## Assignment 3: Pivot Tables and Linear Programming

Date Due: September 18, 2013
Instructor: Trani

## Problem 1

A bus leasing company has hundreds os buses placed in 5 cities around the country. The bus data is shown below and provided to you in a file called BusCompany_2013.xls.
Table 1. Sample Bus Data for Bus Leasing Company.

| A | B | C | D | E |
| :--- | :--- | ---: | ---: | ---: |
| City | Bus Type | Age | Miles | Route length |
| Charlotte | New Flyer MiDi35 | 7 | $398,324.0$ | 25.6 |
| Charlotte | New Flyer MiDi35 | 8 | $335,600.0$ | 42.0 |
| Seattle | New Flyer XDE40 | 9 | $552,353.4$ | 34.0 |
| Charlotte | New Flyer MiDi35 | 10 | $462,929.5$ | 42.0 |
| Salt Lake City | New Flyer XDE40 | 6 | $219,703.2$ | 46.0 |
| Salt Lake City | New Flyer XDE40 | 5 | $194,856.5$ | 33.7 |
| Atlanta | New Flyer XDE40 | 8 | $531,048.3$ | 24.6 |
| Seattle | New Flyer XDE40 | 8 | $287,000.0$ | 32.3 |
| Salt Lake City | New Flyer XDE40 | 4 | $143,900.0$ | 33.6 |

where:
Column A = City where leased bus is located
Column B = Bus type and model
Column C = Age of the vehicle
Column $\mathrm{D}=$ Miles in the odomoter
Column E = Average route length (miles) for the vehicle
Task 1:
Using Excel create a Pivot Table to display the average number of miles in the odometer as a function of bus type, age of vehicle and city. Show a screen capture of the Pivot Table created.

|  |  | Data |  |
| :---: | :---: | :---: | :---: |
| Bus Type $\quad$ - | City $\quad$ - | Average of Miles | Average of Age |
| $\square$ New Flyer Mi | Atlanta | 334875.676 | 6.528301887 |
|  | Charlotte | 368674.3259 | 7.384615385 |
|  | Los Angeles | 352062.3101 | 7.177777778 |
|  | Salt Lake City | 369084.2101 | 7.365853659 |
|  | Seattle | 367258.1594 | 7.186440678 |
| New Flyer MiDi35 Total |  | 358251.855 | 7.116 |
| $\square$ New Flyer XL | Atlanta | 391434.4136 | 7.538461538 |
|  | Charlotte | 369911.3251 | 7.52 |
|  | Los Angeles | 399861.3698 | 7.416666667 |
|  | Salt Lake City | 335907.4078 | 6.916666667 |
|  | Seattle | 321361.9426 | 6.757575758 |
| New Flyer XDE40 Total |  | 361276.3079 | 7.204545455 |
| $\square$ Orion VII | Atlanta | 382200.2304 | 7.428571429 |
|  | Charlotte | 350600.6871 | 7 |
|  | Los Angeles | 341347.9909 | 6.941176471 |
|  | Salt Lake City | 345538.6362 | 7.206896552 |
|  | Seattle | 367087.9107 | 7.25862069 |
| Orion VII Total |  | 358621.9373 | 7.180232558 |
| $\square$ Van Hol AGG: | Atlanta | 361961.1911 | 7.806451613 |
|  | Charlotte | 349786.412 | 7.173913043 |
|  | Los Angeles | 332114.5637 | 7.111111111 |
|  | Salt Lake City | 350709.9272 | 7 |
|  | Seattle | 374472.1269 | 7.384615385 |
| Van Hol AGG300 Total |  | 353723.8319 | 7.278074866 |
| Grand Total |  | 357733.8275 | 7.187584345 |

Task 2:
Using the Pivot Table created in Task 1 find the average mileage for all bus types in the fleet.
Answer:
357733.8275 mile

Task 3:
Using Excel create a Pivot Table to display the average number of miles in the odometer as a function of bus type, age of vehicle for the City of Charlotte. Show a screen capture of the Pivot Table created.

| City | Charlotte | $\boldsymbol{T}$ |  |
| :--- | :--- | ---: | ---: |
|  |  |  |  |
|  | Data |  |  |
| Bus Type | Average of Miles | Average of Age |  |
| New Flyer MiDi35 | 368674.3259 | 7.384615385 |  |
| New Flyer XDE40 | 369911.3251 | 7.52 |  |
| Orion VII | 350600.6871 | 7 |  |
| Van Hol AGG300 | 349786.412 | 7.173913043 |  |
| Grand Total | 360087.9372 | 7.280821918 |  |

Task 4:
Using the Pivot Table created in Task 3 find the average route length traveled by the Van Hol AGG300 buses leased by the City of Seattle.

