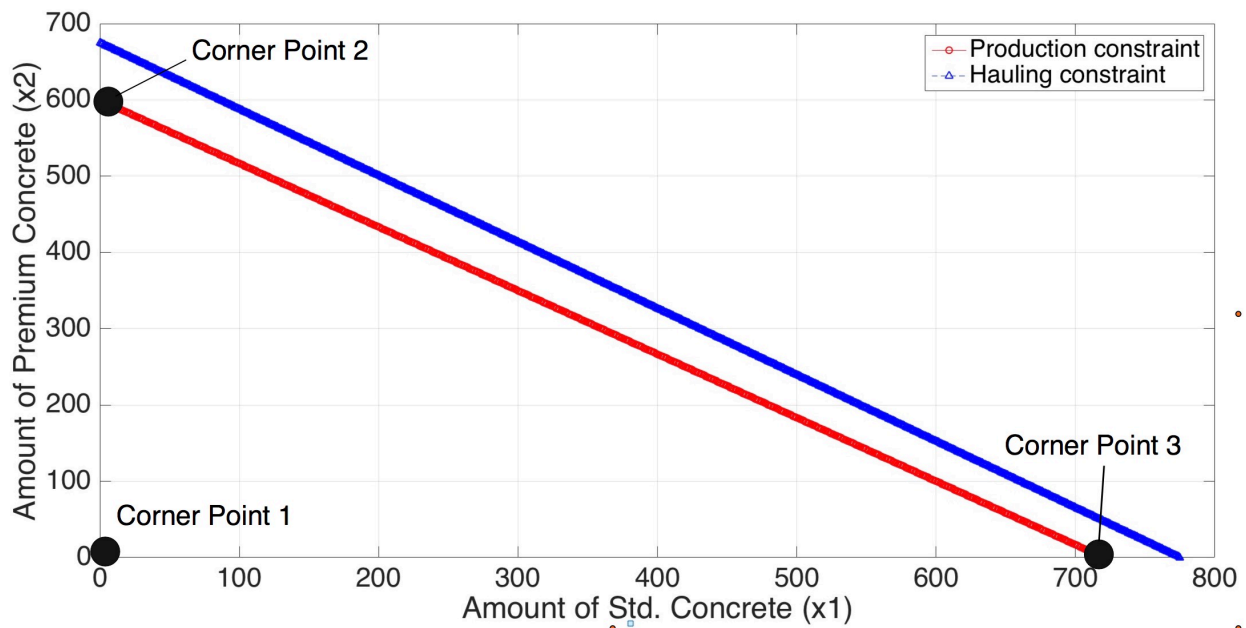
Re-arrange the constraint equations:

$$x_2 + \frac{600}{720}x_1 \leq 600$$

$$x_2 + \frac{675}{775}x_1 \leq 675$$

Task 2:

Solve the problem graphically. Clearly indicate corner points and plot the lines of constant Z value.



Graphical solution. Clearly, the production constraint is dominant for this problem. Producing only standard concrete provides a net value of $Z = \$468,000$ (corner point 3). Producing all premium concrete makes the objective $Z = \$459,000$ (corner point 2). **The optimal solution is corner point 3.**

Problem 2

Task 1:

Solve the water management pollution control problem stated in the class Notes # 7 (pages 34 through 39) if the total pollution removal is 63,000 kg. In solving the new problem, assume the city invested in new pollution treatment plant and technology (\$30,000,000 cost) and generates 9,500 kg/year of pollutant per year. The cost of removing pollutants from the city using the new technology is \$1.23 per kilogram. Find out the total cost of pollution removal for this task. In this solution, assume that for political reasons we would like to remove at least 4,000 kg of pollution from all sources.

Task 2:

Using the solution on Task 1, suppose a new (stricter) environmental law takes effect and it is desired to reduce the total pollution discharge to the lake to 93,000 kg/yr instead. Estimate the cost of removal and the amounts to be removed from each pollution source. Contrast the removal cost in tasks 1 and 2. Comment.

Problem 3

A group in your company develops the following Linear Programming problem to minimize the cost of producing two types of steel doublers commonly used in buildings. The objective function is the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.

Objective Maximize $Z = 110X_1 + 130 X_2$

Subject to

$$X_2 - 1.2 X_1 \leq 250$$
$$3X_2 + X_1 \leq 1200$$
$$X_2 + 3X_1 \leq 1500$$
$$X_1, X_2 \geq 0 \quad (\text{non-negativity conditions})$$

For each task below, use screen captures of your setup using Excel Solver. Show the formulas of the cells to make out task simpler in grading.

Task 1

Solve the problem using Excel Solver. State the exact solution found by Excel for the two decision variables. State the value of the objective function for the optimal solution found.

Task 2

Since number of beams to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.

Task 3

Solve the problem by hand using the Simplex method. Show all the tables in each iteration. Also, for each iteration, indicate the Basic Variables (in the table) and the current solution for Z.

Let :

x_1 = amount of steel doublers of type 1

x_2 = amount of steel doublers of type 2

$$\text{Max } Z = 110x_1 + 130x_2$$

Subject to:

$$-1.2x_1 + x_2 \leq 250$$

$$x_1 + 3x_2 \leq 1200$$

$$3x_1 + x_2 \leq 1500$$

$$x_1, x_2 \geq 0$$

Convert the problem to standard form. Add slack and artificial variables as needed.