| City | Seattle |
| :--- | :---: |
| $\boldsymbol{T}$ |  |
| Average of Route length |  |
| Bus Type | $\boldsymbol{\top}$ |
| Van Hol AGG300 | Total |
| Grand Total | 37.84181715 |

Task 5:
Using Excel create a Pivot Table to count the number of buses as a function of bus type for all cities. Show a screen capture of the Pivot Table created.

| City | (All $)$ |
| :--- | ---: |
|  |  |
| Count of Bus Type |  |
| Bus Type | Total |
| New Flyer MiDi35 | 250 |
| New Flyer XDE40 | 132 |
| Orion VII | 172 |
| Van Hol AGG300 | 187 |
| Grand Total | 741 |

Task 6:
Using the Pivot Table created in Task 5 find the city with the highest number of New Flyer MiDi35 buses with 6 years of age.
Answer:
Atlanta and Seattle has the highest number of New Flyer MiDi35 buses with 6 years of age, which is 11 for both.
Task 7:
Using Excel create a Pivot Table to summarize the total miles traveled by the buses as a function of bus type and city. Show a screen capture of the Pivot Table created.

| Sum of Miles |  |  |
| :---: | :---: | :---: |
| City | Bus Type $\quad$ | Total |
| $\boxminus$ Atlanta | New Flyer MiDi35 | 17748410.83 |
|  | New Flyer XDE40 | 10177294.75 |
|  | Orion VII | 10701606.45 |
|  | Van Hol AGG300 | 11220796.92 |
| Atlanta Total |  | 49848108.96 |
| $\square$ Charlotte | New Flyer MiDi35 | 19171064.95 |
|  | New Flyer XDE40 | 9247783.127 |
|  | Orion VII | 8063815.803 |
|  | Van Hol AGG300 | 16090174.95 |
| Charlotte Total |  | 52572838.83 |
| $\square$ Los Angeles | New Flyer MiDi35 | 15842803.95 |
|  | New Flyer XDE40 | 9596672.876 |
|  | Orion VII | 11605831.69 |
|  | Van Hol AGG300 | 11956124.29 |
| Los Angeles Total |  | 49001432.81 |
| $\square$ Salt Lake City | New Flyer MiDi35 | 15132452.62 |
|  | New Flyer XDE40 | 8061777.787 |
|  | Orion VII | 10020620.45 |
|  | Van Hol AGG300 | 12274847.45 |
| Salt Lake City Total |  | 45489698.31 |
| $\square$ Seattle | New Flyer MiDi35 | 21668231.41 |
|  | New Flyer XDE40 | 10604944.11 |
|  | Orion VII | 21291098.82 |
|  | Van Hol AGG300 | 14604412.95 |
| Seattle Total |  | 68168687.28 |
| Grand Total |  | 265080766.2 |

Task 8:
Using the Pivot Table created in Task 7 find the city with the highest mileage on Orion VII buses.
Answer:
Seattle is the city with the highest mileage on Orion VII buses, which is 21291098.8 mile.

## Problem 2

A company makes two Portland Cement Concrete (PCC) mixes for highway construction projects. Product Xcel is a premium PCC mix that sells for $\$ 175$ per cubic meter. Product Performer ST is a PCC mix that sells for $\$ 154$ per cubic meter.

With the mixing hardware available, the company can produce up to 640 cubic meters of the Xcel PCC mix per day or up to 780 cubic meters of the Performer ST product. Because the PCC concrete mixes are produced using the same machinery, linear combinations of both products not exceeding their maximum individual productions can be produced in one day. For example, the company could produce 530 cu. meters of the Perfomer ST mix and 205 cu . meters of the Xcel mix on the same day if we assume a linear combination of the individual production rates of the two PCC mixes. The company employs special trucks to deliver the concrete to various clients in the region. Because the specific weight of both products is not the same, the delivery trucks can haul up to 700 cu. meters of the Xcel PCC mix per day or up to 730 cu. meters per day of the Perfomer ST PCC mix. Linear combinations of both products not exceeding their maximum individual hauling rates can be delivered in one day. For example, the company could haul 360 cu. meters of the Perfomer ST mix and 350 cu . meters of the Xcel mix on the same day if we assume a linear combination of the individual hauling rates of the two distinct PCC mixes.

Task 1:
Formulate the problem as a linear programming problem. Assume the company wants to maximize the revenue for the company.
Answer:
Variables:
The amount of Xcel PCC mix produced per day is $\mathrm{X}_{1}$
The amount of Performer ST produced per day is $\mathrm{X}_{2}$
Objective:
Maximize avenue: $\mathrm{Z}=175 \times \mathrm{X}_{1}+154 \times X_{2}$
S.T.
$\frac{X_{1}}{640}+\frac{X_{2}}{780} \leq 1$
$\frac{X_{1}}{700}+\frac{X_{2}}{730} \leq 1$
$\mathrm{X}_{1}, \mathrm{X}_{2}$ are non-negative
Task 2:
Solve the problem graphically. Clearly indicate corner points and plot the lines of constant $Z$ value.


Optimal solution:
X1 $=284.26, \mathrm{X} 2=433.55, Z=116513.4$
Task 3:
Solve the problem by hand using the Simplex Method. Clearly show your tables and indicate which variables are the basic variables in the current solution. Assume all constrains are of the type $<=$ like the problem solved in class.

| BV | Z |  | X1 |  | X2 | X3 | X4 | RHS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Z |  | 1 |  | -175 | -154 | 0 | 0 | 0 |
| X3 |  | 0 |  | 780 | 640 | 1 | 0 | 499200 |
| X4 |  | 0 |  | 730 | 700 | 0 | 1 | 511000 |
| BV | Z |  | X1 |  | X2 | X3 | X4 | RHS |
| Z |  | 1 |  | 0 | 13.80822 | 0 | 0.239726 | 122500 |
| X3 |  | 0 |  | 0 | -107.945 | 1 | -1.06849 | -46800 |
| X1 |  | 0 |  | 1 | 0.958904 | 0 | 0.00137 | 700 |
| BV | Z |  | X1 |  | X2 | X3 | X4 | RHS |
| Z |  | 1 |  | 0 | 0 | 0.127919 | 0.103046 | 116513.4 |
| X2 |  | 0 |  | 0 | 1 | -0.00926 | 0.009898 | 433.5533 |
| X1 |  | 0 |  | 1 | 0 | 0.008883 | -0.00812 | 284.264 |

## Task 4:

Solve the PCC mix problem using Excel Solver. Comment on the results obtained in Tasks 3 and 4. Do they agree?


Result:

| Variable |  |
| :--- | :--- |
| X_1 | 284.2639 |
| X_2 | 433.5533 |
| obj |  |
| $Z=175 \times X \_1+154 \times X \_2$ | 116513.4 |

## Comment:

These three methods agree to each other.

## Problem 3

Modify the water management pollution control problem described in the class notes and explained in class. New removal costs are presented in Table 1.
Table 1. Removal Costs and Pollution Values for Water Pollution Control Problem.

| Source | Removal Cost (\$/kg) | Pollution to Lake (kg) |
| :---: | :---: | :---: |
| River A | 35 | 23,400 |
| River B | 46 | 12,800 |
| River C | 50 | 28,600 |
| City | 87 | 17,400 |
| Airport | 76 | 19,400 |

Assume that under a new water mandate by EPA we would like to remove at least $62,000 \mathrm{~kg}$. of the baseline pollution going into the lake. Moreover, airport and city managers want to participate in the pollution removal program by removing at least $50 \%$ of their baseline pollution allocations per year. The pollution processing plants at all three rivers need to remove at least a quarter of their pollutants as a minimum according to a new environmental law.
a) Formulate the problem as a linear programming problem. Solve the new problem using Excel Solver and state the optimal cost.

Answer;
Variables:
Amount of removal pollutants from river A is X 1 kg ;
Amount of removal pollutants from river $B$ is X 2 kg ;
Amount of removal pollutants from river C is X 3 kg ;
Amount of removal pollutants from City is X 4 kg ;
Amount of removal pollutants from Airport is X 5 kg ;
Objective:
Minimize cost: $\mathrm{Z}=35 \times \mathrm{X}_{1}+46 \times X_{2}+50 \times X_{3}+87 \times X_{4}+76 \times X_{5}$
S.T.
$\mathrm{X}_{1}+X_{2}+X_{3}+X_{4}+X_{5} \geq 62000$
$23400 \geq \mathrm{X}_{1} \geq \frac{23400}{4}=5850$
$12800 \geq \mathrm{X}_{2} \geq \frac{12800}{4}=3200$
$28600 \geq \mathrm{X}_{3} \geq \frac{28600}{4}=7150$
$17400 \geq \mathrm{X}_{4} \geq \frac{17400}{2}=8700$
$19400 \geq \mathrm{X}_{5} \geq \frac{19400}{4}=9700$

Solver:


Result:

| obj |  |
| :--- | ---: |
| $Z=35 \times X \_1+46 \times X \_2+50 \times X \_3+87 \times X \_4+76 \times X \_5$ | 3271900 |
| variable |  |
| X_1 | 23400 |
| X_2 | 12800 |
| X_3 | 7400 |
| X_4 | 8700 |
| X_5 | 9700 |

The airport manager would like to invest in a deicing fluid system able to recycle $50 \%$ of the pollutants produced by the airport. The new plant is expected to cost $\$ 14,000,000$ and has a nominal life cycle of at least 15 years.
b) Using principles of engineering economics and Excel, calculate the yearly payments from the airport authority to a bank to buy the recycling system and pay it off at the end of 15 years. Assume the bank charges $4 \%$ yearly over the loan period.