Add slack variables

$$-1.2x_1 + x_2 + x_3 = 250$$

$$x_1 + 3x_2 + x_4 = 1200$$

$$3x_1 + x_2 + x_5 = 1500$$

Construct the first table.

BV	Z	x1	x2	x3	x4	x5	RHS
Z		1	-110	-130	0	0	0
x3		0	-1.2	1	1	0	250
x4		0	1	3	0	1	1200
x5		0	3	1	0	0	1500

Step 1. Decide on the entering variable to the solution. x2 enters the solution first (highest negative coefficient in Z-row).

Step 2. Apply ratio test to decide which Basic Variable leaves the solution first. Variable x3 has the lowest ratio. Therefore leaves the solution and x2 enters the solution.

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio
Z		1	-110	-130	0	0	0	
x3		0	-1.2	1	1	0	250	250
x4		0	1	3	0	1	1200	400
x5		0	3	1	0	0	1500	1500

Step 3. Perform row operations using the pivot row to eliminate all the coefficients of the pivot column.

BV	Z	x1	x2	x3	x4	x5	RHS
Z		1	-266	0	130	0	32500
x2		0	-1.2	1	1	0	250
x4		0	4.6	0	-3	1	450
x5		0	4.2	0	-1	0	1250

In this solution, variable x2 has replaced x3. The current solution has a value of Z=32,500 with variables x1=0, x3=0, x2=250, x4=450 and x5=1250.

Step 4. Repeat until the optimal solution is found. Note the coefficient of x1 is negative, therefore the solution can be improved by bringing x1 to the solution.

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio
Z	1	-266	0	130	0	0	32500	
x2	0	-1.2	1	1	0	0	250	-208.33333
x4	0	4.6	0	-3	1	0	450	97.8
x5	0	4.2	0	-1	0	1	1250	297.6

Variable x1 enters the solution and variable x4 leaves the solution (Basic Variable set). Note that we do not consider negative ratios to check which variable leaves the solution first as x1 increases. Perform row operations to zero out cells in pivot column. First divide row 3 (x4) by 4.6 to facilitate the computations.

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio
Z	1	-266	0	130	0	0	32500	
x2	0	-1.2	1	1	0	0	250	-208.33333
x4	0	4.6	0	-3	1	0	450	97.8
x5	0	4.2	0	-1	0	1	1250	297.6

The solution is now. Now do remaining row operations.

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1	-266	0	130	0	0	32500
x2	0	-1.2	1	1	0	0	250
x4	0.00	1.00	0.00	-0.65	0.22	0.00	97.83
x5	0	4.2	0	-1	0	1	1250

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1.00	0.00	0.00	-43.48	57.83	0.00	58521.74
x2	0.00	0.00	1.00	0.22	0.26	0.00	367.39
x1	0.00	1.00	0.00	-0.65	0.22	0.00	97.83
x5	0.00	0.00	0.00	1.74	-0.91	1.00	839.13

The solution is not optimal because the coefficient of x3 is still negative.

BV	Z	x1	x2	x3	x4	x5	RHS	Ratio
Z	1.00	0.00	0.00	-43.48	57.83	0.00	58521.74	
x2	0.00	0.00	1.00	0.22	0.26	0.00	367.39	1690.00
x1	0.00	1.00	0.00	-0.65	0.22	0.00	97.83	-150.00
x5	0.00	0.00	0.00	1.7391	-0.91	1.00	839.13	482.50

Variable x3 enters the solution and variable x5 leaves the solution (Basic Variable set). Note that we do not consider negative ratios to check which variable leaves the solution first as x3 increases. Perform row operations to zero out cells in pivot column. First divide row 4 (x5) by 1.74 to facilitate the computations.

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1.00	0.00	0.00	-43.48	57.83	0.00	58521.74
x2	0.00	0.00	1.00	0.22	0.26	0.00	367.39
x1	0.00	1.00	0.00	-0.65	0.22	0.00	97.83
x5	0.00	0.00	0.00	1.00	-0.52	0.58	482.51

BV	Z	x1	x2	x3	x4	x5	RHS
Z	1.00	0.00	0.00	0.02	35.08	25.00	79501.20
x2	0.00	0.00	1.00	-0.00	0.38	-0.13	261.24
x1	0.00	1.00	0.00	0.00	-0.12	0.37	411.46
x3	0.00	0.00	0.00	1.00	-0.52	0.58	482.51

This is the optimal solution (all coefficients in the objective function row (Z-row) are zero or positive numbers.

$x_1=411.46$, $x_2 = 261.24$, $x_3 = 482.51$, $x_4 = 0$ and $x_5 = 0$

Objective function is: 79,501.2

Note that the solution is not integer. For this class we can round the solution to the nearest integer values.

Solution Using Excel Solver. Note that the solution is close because while doing row operations I carried some round off error in my calculations.

Optimization problem		
Decision Variables		
x1	412.5	X1 variable
x2	262.5	X2 variable
Objective Function		
110 x1 + 130 x2	79500	
Constraint Equations		
	Formula	
$(-1.2)x_1 + x_2 \leq 250$	$-232.5 \leq$	250
$x_1 + 3x_2 \leq 1200$	$1200 \leq$	1200
$3x_1 + x_2 \leq 1500$	$1500 \leq$	1500
$x_1 \geq 0$	$412.5 \geq$	0
$x_2 \geq 0$	$262.5 \geq$	0