Answer:

| Total loan | 14000000 USD |  |
| :--- | ---: | :---: |
| Period | 15 years |  |
| APR | $4 \%$ |  |
|  |  |  |
| Yearly Pa | $(\$ 1,210,745.58)$ |  |

c) Assume that city in question increases in population at a rate of $2 \%$ per year. Assume the pollution increases proportionally to population. How much pollution does the city needs to remove after 15 years?
Answer:

$$
P_{15}=P_{0} \times(1+0.02)^{15}
$$

| Source | Removal Cost (\$/kg) | Pollution to Lake (kg) | Annual Increased Rate | Year | 15 Year Polution |
| :---: | :---: | :---: | :---: | :---: | :---: |
| River A | 35 | 23,400 | 0.02 | 15 | 31493.31912 |
| River B | 46 | 12,800 | 0.02 | 15 | 17227.11473 |
| River C | 50 | 28,600 | 0.02 | 15 | 38491.83448 |
| City | 87 | 17,400 | 0.02 | 15 | 23418.10909 |
| Airport | 76 | 19,400 | 0.02 | 15 | 26109.84576 |

## Problem 4

A group of engineers in your company setups the following Linear Programming problem to minimize the cost of producing three types of steel beams commonly used in buildings. The objective function is the profit for the company (in dollars per production batch). The company would like to maximize the profit in solving this problem.
Objective $\quad$ Maximize $Z=150 X_{1}+200 X_{2}+210 X_{3}$
Subject to

$$
1.1 \mathrm{X}_{1}+1.2 \mathrm{X}_{2}+1.8 \mathrm{X}_{3}<=2400
$$

$$
0.6 \mathrm{X}_{1}+0.7 \mathrm{X}_{3}<=700
$$

$$
X_{2}-0.9 X_{3}<=1300
$$

$$
1.4 \mathrm{X}_{1}+2 \mathrm{X}_{2}+2.8 \mathrm{X}_{3}>=3700
$$

$$
X_{1}, X_{2} \text { and } X_{3}>=0 \quad \text { (non-negativity conditions) }
$$

For each task below, use screen captures of your setup using Excel Solver. Show the formulas of the cells to make out task simpler in grading.

## Task 1

Solve the problem using Excel Solver. State the exact solution found by Excel for all three decision variables. State the value of the objective function for the optimal solution found.

Answer:


| obj |  |  |
| :--- | ---: | ---: |
| Z=150X1+200X2+210X3 | 374455.25 |  |
| s.t. |  |  |
| $1.1 \mathrm{X} 1+1.2 \mathrm{X} 2+1.8 \mathrm{X} 3$ | 2400 | 2400 |
| $0.6 \mathrm{X1}+0.7 \mathrm{X} 3$ | 429.37743 | 700 |
| X2 - 0.9 X3 | 1300 | 1300 |
| 1.4 X1 + 2 X2 + 2.8 X3 | 3700 | 3700 |
| variable |  |  |
| X1 | 677.0428 |  |
| X2 | 1329.7665 |  |
| X3 | 33.07393 |  |

Three decision variables:

| X1 | 677.0428 |
| :--- | ---: |
| X2 | 1329.7665 |
| X3 | 33.07393 |

The value of the objective function: 374455.25
Z=150X1+200X2+210X3 374455.25

## Task 2

Since number of beams to be produced needs to be an integer solution, solve the problem with Excel to obtain an integer solution. State the value of the objective function for the optimal solution found.
Answer:

obj

| Z=150X1+200X2+210X3 | 374380 |  |
| :--- | ---: | ---: |
| s.t. |  |  |
| 1.1X1+1.2X2+1.8X3 | 2400 | 2400 |
| 0.6 X1 +0.7 X3 | 407.8 | 700 |
| X2 -0.9 X3 | 1299.8 | 1300 |
| 1.4 X1+2 X2 +2.8 X 3 | 3723.2 | 3700 |
| variable |  |  |
| X1 | 612 |  |
| X2 | 1352 |  |
| X3 | 58 |  |

The value of the objective function: 374380

## Task 3

Examine the final integer solution and state which constraint(s) are limiting the solution to the problem. This means which constrains bound the solution of the problem.

Answer:
The constraints showed below are limiting the solution.

$$
\begin{aligned}
& 1.1 \mathrm{X}_{1}+1.2 \mathrm{X}_{2}+1.8 \mathrm{X}_{3}<=2400 \\
& \mathrm{X}_{2}-0.9 \mathrm{X}_{3}<=1300
\end{aligned}
$$

